Compactification of pointed 1-movable spaces

by

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Abstract. Let X be a locally compact metrizable space with a locally finite cover consisting of pointed 1-movable continua. It is proved that if αX is a metrizable compactification of X such that each component of $\alpha X - X$ is pointed 1-movable, then αX is pointed 1-movable. This fact does not generally hold in case 1-movability is replaced by r-movability, $r > 1$.

§ 1. Introduction. Let X be a locally compact metrizable space and let αX be a metrizable compactification of X. It is known that many of topological properties of X are not preserved by αX. For example, as shown by the curve "$\sin 1/x$", even if X and $\alpha X - X$ are both AR, the local connectedness is not preserved.

In this paper we shall prove that if X is locally pointed 1-movable and αX is a metrizable compactification of X such that each component of the remainder $\alpha X - X$ is pointed 1-movable, then αX is pointed 1-movable. Thus a Freudenthal compactification or one point compactification of a locally pointed 1-movable space is pointed 1-movable. Also, a continuum being a disjoint union of locally connected subspaces one of which is compact is pointed 1-movable. Since there is a metrizable compactification of a real line which is not 1-movable, the condition "pointed 1-movability of the remainder" can not be omitted in these results. Finally, for $r > 1$, it is shown that the pointed r-movability is not generally preserved by a Freudenthal compactification even in case X is a locally compact AR.

The author wishes to thank the referee for a simple proof of Theorem 3 and a correction of Example 1.

Throughout this paper, all of topological spaces are Hausdorff and maps are continuous. We mean by a continuum a compact connected metric space and by AR and ANR those for metrizable spaces.

§ 2. Pointed 1-movability. Let X be a continuum and let x_0 be a point of X. Consider X as a subset of the Hilbert cube Q. Then X is said to be *pointed r-movable* if for every neighborhood U of X in Q there exists a neighborhood V of X in Q satisfying the following conditions: Let (Y, y_0) be a pointed CW-complex with $\dim Y \leqslant r$ and let $f\colon (Y, y_0) \to (U, x_0)$ be a map; then for every neighborhood W of X in Q there exists a homotopy $H\colon Y \times I \to U$ such that $H(y, 0) = f(y)$,

$H(y, 1) \in W$ for $y \in Y$ and $H(y_0, t) = x_0$ for $t \in I$. Obviously this definition is equivalent to the original one by Borsuk [2, p. 171].

In the paper [3] we gave the following characterization of pointed 1-movability (Theorem 1) which is used in the proof of main results (§ 4). Let T be the half open interval $[0, 1)$. Denote by E the product space $I \times T$. Consider the Stone–Čech compactification βE of E. The remainder $\beta E - E$ is denoted by E^*. Put

$$E_0 = E^* \cap \mathrm{Cl}_{\beta E}(\{0\} \times T)$$

and $E_1 = E^* \cap \mathrm{Cl}_{\beta E}(\{1\} \times T)$. Here Cl_A means the closure in the space A. Each E_i, $i = 0, 1$, is homeomorphic to the remainder $\beta T - T$, where βT is the Stone–Čech compactification of T. We call E^* a *Čech* 1-*cell*, and E_i, $i = 0, 1$, an *end* of E^*.

THEOREM 1 ([3]). *Let X be a continuum. The following are equivalent.*

(1) *X is pointed* 1-*movable.*

(2) *For any two points x_i, $i = 0, 1$, of X, there exists a map f: $E^* \to X$ such that $f(E_i) = x_i$, $i = 0, 1$.*

(3) *Every map h: $E_0 \cup E_1 \to X$ is extendable over E^*.*

The proof is given in [3]. Following [3], a map f in Theorem 1 (2) is said to be a *Čech-path* in X connecting the points x_0 and x_1.

The following theorem was proved by Krasinkiewicz.

THEOREM 2 (Krasinkiewicz [5, 1.8]). *If X is a continuum which is a union of a finite number of pointed* 1-*movable continua, then X is pointed* 1-*movable.*

Note that any two points in a continuum X as in Theorem 2 are connected by a Čech path in X.

§ 3. **Lemmas in compactification.** Throughout this section we assume that

(3.1) *X is a connected metrizable space and $\mathscr{F}$ is a locally finite cover of X consisting of continua.*

Note that X is locally compact and its Freudenthal compactification FX is metrizable.

Let Z be a closed set of X. The inclusion map f: $Z \to X$ induces the map Ff: $FZ \to FX$ such that $Ff(FZ - Z) \subset FX - X$, where FZ is the Freudenthal compactification of Z. Following Ball [1, p. 180] Z is said to be *strongly properly embedded in X* if $Ff|FZ - Z$: $FZ - Z \to FX - X$ is a homeomorphism onto.

LEMMA 1. *Let Z be a closed set of X such that the covering $\{F \cap Z: F \in \mathscr{F}\}$ of Z consists of continua and is similar to F. Then Z is strongly properly embedded in X and hence FZ is identified with the closure $\mathrm{Cl}_{FX} Z$ of Z in FX.*

For the proof we refer § 2 of Ball [1] and need a couple of lemmas.

Let $\mathscr{F} = \{F_\tau: \tau \in \Lambda\}$. Then each F_τ is a continuum of X. Choose a point x_τ of $F_\tau \cap Z$ for each $\tau \in \Lambda$. Following Ball [1], we denote by $\mathscr{A}_X$ the set of all admissible sequences in X.

LEMMA 2. *Given sequence $\alpha = \{y_i: i = 1, 2, ...\} \in \mathscr{A}_X$, let $\beta = \{z_i: i = 1, 2, ...\}$ be a sequence such that for each i $z_i \in \{x_\tau: \tau \in \Lambda\}$ and both y_i and z_i belong to the same member of $\mathscr{F}$. Then $\beta \in \mathscr{A}_X$ and $\alpha \sim \beta$* (cf. [1, p. 179]).

Proof. Let $\gamma = \{r_i: i = 1, 2, ...\}$ be the sequence defined by $r_{2i-1} = y_i$ and $r_{2i} = z_i$, $i = 1, 2, ...$ It is enough to prove that $\gamma \in \mathscr{A}_X$. Suppose that $\gamma \notin \mathscr{A}_X$, that is, there exist two infinite subsequences γ_1 and γ_2 of γ separated by some compact set C of X. Then $X - C$ is a union of two disjoint open sets U_1 and U_2 such that $\bigcup \gamma_1 \subset U_1$ and $\bigcup \gamma_2 \subset U_2$. Let $\Lambda' = \{\tau: F_\tau \cap C \neq \emptyset, \tau \in \Lambda\}$. Since $\mathscr{F}$ is locally finite, Λ' is finite. Since each member of $\mathscr{F}$ is a continuum, if $\tau \in \Lambda - \Lambda'$ either $F_\tau \subset U_1$ or $F_\tau \subset U_2$. Hence, for each $j = 1, 2$, $r_{2i-1} = y_i \in \gamma_j$ if and only if $r_{2i} = z_i \in \gamma_j$. Let $\alpha_j = \{y_i: y_i \in \gamma_j\}$, $j = 1, 2$. Then α_1 and α_2 are infinite subsequence of α and separated by C. This contradicts that $\alpha \in \mathscr{A}_X$.

LEMMA 3. *Let $\beta = \{z_i: i = 1, 2, ...\}$ be a sequence taken from $\{x_\tau: \tau \in \Lambda\}$. Then $\beta \in \mathscr{A}_X$ if and only if $\beta \in \mathscr{A}_Z$. Here $\mathscr{A}_Z$ is the set of all admissible sequences in the space Z.*

Proof. The if part follows from [1, Lemma 2.7]. To prove the only if part, let $\beta \notin \mathscr{A}_Z$. Then there exist a compact set C of Z and two infinite subsequences β_1 and β_2 of β separated by C in Z. Let $\Lambda' = \{\tau: F_\tau \cap C \neq \emptyset, \tau \in \Lambda\}$ and let $E = \bigcup \{F_\tau: \tau \in \Lambda'\}$. Then E is compact. Put $\beta_j' = \{z_i: z_i \in \beta_j$ and $z_i \notin E\}$, $j = 1, 2$. Obviously β_j' is infinite. Let us prove that β_1' and β_2' are separated by E in X. Suppose they are not separated by E. Then, by the local finiteness of $\mathscr{F}$ and the connectedness of each member of $\mathscr{F}$, there exist points $z_k \in \beta_1'$ and $z_m \in \beta_2'$, and a finite chain $\{F_{\tau_i}: i = 1, 2, ..., n\}$ in $\mathscr{F}$ such that $z_k \in F_{\tau_1}$, $z_m \in F_{\tau_n}$, $\tau_i \in \Lambda - \Lambda'$ and $F_{\tau_i} \cap F_{\tau_{i+1}} \neq \emptyset$ for each i. Since $\{F_\tau \cap Z\}$ and $\{F_\tau\}$ are similar,

$$\{F_{\tau_i} \cap Z: i = 1, 2, ..., n\}$$

is a chain in $\{F_\tau \cap Z\}$ such that $z_k \in F_{\tau_1} \cap Z$, $z_m \in F_{\tau_n} \cap Z$, $(F_{\tau_i} \cap Z) \cap (F_{\tau_{i+1}} \cap Z) \neq \emptyset$ and $F_{\tau_i} \cap C = \emptyset$ for each i. Since $F_{\tau_i} \cap Z$ is a continuum, this implies that β_1' and β_2' are not separated by C in Z. This contradiction means that β_1' and β_2' are separated by E in X. Thus $\beta \notin \mathscr{A}_X$.

Proof of Lemma 1. It follows from Lemmas 2, 3 and Ball [1, Theorem 2.8].

LEMMA 4. *Let αX be a metrizable compactification of X and let C be a component of the remainder $\alpha X - X$. Then there exists a sequence $\{F_i: i = 1, 2, ...\}$ in $\mathscr{F}$ such that*

(3.2) $F_i \cap F_{i+1} \neq \emptyset$ *for* $i = 1, 2, ...,$

(3.3) $\mathrm{Lim} F_i \subset C$, *where* $\mathrm{Lim} F_i$ *is the limit of* $\{F_i\}$ ([4, p. 339]).

Proof. Let γX be the quotient space obtained from αX by contracting each component of $\alpha X - X$ to a point. Then γX is a metrizable compactification of X. Let f: $\alpha X \to \gamma X$ be the projection. Since $\mathrm{ind}\, \gamma X - X = 0$, by the maximality of Freudenthal compactification there is a projection g: $FX \to \gamma X$. Let $c \in g^{-1} f(C)$. We shall prove that

(3.4) there exists a sequence $\{F_i\}$ in $\mathscr{F}$ such that (3.2) is satisfied and

(3.5) $\mathrm{Lim} F_i = \{c\}$.

Since f and g are proper maps, this completes the proof. To show (3.4), let K be the 1-skeleton of the nerve of $\mathscr{F}$. Then K is a locally finite simplicial complex and hence locally compact. Let $\{v_\tau:\ \tau\in\Lambda\}$ be the set of vertices of K, where v_τ corresponds F_τ for each $\tau\in\Lambda$. Choose a point x_τ of F_τ for $\tau\in\Lambda$. Let Y be the quotient space obtained by identifying the points x_τ and v_τ for each $\tau\in\Lambda$ from a topological sum $X\oplus K$. Then Y is a connected and locally compact metrizable space. We consider X and K as closed sets in Y and $X\cap K=\{x_\tau\}=\{v_\tau\}$. Let K' be a barycentric subdivision of K. For $\tau\in\Lambda$, put $H_\tau=F_\tau\cup \mathrm{St}v_\tau$, where $\mathrm{St}v_\tau$ is the closed star of v_τ in K'. Then each H_τ is a continuum and $\{H_\tau:\ \tau\in\Lambda\}$ forms a locally finite cover of Y. Since $H_\tau\cap X=F_\tau$ and $H_\tau\cap K=\mathrm{St}v_\tau$, the collections $\{H_\tau\}$, $\{H_\tau\cap X\}$ and $\{H_\tau\cap K\}$ are similar to each other. By Lemma 1 we can consider that $FY=FX\cup FK$ and $FY-Y=FX-X=FK-K$. To complete the proof, let S be a maximal tree of K. The inclusion of S into Y induces the map $h\colon FS\to FY$ such that $h(FS-S)=FY-Y$. Let c' be a point of $h^{-1}(c)$. Since FS is an AR and $FS-S$ is unstable by Sher [7, Lemma (2.2)], there is an into homeomorphism $k\colon I\to FS$ such that $k(0)=c'$, $k(1/i)=v_{\tau_i}$ is a vertex of S and $k([1/i+1,1/i])$ is a 1-simplex of S for $i=1,2,\dots$ Put $F_i=F_{\tau_i}$ and consider the sequence $\{F_i:\ i=1,2,\dots\}$ in $\mathscr{F}$. Obviously $\{F_i\}$ satisfies (3.2). To see (3.5), it is enough to note that

$$(\mathrm{Cl}_{FX}\bigcup_{i=1}^{\infty}F_i)-X=(\mathrm{Cl}_{FY}\bigcup_{i=1}^{\infty}H_i)-Y=(\mathrm{Cl}_{FK}\bigcup_{i=1}^{\infty}\mathrm{St}v_{\tau_i})-K$$
$$=(\mathrm{Cl}_{FK}\bigcup_{i=1}^{\infty}\mathrm{St}v_{\tau_i})-S=hk(I)-S=\{h(c')\}=\{c\}.$$

This completes the proof.

§ 4. Main results.

THEOREM 3. *Let X be a connected and locally compact metrizable space, and let $\mathscr{F}$ be a locally finite cover of X consisting of pointed 1-movable continua. If αX is a metrizable compactification of X such that each component of the remainder $\alpha X-X$ is pointed 1-movable, then αX is pointed 1-movable.*

Proof. By Theorem 1, it is enough to show that any two points of αX are connected by a Čech path. Let p and q be points of αX. If both p and q are contained in X, then they are connected by a Čech path in X. This is done by Theorem 1(2) and Theorem 2, because there is a chain $\{F_i:\ i=1,2,\dots,n\}$ in $\mathscr{F}$ such that $p\in F_1$, $q\in F_n$ and $F_i\cap F_{i+1}\neq\emptyset$ for $i=1,2,\dots,n-1$, and each F_i is pointed 1-movable. Thus we can assume that $p\in X$ and $q\in\alpha X-X$. Let C be a component of $\alpha X-X$ containing q. By Lemma 4, there exists a sequence $\{F_i:\ i=1,2,\dots\}$ in $\mathscr{F}$ satisfying (3.2) and (3.3). Without loss of generality, we can assume that $p\in F_1$. We shall prove that the union $\bigcup_{i=1}^{\infty}F_i\cup C$ is pointed 1-movable. The referee has pointed out that this fact is a consequence of Theorem 3.1 of Krasinkiewicz and Minc [6]. Since we generalize this fact slightly in Theorem 4, we shall give a direct and elementary proof to this. The proof is divided into three steps.

Step 1. Let R_+ be the half line $\{x:\ 1\leqslant x<\infty\}$ and let $A=R_+\times R_+$. Put $A_n=[n,n+1]\times R_+$ for $n=1,2,\dots$ Then $A=\bigcup_{n=1}^{\infty}A_n$. Consider the Stone–Čech compactification βA_n of A_n. For each n, $\mathrm{Cl}_{\beta A_n}\{n+1\}\times R_+$ and $\mathrm{Cl}_{\beta A_{n+1}}\{n+1\}\times R_+$ are naturally identified because each of them is homeomorphic to βR_+. Let B be the space obtained by identifying $\mathrm{Cl}_{\beta A_n}\{n+1\}\times R_+$ and $\mathrm{Cl}_{\beta A_n}\{n+1\}\times R_+$ for each n from a topological sum $\bigoplus_{n=1}^{\infty}\beta A_n$. Then A and βA_n, $n=1,2,\dots$, are considered as subspaces of B and $\beta A_n\cap\beta A_{n+1}=\mathrm{Cl}_{\beta A_n}\{n+1\}\times R_+=\mathrm{Cl}_{\beta A_{n+1}}\{n+1\}\times R_+$. Put $B_1=\mathrm{Cl}_B\{1\}\times R_+$ and $B_{n+1}=\beta A_n\cap\beta A_{n+1}$, $n=1,2,\dots$ Since B is σ-compact, it is obvious that

(4.1) $\quad B$ is Lindelöf and $\bigcup_{n=1}^{\infty}(\beta A_n-A_n)\cup\bigcup_{n=1}^{\infty}B_n$ is closed in B.

Step 2. By (3.3) $C\cup\bigcup_{i=1}^{\infty}F_i$ is a continuum. Imbed $C\cup\bigcup_{i=1}^{\infty}F_i$ into the Hilbert cube Q. Choose a point x_i of F_i for each $i=1,2,\dots$ Construct a map $f\colon\bigcup_{n=1}^{\infty}(\beta A_n-A_n)\cup\bigcup_{n=1}^{\infty}B_n\to Q$ as follows: $f(B_n)=x_n$ and $f(\beta A_n-A_n)\subset F_n\cup F_{n+1}$, $n=1,2,\dots$ Since $F_n\cap F_{n+1}\neq\emptyset$ and $F_n\cup F_{n+1}$ is pointed 1-movable by Theorem 2, such a map f exists. Since B is Lindelöf by (4.1) and Q is an AR, f is extended to a map $g\colon B\to Q$. Let U be an open set of Q such that

(4.2) $\quad\bigcup_{i=1}^{\infty}F_i\subset U\subset Q-C$ and $d(F_i,Q-U)<1/i$ for $i=1,2,\dots$, where d is a metric on Q.

Then $g^{-1}(U)$ is an open set of B containing $\bigcup_{n=1}^{\infty}(\beta A_n-A_n)$. There is a map $h\colon R_+\to R_+$ such that $D=\{(x,y):\ (x,y)\in A$ and $h(x)\leqslant y\}\subset g^{-1}(U)\cap A$. Note that D is homeomorphic to $R_+\times R_+$ and hence to $I\times[0,1)$. Let D_0 be the subspace $\{(x,h(x)):\ x\in R_+\}$ of D. Consider the map $\gamma=g|D\colon D\to Q$ and its extension $\tilde\gamma\colon\beta D\to Q$. By (4.2)

(4.3) $\quad\tilde\gamma(\beta D-D)\subset C\cup\bigcup_{i=1}^{\infty}F_i\quad$ and $\quad\tilde\gamma(\mathrm{Cl}_{\beta D}D_0-D_0)\subset C.$

Step 3. Let E be the product space $I\times T$ defined in § 2 and let E_0 and E_1 be the ends of a Čech 1-cell $E^*=\beta E-E$. Since D_0 is homeomorphic to the subspace $\{0\}\times T$ of E, there is a homeomorphism $\alpha\colon E_0\to\mathrm{Cl}_{\beta D}D_0-D_0$. Define a map $\xi\colon E_0\cup E_1\to C$ by $\xi|E_0=\tilde\gamma\alpha$ and $\xi(E_1)=q$. By (4.3) ξ is well defined. Since C is pointed 1-movable, by Theorem 1(3) the map ξ is extended to the map $\tilde\xi\colon E^*\to C$. Consider two maps $\tilde\gamma|\beta D-D\colon\beta D-D\to C\cup\bigcup_{i=1}^{\infty}F_i$ and $\tilde\xi\colon E^*\to C$. Note that $\beta D-D$ is a Čech 1-cell. By identifying E_0 and $\mathrm{Cl}_{\beta D}D_0-D_0$ by the homeomorphism α and by making use of the maps $\tilde\alpha$ and $\tilde\xi$, we can construct a Čech path connecting two points p and q in $C\cup\bigcup_{i=1}^{\infty}F_i$. This completes the proof.

Next, we consider the pointed 1-movability of a continuum which is a union of disjoint pointed 1-movable spaces. Suppose that

(4.4) Y is a continuum which is a union of two disjoint subsets X and Z satisfying the following conditions;

(i) Z is a compact set each component of which is pointed 1-movable,

(ii) X is connected and locally compact and has a locally finite covering $\mathscr{F}$ such that for each member F of $\mathscr{F}$ $\mathrm{Cl}_X F$ is compact and any two points of F are connected by a Čech path in F.

We shall show that the space Y in (4.4) is pointed 1-movable. This is done by the same argument as in the proof of Theorem 3. First, that each member of $\mathscr{F}$ is a compactum is not necessary in the proof of Theorem 3. We need only that for each member F of $\mathscr{F}$ $\mathrm{Cl}_X F$ is compact and any two points of F are connected by a Čech path in F. Second, if C is a component of Z then $C \cap \mathrm{Cl}_Y X \neq \emptyset$. To see it, let $\tilde{Y}$ be the quotient space obtained from Y by contracting each component of Z to a point. Since Y is a continuum, $\tilde{Y}$ is a metrizable compactification of X. If $h\colon Y \to \tilde{Y}$ is a projection, then $h(\mathrm{Cl}_Y X) = \tilde{Y}$. Therefore $C \cap \mathrm{Cl}_Y X \neq \emptyset$ for each component C of Z. By Lemma 4 it is known that for a given component C of Z there exists a sequence $\{F_i\}$ in $\mathscr{F}$ such that $\mathrm{Lim} F_i \subset C$ and $F_i \cap F_{i+1} \neq \emptyset$ for each i. Thus the following theorem is reduced to Theorem 3.

THEOREM 4. *If Y is a continuum satisfying the conditions in* (4.4), *then Y is pointed* 1*-movable.*

The following corollaries follow from Theorems 3 and 4.

COROLLARY 1. *Let X be a connected and locally compact metrizable space which has a locally finite cover consisting of pointed* 1*-movable continua. Then the Freudenthal compactification and the one point compactification of X are pointed* 1*-movable.*

COROLLARY 2. *Let X be a connected, locally connected and locally compact metrizable space. If αX is a metrizable compactification of X such that each component of $\alpha X - X$ is locally connected, then αX is pointed* 1*-movable.*

COROLLARY 3. *Let X be a connected and locally compact* ANR. *If αX is a metrizable compactification of X such that each component of $\alpha X - X$ is pointed* 1*-movable, then αX is pointed* 1*-movable.*

COROLLARY 4. *Let X be a continuum being a disjoint union of two locally connected subspaces X_1 and X_2 one of which is compact. Then X is pointed* 1*-movable. In particular, if X is a continuum being a disjoint union of two* ANR's *one of which is compact, then X is pointed* 1*-movable.*

Corollaries 1, 2 and 3 are immediate consequences of Theorems 3 and 4. Let us prove Corollary 4. Suppose that X_1 is compact. Since X_1 is locally connected, there exists only a finite number of components of X_1. Let us denote them by $A_1, A_2, \ldots, A_n$. Denote the components of X_2 by $\{H_\tau\}$. Put $B_k = \bigcup \{H_\tau\colon A_k \cap \mathrm{Cl}_X H_\tau \neq \emptyset\} \cup A_k$, $k = 1, 2, \ldots, n$. Then $\{B_k\}$ forms a finite cover of X. It is enough to prove that any two points p and q of B_k are connected by a Čech path in B_k. For it, let us consider the case where $p \in A_k$ and q belongs to a component H of X_2 such that $A_k \cap \mathrm{Cl}_X H \neq \emptyset$. The proof of the other case is similar. Since H is locally connected, connected and locally compact, there is an open cover $\mathscr{F}$ of H such that $\mathscr{F}$ is locally finite in H and for each member F of $\mathscr{F}$ F is arcwise connected and $\mathrm{Cl}_H F$ is compact. Then the proof follows from one of Theorem 4.

In the following examples, it is shown that the condition "pointed 1-movability of the remainder" in Theorems or Corollaries can not be omitted.

EXAMPLE 1 (Krasinkiewicz and Minc [6]). Let M be a dyadic solenoid and let C be a composant of M. There is a 1:1 continuous map f from a real line R onto C. Consider the product space $M \times I$ and its subspace $Y = \{(f(x), 1/(1+|f(x)|))\colon x \in R\}$. Obviously Y is homeomorphic to R and hence a locally compact AR. Put $\alpha Y = \mathrm{Cl}_{M \times I} Y$. Then αY is a metrizable compactification of Y and the remainder $\alpha Y - Y = M \times \{0\}$ is not 1-movable. Since there is a retraction from αY onto $\alpha Y - Y$, αY is not 1-movable.

EXAMPLE 2. Let r be a positive integer >1. Let S^r be an r-sphere. Consider an inverse sequence $\{S_i, f_{i,i+1}\colon i = 1, 2, \ldots\}$ such that S_1 is a point, f_{12} is a constant map, for $i > 1$ S_i is a copy of S^r and each bonding map $f_{i,i+1}$ has a fixed degree >1. Denote by M_i the mapping cylinder of $f_{i,i+1}$. Then M_i contains S_i and S_{i+1} as closed sets. Let X be a telescope associating with the inverse sequence $\{S_i\}$, that is, the space obtained by identifying S_{i+1}'s in M_i and M_{i+1}, $i = 1, 2, \ldots$, from a topological sum $\bigoplus_{i=1}^{\infty} M_i$. Then X is a locally compact AR. Consider a one point compactification cX of X with the added point c. Since X is a union of an increasing sequence of continua with connected boundaries, cX is the Freudenthal compactification of X. By computing a local cohomology of cX about the point c, it is easy to see that cX is not r-movable.

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Accepté par la Rédaction le 30. 10. 1980