contains an arc. Since $H_m(X)$ is locally homeomorphic to X^m, this implies that X contains an arc. ∎

5.7. Remark. Let $X \subset Q$ be a free pseudo-arc. Then $\hat{X}$ is a boundary set for $\tilde{X} \approx Q$ containing no arcs. It might be interesting to point out that $\hat{X}$ is countable dimensional, i.e. a union of countable many zero-dimensional subsets. It is also easy to construct a boundary set containing no arcs which is strongly infinite dimensional. Let $X \subset Q$ be a free strongly infinite dimensional continuum containing no arcs. Then $\hat{X} \subset \tilde{X}$ is as required. We do not have an example of a boundary set $B \subset Q$ so that either $\dim A = 0$ or $\dim A = \infty$ for all $A \subset B$. If there is a continuum X with the property that for any $n \in N$ and $A \subset X^n$ either $\dim A = 0$ or $\dim A = \infty$ then it is possible to construct a "hereditary infinite dimensional" boundary set. It is unknown whether such a continuum exists. Notice however that there is a continuum with no n-dimensional ($n \geqslant 1$) subsets, [11].

Let M be a Q-manifold. Using the fact that $M \times [0, 1)$ embeds in Q as an open subset, it is easy to show that M contains a σ-compact σ-Z-set B such that B contains no arcs and $M - B$ is an l_2-manifold.

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Zero-dimensional countable dense unions of Z-sets in the Hilbert cube

by

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Abstract. We show that every σ-compact, nowhere locally compact, zero-dimensional metric space can be imbedded in the Hilbert cube as a countable dense union of Z-sets, and that there are exactly three such spaces for which all such imbeddings are topologically equivalent.

§ 0. Introduction. It is well know that the Hilbert cube I^∞ is countable dense homogeneous: for any two countable dense subsets D and E, there exists a homeomorphism $h\colon I^\infty \to I^\infty$ with $h(D) = E$. Thus, all dense imbeddings of the space Q of rationals into I^∞ are topologically equivalent. It seems natural to ask which other σ-compact, 0-dimensional metric spaces share this property. It is easily shown that such a space X admits a dense imbedding into I^∞ if and only if it is nowhere locally compact. Furthermore, to obtain positive results in the general case when X is uncountable, we consider only imbeddings as countable unions of Z-sets (see § 1). Thus, the question we ask is: which σ-compact, nowhere locally compact, 0-dimensional metric spaces X have the property that all imbeddings of X into the Hilbert cube as countable dense unions of Z-sets are topologically equivalent? In this note we show that there are exactly three such spaces: the space of rationals, the product of the rationals and the Cantor set, and the space which is the union of a copy of the rationals and a nowhere dense Cantor set.

Actually, the question of equivalence of imbeddings $f_1\colon X \to I^\infty$ and $f_2\colon X \to I^\infty$ of a 0-dimensional space X reduces to the question of whether the complements $I^\infty \backslash f_1(X)$ and $I^\infty \backslash f_2(X)$ are homeomorphic (see § 4). This rather curious result is of course strictly limited to the 0-dimensional case (compare for instance with Chapman's complement theorem for Z-sets in I^∞ [3], or with the fact that the complements of both capsets and fd-capsets in I^∞ are homeomorphic to l^2 [1]).

§ 1. Preliminaires. All spaces considered are separable metric. We shall frequently use the following classical characterizations for certain 0-dimensional spaces (for techniques of proof and references, see [6]):

1.1. Lemma. *$X \approx Q$, the space of rationals, if and only if X is countable and has no isolated points.*

1.2. LEMMA. *$X \approx Q \times C$, the product of the rationals and the Cantor set, if and only if X is σ-compact, nowhere locally compact, nowhere locally countable, and 0-dimensional.*

A 0-dimensional space is called *strongly homogeneous* if it is homeomorphic to every nonempty clopen subspace. By the above characterizations, both Q and $Q \times C$ are strongly homogeneous.

1.3. LEMMA. *Let A and B be homeomorphic, closed, nowhere dense subspaces of X and Y respectively, such that $X \setminus A$ and $Y \setminus B$ are homeomorphic, strongly homogeneous, 0-dimensional spaces. Then every homeomorphism $f\colon A \to B$ extends to a homeomorphism $\tilde{f}\colon X \to Y$.*

Proof. The technique is similar to that used in [4]. In brief, one constructs covers $\{V_i\}_1^\infty$ and $\{W_i\}_1^\infty$ of $X \setminus A$ and $Y \setminus B$ by disjoint clopen subsets of X and Y, respectively, such that:

1) $\operatorname{diam} V_i < d(V_i, A)$ and $\operatorname{diam} W_i < d(W_i, B)$ for each i;

2) there exist sequences $\{a_i\}$ and $\{b_i\}$ in A and B, respectively, with $f(a_i) = b_i$, such that

$$\lim_{i\to\infty} d(a_i, V_i) = 0 = \lim_{i\to\infty} d(b_i, W_i).$$

For each i, choose an arbitrary homeomorphism $f_i\colon V_i \to W_i$. Then $\tilde{f}\colon X \to Y$, defined by $\tilde{f}_{|A} = f$ and $\tilde{f}_{|V_i} = f_i$ for each i, is a homeomorphism extending f.

We consider the Hilbert cube $I^\infty = \prod_1^\infty [-1, 1]_i$ with the metric $d((x_i), (y_i)) = \sum_1^\infty 2^{-i}|x_i - y_i|$. A closed subset $F \subset I^\infty$ is a *Z-set* in I^∞ if there exist maps $\eta\colon I^\infty \to I^\infty \setminus F$ arbitrarily close to the identity map. Note that every endface of I^∞ (a subset of the form $\pi_i^{-1}(-1)$ or $\pi_i^{-1}(1)$, where $\pi_i\colon I^\infty \to [-1, 1]_i$ is the projection map) is a Z-set. A countable union of Z-sets is called a *σ-Z-set*. An imbedding $h\colon X \to I^\infty$ is called a *Z-imbedding* if $h(X)$ is a Z-set; similarly for a *σ-Z-imbedding*. For a discussion of Z-sets, and proofs of the following basic results of Anderson, see [3].

1.4. HOMEOMORPHISM EXTENSION THEOREM. *Let $h\colon A \to B$ be a homeomorphism between Z-sets in I^∞, with $d(h, \mathrm{id}) < \varepsilon$. Then h extends to a homeomorphism $\tilde{h}\colon I^\infty \to I^\infty$ with $d(\tilde{h}, \mathrm{id}) < \varepsilon$.*

1.5. INDUCTIVE CONVERGENCE CRITERION. Suppose a sequence $\{h_n\}_1^\infty$ of homeomorphisms of I^∞ is chosen inductively so that each h_n is sufficiently close to the identity. Then $\lim_{n\to\infty} h_n \circ \ldots \circ h_1$ exists and is a homeomorphism of I^∞.

1.6. LEMMA. *Every σ-compact, nowhere locally compact, 0-dimensional space X admits a dense, σ-Z-imbedding into I^∞.*

Note. We do not know whether the 0-dimensional hypothesis can be omitted.

Proof. We first consider the special case $X \approx Q \times C$. Let $\{K_i\}_1^\infty$ be a sequence of Z-set Cantor sets in I^∞ such that $Y = \bigcup_1^\infty K_i$ is dense in I^∞. Then Y is nowhere dense, hence nowhere locally compact, and it follows from the characterization Lemma 1.2 that $Y \approx Q \times C$.

In the general case, let $\tilde{X}$ be a 0-dimensional compactification of X. Since X has no isolated points, neither does $\tilde{X}$, and therefore $\tilde{X}$ is a Cantor set. Let $\{C_i\}_1^\infty$ be a sequence of nowhere dense Cantor sets in $\tilde{X}$ whose union is dense. Since X is nowhere locally compact, X is also nowhere dense in $\tilde{X}$. Thus $Y = \bigcup_1^\infty C_i \cup X$ is a σ-compact, nowhere locally compact, nowhere locally countable, 0-dimensional space, and again by Lemma 1.2, $Y \approx Q \times C$. Since X is dense in Y, the general result follows from the special case.

The reason for considerings only σ-Z-imbedding in I^∞ is now apparent. Since the Cantor set can be imbedded in I^∞ as a non-Z-set [7], there exists, for every *uncountable* 0-dimensional space as above, both a dense σ-Z-imbedding and a dense non-σ-Z-imbedding, which obviously are not equivalent in I^∞; for details see [5].

§ 2. A 0-dimensional capset.

The idea of a *capset* (abbreviation for *set with the compact absorption property*) for a given topological class $\mathscr{K}$ of compacta is due independently to Anderson [1] and Bessaga–Pełczyński [2]. Let $\mathscr{K}$ be a topological class of Z-sets in I^∞. A dense σ-Z-set $K \subset I^\infty$ is called a *capset for $\mathscr{K}$* if $K = \bigcup_1^\infty K_i$, for some tower $K_1 \subset K_2 \subset \ldots$ of members of $\mathscr{K}$ with the following property:

> for each $\varepsilon > 0$, each integer m, and each $F \in \mathscr{K}$, there exists a homeomorphism $h\colon I^\infty \to I^\infty$ with $d(h, \mathrm{id}) < \varepsilon$, $h|_{K_m} = \mathrm{id}$, and $h(F) \subset K_n$, for some $n > m$.

Examples of capsets for various classes of compacta are well-known. These include: the class of all compacta, with a capset $\Sigma = \{(x_i) \in I^\infty\colon \sup|x_i| < 1\}$; the class of finite-dimensional compacta, with a capset $\sigma = \{(x_i) \in I^\infty\colon x_i = 0$ for almost all $i\}$; and the class of finite spaces, with every countable dense set a capset. We show here that every dense σ-Z-set copy in I^∞ of the space $Q \times C$ is a capset for the class of 0-dimensional compacta.

An elementary argument using the Inductive Convergence Criterion 1.5 shows that capsets are topologically unique and in fact, for any two capsets K and M in I^∞ (for the same class $\mathscr{K}$), and $\varepsilon > 0$, there exists a homeomorphism $h\colon I^\infty \to I^\infty$ with $d(h, \mathrm{id}) < \varepsilon$ and $h(K) = M$. Thus, the above capset characterizations show that the Hilbert cube is "dense homogeneous" with respect to the spaces Q and $Q \times C$.

2.1. THEOREM. *Let $K \subset I^\infty$ be a dense σ-Z-set homeomorphic to $Q \times C$. The K is a capset for the class of 0-dimensional compacta.*

Proof. Clearly, Q may be written as the union of a tower of compacta $F_1 \subset F_2 \subset \ldots$ such that each F_i is nowhere dense in F_{i+1}. Taking products with C, we obtain $K = \bigcup_1^\infty K_i$, with each K_i a Cantor set which is nowhere dense in K_{i+1}. Let a 0-dimensional Z-set $A \subset I^\infty$, $\varepsilon > 0$, and an integer m be given. Choose $n > m$ such that $d(x, K_n) < \frac{1}{4}\varepsilon$ for each $x \in A$. Let $\mathscr{D}$ be an open cover of $A \setminus K_m$ by disjoint, compact subsets such that for each $D \in \mathscr{D}$, $\operatorname{diam} D < \min\{d(D, K_m), \frac{1}{4}\varepsilon\}$. For each D pick a point $x(D) \in K_n \setminus K_m$ such that $d(x(D), D) < \min\{2d(D, K_m), \frac{1}{2}\varepsilon\}$, and with $x(D) \neq x(D')$ if $D \neq D'$. This is possible since K_m is nowhere dense in K_n. For each D, choose a Cantor set $C(D) \subset K_n \setminus K_m$ containing $x(D)$ such that $\operatorname{diam} C(D) < \min\{d(x(D), K_m), \frac{1}{4}\varepsilon\}$, and such that $C(D) \cap C(D') = \emptyset$ if $D \neq D'$. For each $D \in \mathscr{D}$, choose an arbitrary imbedding $f_D\colon D \to C(D)$. Define an imbedding $f\colon A \cup K_m \to K_n$ by

$$f(x) = \begin{cases} f_D(x) & \text{if } x \in D \in \mathscr{D}; \\ x & \text{if } x \in K_m. \end{cases}$$

Clearly, f is an imbedding, $f|_{K_m} = \mathrm{id}$, and $d(f, \mathrm{id}) < \frac{1}{4}\varepsilon + \frac{1}{2}\varepsilon + \frac{1}{4}\varepsilon = \varepsilon$. By the Homeomorphism Extension Theorem 1.4, f extends to a homeomorphism $h\colon I^\infty \to I^\infty$ with $d(h, \mathrm{id}) < \varepsilon$.

§ 3. Dense homogeneity of the Hilbert cube. If X is a σ-compact, nowhere locally compact space which admits a dense σ-Z-imbedding into I^∞, we say that I^∞ is *dense homogeneous with respect to* X, if, for any two dense σ-Z-set copies X_1 and X_2 of X in I^∞, there exists a homeomorphism $h\colon I^\infty \to I^\infty$ with $h(X_1) = X_2$. (Of course, this does not require that every homeomorphism $f\colon X_1 \to X_2$ extend to a homeomorphism $h\colon I^\infty \to I^\infty$).

By Lemma 1.3, the space which is the union of a copy of the rationals and a nowhere dense Cantor set is topologically unique. We denote this space by L.

3.1. THEOREM. *The 0-dimensional spaces with respect to which I^∞ is dense homogeneous are Q, $Q \times C$, and L.*

Proof. As previously noted, the dense homogeneity of I^∞ with respect to Q and $Q \times C$ follows from the fact that all dense σ-Z-set copies of these spaces in I^∞ are capsets, for the class of finite compacta and the class of 0-dimensional compacta, respectively.

Let L_1 and L_2 be dense σ-Z-set copies in I^∞ of the space L. We may suppose that $L_i = C_i \cup Q_i$, where C_i is a nowhere dense Cantor set in L_i, Q_i is a copy of Q, and $C_i \cap Q_i = \emptyset$, $i = 1, 2$. By the Homeomorphism Extension Theorem 1.4, there exists a homeomorphism $f\colon I^\infty \to I^\infty$ with $f(C_1) = C_2$. Using 1.4 again, we may construct a sequence $\{h_n\}$ of homeomorphisms of I^∞ such that $h_n|_{C_2} = \mathrm{id}$ for each n, $h = \lim_{n\to\infty} (h_n \circ \ldots \circ h_1)$ is a homeomorphism by the Inductive Convergence Criterion 1.5, and $h(f(Q_1)) = Q_2$. Then $h \circ f$ is a homeomorphism of I^∞ taking L_1 onto L_2 (this technique of proof is known of course, see e.g. [2], pp. 123 and 139).

We now consider a σ-compact, nowhere locally compact, 0-dimensional space X which is *not* homeomorphic to Q, $Q \times C$, or L. Let $V = \{x \in X\colon X$ is locally countable at $x\}$. Since $X \not\approx Q \times C$, $V \neq \emptyset$, and since $X \not\approx Q$, $V \neq X$.

Suppose first that $\overline{V} = X$. Then $B = \overline{V} \setminus V$ can have no isolated points. If B is compact, then $B \approx C$ and $X \approx L$. Thus B is non-compact. Consider two non-homeomorphic compactifications $\tilde{B}_1$ and $\tilde{B}_2$ of B (for instance, let $\tilde{B}_1$ be a Cantor set compactification and $\tilde{B}_2$ an infinite-dimensional compactification containing a copy of I^∞). Let $f_i\colon \tilde{B}_i \to I^\infty$ be Z-imbeddings, and $Q_i \subset I^\infty \setminus f_i(\tilde{B}_i)$ countable dense sets, $i = 1, 2$. By Lemma 1.3, $f_1(B) \cup Q_1 \approx f_2(B) \cup Q_2 \approx B \cup V = X$, and the sets $X_i = f_i(B) \cup Q_i$, $i = 1, 2$, are dense σ-Z-set copies of X in I^∞. If there exists a homeomorphism $h\colon I^\infty \to I^\infty$ taking X_1 onto X_2, then $h(f_1(B)) = f_2(B)$, and since $f_i(\tilde{B}_i) = \overline{f_i(B)}$, we must have $hf_1(\tilde{B}_1) = f_2(\tilde{B}_2)$, impossible since $\tilde{B}_1 \not\approx \tilde{B}_2$.

Now suppose that $\overline{V} \neq X$, and set $W = X \setminus \overline{V}$. By Lemma 1.2, $W \approx Q \times C$. Let $\pi_1\colon I^\infty \to [-1, 1]$ be the projection onto the first coordinate factor. Let $f\colon \overline{V} \to I^\infty$ be a σ-Z-imbedding such that $f(\overline{V} \setminus V) \subset \pi_1^{-1}(\{0\})$ and $f(V)$ is a dense subset of $\pi_1^{-1}([-1, 0))$. Let $g\colon \overline{W} \to I^\infty$ be a σ-Z-imbedding such that $g|_{\overline{W} \setminus W} = f|_{\overline{W} \setminus W}$ and $g(W)$ is a dense subset of $\pi_1^{-1}((0, 1])$. These imbeddings, taken together, give a dense σ-Z-imbedding $h_1\colon X \to I^\infty$ such that $\overline{h_1(V)} = \pi_1^{-1}([-1, 0])$ is connected. Similarly, we may construct a dense σ-Z-imbedding $h_2\colon X \to I^\infty$ such that $h_2(\overline{V} \setminus V) \subset \pi_1^{-1}(\{0\})$, $h_2(V) \subset \pi_1^{-1}([-1, 0) \cup (\frac{1}{2}, 1])$, and $h_2(W) \subset \pi_1^{-1}((0, \frac{1}{2}))$. Then $\overline{h_2(V)} = \pi_1^{-1}([-1, 0] \cup [\frac{1}{2}, 1])$ is not connected. Thus there is no homeomorphism of I^∞ taking $h_1(X)$ onto $h_2(X)$. This completes the proof of the theorem.

Remark. The inequivalent imbeddings $f_i\colon X \to I^\infty$ constructed in the above proof clearly do not have the property that, for arbitrary $\varepsilon > 0$, there exists a homeomorphism $g\colon f_1(X) \to f_2(X)$ with $d(g, \mathrm{id}) < \varepsilon$. One might conjecture that two dense σ-Z-imbeddings *with* this property would necessarily be equivalent. The following example shows the conjecture to be false.

Let J be a Z-set arc in I^∞, let K be a countable dense union of Cantor sets in $J \setminus \partial J$, and let E be a countable dense subset of $I^\infty \setminus J$. Set $X_1 = K \cup E$ and $X_2 = (K \cup \partial J) \cup E$. Since $K \approx Q \times C \approx K \cup \partial J$, we have $X_1 \approx X_2$, and each X_i is a dense σ-Z-set in I^∞. Moreover, for arbitrary $\varepsilon > 0$, there exists a homeomorphism $\gamma\colon K \to K \cup \partial J$ with $d(\gamma, \mathrm{id}) < \varepsilon$. By the proof of Lemma 1.3, γ extends to a homeomorphism $g\colon X_1 \to X_2$ with $d(g, \mathrm{id}) < \varepsilon$. However, there does *not* exist a homeomorphism $h\colon I^\infty \to I^\infty$ with $h(X_1) = X_2$, since then $h(J) = h(\overline{K}) = \overline{h(K)} = \overline{K \cup \partial J} = J$, and $h(\partial J) = \partial J$, impossible.

§ 4. **Complements of 0-dimensional sets in I^∞.** As remarked in the introduction, the question of equivalence of imbeddings of a 0-dimensional space into I^∞ reduces to the question of homeomorphism of their complements.

4.1. THEOREM. *Let $X, Y \subset I^\infty$ be 0-dimensional, and $f\colon I^\infty \setminus X \to I^\infty \setminus Y$ a homeomorphism. Then f extends to a homeomorphism $\bar{f}\colon I^\infty \to I^\infty$.*

Proof. For each $x \in X$ and $i \geqslant 1$, let $N_i(x)$ be the open $1/i$-neighborhood of x in I^∞. We claim that $\bar{f}(x) = \bigcap_{i=1}^{\infty} \overline{f(N_i(x)\setminus X)}$ defines an extension of f. Since each $N_i(x)$ is open and connected, and X is 0-dimensional, $N_i(x)\setminus X$ is nonempty and connected. Thus $\bar{f}(x)$ is the intersection of a decreasing sequence of continua, and is therefore a continuum which must lie in Y. Since Y is 0-dimensional, $\bar{f}(x)$ is a point. Clearly, this defines a continuous extension $\bar{f}\colon I^\infty \to I^\infty$ of f. Similarly, one defines a continuous extension $\bar{g}\colon I^\infty \to I^\infty$ of $g = f^{-1}$. Since X and Y are nowhere dense in I^∞, each of the compositions $\bar{f}\circ\bar{g}$ and $\bar{g}\circ\bar{f}$ is the identity map, and $\bar{f}$ is a homeomorphism.

Remark. It is clear from the proof that this theorem can be generalized, by substituting for I^∞ any locally connected continuum Z, and requiring only that X and Y be totally disconnected and locally non-separating in Z.

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Chains in Ehrenfeucht-Mostowski models

by

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Abstract. We study the relationship between the chain of indiscernibles in an Ehrenfeucht-Mostowski model and the subchains of the model. As an application we construct large families of almost disjoint models for some theories.

Introduction. This work arises from the following considerations: by means of Ehrenfeucht–Mostowski models, the class of chains can be represented in the class of models of any theory. In some cases, especially unstable theories this method allows the construction of a large number of models (see S. Shelah [10]). However the sort of relationship between these models seems to be interesting, also. And this representation of classes of chains in classes of models allows one to think, in particular, that the complexity of comparing models is as high as the complexity of comparing chains. This paper is an attempt to work out this idea.

There is one basic question: if for a theory T two Ehrenfeucht–Mostowski models $M(C)$, $M(C')$ are comparable in some sense (by extension or elementary extension), what is the relationship between the chains C and C' which generated them? the same question arises when C' is a subchain of $M(C)$. Here we give a partial answer to this question, assuming that $M(C)$ is partially ordered by a formula φ, we prove: *if C' is a chain of regular power $\varkappa$ in this partial order then there is some subchain C'' of C', with power $\varkappa$, which is isomorphic to a subchain of C or its converse C^** (Theorem II–1). This is not the best possible result, however, as we get that C' is a countable union of chains, each of them being isomorphic to a subchain of some finite lexicographical product of copies of C or C^*.

Nevertheless this result is enough to transfer some properties of chains to models. Let (P) be the following property of two chains C and C': "C and C' are of same power $\varkappa$ and there is no chain of power $\varkappa$ order or antiorder isomorphic to subchains in both C and C'". If we take two chains C and C' with the property (P), then the two Ehrenfeucht–Mostowski models generated by them also have the corresponding property for models. So if the orderings on C and C' are definable in the Ehrenfeucht–Mostowski models to which they give rise, these models are uncomparable. The situation occurs when the theory T has some model containing an infinite chain; in this case large families of chains satisfying pairwise the property (P) give rise to large families of uncomparable models with the corresponding property. We can get such families of chains by using the