Thus the map $g_n \circ p_{\varphi(n)}^{\varphi(n+s)}$ is homotopic (relatively to the base point) to a map which coincides the map $q_{n-r}^{n+s-r} \circ g_{n+s}$ on the k-skeleton $X_{\varphi(n+s)}^{(k)}$ of $X_{\varphi(n+s)}$. By Corollary (8.2) the maps $q_{n-s}^{n-r} \circ g_n \circ p_{\varphi(n)}^{\varphi(n+s)}$ and $q_{n-s}^{n+s-r} \circ g_{n+s}$ are homotopic (relatively to the base point). Thus the maps $\tilde{g}_n \circ p_{\varphi(n)}^{\varphi(n+s)}$ and $q_{n-s}^{n} \circ \tilde{g}_{n+s}$ are homotopic (relatively to the base point), where $\tilde{g}_m = q_{m-s}^{m-r} \circ g_m$.

Thus the maps $\tilde{g}_n$, where $n = k \cdot s+1$, $k \in \boldsymbol{J}$, define a morphism (in procategory homotopy) of X in a subsequence of Y. Since.

$$\pi_k(\tilde{g}_n) = \pi_k(q_{n-s}^{n}) \circ \varphi_n ,$$

we can define required f.

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On a Problem of Silver

by

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Abstract. We show that it is consistent, relative to an ω sequence of measurable cardinals, for $\aleph_\omega$ to be a Rowbottom cardinal and for $\mathrm{DC}_{\aleph_n}$ to hold, where n is an arbitrary natural number.

Of all of the large cardinal axioms which are currently known, the axioms which assert the existence of Rowbottom and Jonsson cardinals are amongst the more interesting hypotheses. Most large cardinal axioms assert, at least when the Axiom of Choice is true, that the cardinal in question is strongly inaccessible. This, however, is not true about Rowbottom and Jonsson cardinals. Indeed, Devlin has shown [3] that it is relatively consistent for $2^{\aleph_0}$ be a Jonsson cardinal, and Prikry has shown [6] that, assuming the consistency of a measurable cardinal, it is consistent for a Rowbottom cardinal of cofinality ω to exist.

The above results inspire the following question: How large is the least Rowbottom cardinal? Silver in his thesis [7] hypotheses that it is relatively consistent that the answer is $\aleph_\omega$, assuming the Axiom of Choice.

The answer to Silver's question is still not known, and is the only remaining unsolved problem from Silver's thesis. We have obtained a partial answer to Silver's question by showing that it is consistent, relative to the existence of an ω sequence of measurable cardinals, for $\aleph_\omega$ to be a Rowbottom cardinal and for a large portion, though not all, of the Axiom of Choice to be true. Specifically, we have proven the following:

THEOREM 1. *Assume that the theory* "ZFC + *There is an ω sequence of measurable cardinals*" *is consistent. Let $n_0 \in \omega$ be a fixed* (*though arbitrary*) *natural number. Then the theory* "$\mathrm{ZF} + \mathrm{DC}_{\aleph_{n_0}} + \aleph_\omega$ *carries a Rowbottom filter*" *is consistent.*

Note that some strong hypothesis is needed to obtain a model which witnesses Theorem 1 since an unpublished result of Silver shows that if $\aleph_\omega$ is a Rowbottom cardinal, then it must be measurable in some inner model. Note also that other partial results on Silver's problem have been obtained. In particular, Bull in his thesis [2] showed that, assuming the consistency of a measurable cardinal, the theory "$\mathrm{ZF} + \forall n \in \omega [2^{\aleph_n} = \aleph_{n+1}] + \aleph_\omega$ is a Rowbottom cardinal $+ \neg \mathrm{AC}_\omega$" is consistent.

Before beginning the proof of Theorem 1, we briefly mention some background information. Basically, our notation and terminology are fairly standard.

We work in ZF, both with and without AC. Lower case Greek letters $\alpha, \beta, \gamma, \ldots$ denote ordinals, with the letters $\varkappa$, λ, and δ generally being reserved for cardinals. V denotes the universe. For x a set, $|x|$ denotes the cardinality of x, and 2^x denotes the power set of x. For $\alpha<\beta$, $[\beta]^{<\alpha} = \bigcup_{\delta<\alpha}\{f\colon f$ is a strictly increasing function from δ to $\beta\}$. For f a function, $f \restriction x$ is f restricted to x, and $f''x$ is the range of f on x. For α an ordinal, $\varkappa^{+\alpha}$ is the αth least cardinal $>\varkappa$.

When we talk about forcing, $\Vdash$ will mean "weakly forces" and $\|$ will mean "decides". $p\leqslant q$ means that q is *stronger* than p.

For $\varkappa$ a cardinal, $\mathrm{DC}_\varkappa$ is the following assertion: Assume that X is a set and that R is a binary relation on X so that for $\langle x_\alpha\colon \alpha<\beta<\varkappa\rangle$ a sequence of elements of X, there is some $x\in X$ so that $\langle x_\alpha\colon \alpha<\beta<\varkappa\rangle Rx$. There is then a function $f\colon \varkappa\to X$ so that $\forall\alpha<\varkappa[f\restriction\alpha Rf(\alpha)]$.

We recall Levy's notion of forcing, $\mathrm{Col}(\varkappa,\lambda)$, for collapsing an inaccessible cardinal λ to the successor of a regular cardinal $\varkappa$. $\mathrm{Col}(\varkappa,\lambda) = \{f\colon f\colon \varkappa\times\lambda\to\lambda$ is a function so that $|\mathrm{dmn}(f)|<\varkappa$ and so that $\langle\alpha,\beta\rangle\in\mathrm{dmn}(f)\Rightarrow f(\alpha)<\beta\}$, ordered by set-theoretic inclusion. Any compatible collection of conditions of cardinality $<\varkappa$ in $\mathrm{Col}(\varkappa,\lambda)$ thus has an upper bound.

Finally, we assume that the reader is familiar with the notions of measurable cardinal and Rowbottom cardinal for which we refer the reader to [8] and [4]. We only mention what a Rowbottom filter is. For $\varkappa$ a cardinal, we say that F is a Rowbottom filter over $\varkappa$ if F is a filter and for $\delta<\varkappa$, $H\colon [\varkappa]^{<\omega}\to\delta$, there is a set $C\in F$ so that $|C| = \varkappa$ and $|H''[C]^{<\omega}|\leqslant\omega$.

We now turn our attention to the proof of Theorem 1.

Proof of Theorem 1. Let $V\models$ "ZFC+$\langle\varkappa_n\colon n<\omega\rangle$ is an increasing ω sequence of measurable cardinals". Let μ_n be a fixed normal measure on $\varkappa_n$.

Our proof extends ideas found in [1]. First, fix $n_0\in\omega$ an arbitrary natural number n_0. Next, define a sequence $\langle P_n\colon n<\omega\rangle$ of partial orderings by $P_0 = \mathrm{Col}(\aleph_{n_0+1},\varkappa_0)$, and for $n>0$, $P_n = \mathrm{Col}(\varkappa_{n-1}^+,\varkappa_n)$. We then define our partial ordering P as $\prod_{n\in\omega}P_n$ ordered componentwise, i.e., for $p, q\in P$, $p = \langle p_n\colon n<\omega\rangle$, $q = \langle q_n\colon n<\omega\rangle$, $p\leqslant q$ iff $\forall n[p_n\leqslant q_n]$. Note that we are allowing a condition p to be nontrivial infinitely often.

Let G be V-generic on P. By the product lemma for product forcing, the projection of G onto its nth coordinate, G_n, is V-generic on P_n. Also, for $\alpha\in(\aleph_{n_0+1},\varkappa_0)$ or $\alpha\in(\varkappa_{n-1}^+,\varkappa_n)$ (we use standard interval notation here), $G_n\restriction\alpha = \{p\in P_n\colon p\in G_n$ and $\mathrm{dmn}(p)\subseteq\varkappa_{n-1}\times\alpha\}$ is V-generic on $P_n\restriction\alpha$.

We are now in a position to describe the model N, a certain submodel of $V[G]$, which will be the desired model. First, let $I_0 = (\aleph_{n_0+1},\varkappa_0)$ and for $n>0$, $I_n = (\varkappa_{n-1}^+,\varkappa_n)$. Let $I = \prod_{n\in\omega}I_n$, and let $K = \{f\colon f\colon \omega\to I$ so that $f(n)\in I_n\}$. For a particular $f\in K$, let $G\restriction f = \prod_{n\in\omega}G_n\restriction f(n)$. N will be the least model of ZF extending V which contains each $G\restriction f$ for $f\in K$. More formally, we define N as follows: Let $L_1\subseteq L$, where L is the forcing language with respect to P, be a ramified sublanguage which contains a predicate symbol $\underline{V}$ (to be interpreted as $\underline{V}(v)\Leftrightarrow v\in V$), symbols $\underline{v}$ for each $v\in V$, and symbols $G\restriction f$ for each $f\in K$. N is then defined by

$$N_0 = \varnothing,$$
$$N_\lambda = \bigcup_{\alpha<\lambda}N_\alpha \text{ for } \lambda \text{ a limit ordinal},$$
$$N_{\alpha+1} = \{x\subseteq N_\alpha\colon x \text{ is definable over } N_\alpha \text{ by a formula } \varnothing\in L_1 \text{ of rank } \leqslant\alpha\},$$
$$N = \bigcup_{\alpha\in\mathrm{Ordinals}}N_\alpha.$$

Standard arguments show that $N\models$ ZF. Also, note that we may assume that each $\underline{v}$ for $v\in V$ is invariant under any automorphism of P, and that any term τ which only mentions $G\restriction f$ is invariant under any automorphism of P which is generated by a function which is the identity on ordinals α such that $\alpha\in(\varkappa_{n-1}^+,f(n))$ or $\alpha\in(\aleph_{n_0+1},f(0))$.

We now begin the proofs of a sequence of lemmas which show that N is our desired model.

Lemma 1.1. *If x is a set of ordinals so that $x\in N$, then $x\in V[G\restriction f]$ for some $f\in K$.*

Proof of Lemma 1.1. Let τ be a term which denotes x, and assume that $p\Vdash$ "$\tau\subseteq\alpha$" for some α. As τ denotes a set in N, τ will mention only finitely many $G\restriction f$ terms. These can all be coded with one such term; if $G\restriction f_1,\ldots, G\restriction f_m$ are the terms which appear in τ, then we can define $f(n) = \sup_{i\leqslant m}f_i(n)$ and use $G\restriction f$ to define τ. Thus, without loss of generality, we assume that τ mentions one $G\restriction f$.

Let $q\geqslant p$ be so that $q\Vdash$ "$\beta\in\tau$". We claim that $q\restriction f = \langle q_n\restriction f(n)\colon n\in\omega\rangle$ where $q_n\restriction f(n)$, the function q_n restricted to $(\varkappa_{n-1}^+,f(n))$, is such that $q\restriction f\Vdash$ "$\beta\in\tau$". If this is not so, then let $r\geqslant q\restriction f$ be so that $r\Vdash$ "$\beta\notin\tau$". Using the standard properties of Levy conditions, there is an automorphism π_n of P_n which is generated by a function which is the identity on $(\varkappa_{n-1}^+,f(n))$ or $(\aleph_{n_0+1},f(0))$ so that $\pi_n(q_n)$ is compatible with r_n. $\pi = \langle\pi_n\colon n<\omega\rangle$ is then an automorphism of P so that $\pi(q)$ is compatible with r. However, $\pi(q)\Vdash$ "$\beta\in\tau$" and $r\Vdash$ "$\beta\notin\tau$", and this is a contradiction. Thus, in $V[G\restriction f]$ we can define x, assuming $p\in G$, by $x = \{\beta<\alpha\colon \exists q\in G\restriction f[q\Vdash$ "$\beta\in\tau$"$]\}$. This proves Lemma 1.1. ■

Lemma 1.2. *Let $\lambda = \bigcup_{n\in\omega}\varkappa_n$. Then $N\models$ "$\lambda = \aleph_\omega$".*

Proof of Lemma 1.2. By definition of N, for each $\alpha\in(\aleph_{n_0+1},\varkappa_0)$ or $\alpha\in(\varkappa_{n-1}^+,\varkappa_n)$ $G_n\restriction\alpha\in N$. Hence, there are no cardinals present in any of these open intervals. Therefore, $N\models$ "$\lambda\leqslant\aleph_\omega$". Thus, to show that $N\models$ "$\lambda = \aleph_\omega$", it suffices to show that $\aleph_{n_0}^V$ and each $\varkappa_{n-1}$ for $n\geqslant 1$ remain cardinals.

As any set of ordinals in N must be in $V[G\restriction f]$ for some f by Lemma 2.1, if we can show that $V[G\restriction f]\models$ "$\aleph_{n_0}^V$ and each $\varkappa_{n-1}$ for $n\geqslant 1$ are cardinals", then we are done. To do this, let us view $P\restriction f$ (where $P\restriction f$ is defined similarly to $G\restriction f$) as $\prod_{n\in\omega}Q_n$, where Q_n is a subordering of P_n. Each such Q_n will have the same closure properties of P_n, i.e., each Q_n will be so that for $n = 0$, any collection of compatible conditions of cardinality $\aleph_{n_0}$ has an upper bound and for $n>0$, any collection of compatible conditions of cardinality $\varkappa_{n-1}$ has an upper bound.

Now $P \restriction f$ can be viewed as $(\prod_{i<n} Q_i)\times(\prod_{i\geqslant n} Q_i)$. Since any $q \in \prod_{i\geqslant n} Q_i$ is such that its jth coordinate is a function in an ordering which is at least $\varkappa_{n-1}$ closed, and since q can be non-trivial infinitely often, $\prod_{i\geqslant n} Q_i$ is a partial ordering which is $\varkappa_{n-1}$ closed. Thus, forcing with this partial ordering will preserve the fact that $\varkappa_{n-1}$ is a cardinal. By definition of Q_i, $|\prod_{i<n} Q_i|<\varkappa_{n-1}$. Thus, forcing with this partial ordering will preserve the fact that $\varkappa_{n-1}$ is a cardinal. Since by the product lemma, forcing with $P \restriction f$ is the same as first forcing with $\prod_{i\geqslant n} Q_i$ and then forcing with $\prod_{i<n} Q_i$, the above shows that in $V[G \restriction f]$, each $\varkappa_{n-1}$ is a cardinal. The above arguments also show that $P \restriction f$ is $\aleph_{n_0}$ closed, since P_0 is. Thus, $\aleph_{n_0}$ remains a cardinal in $V[G \restriction f]$. This proves Lemma 1.1. ■

LEMMA 1.3. $N \models$ "*$\aleph_\omega$ carries a Rowbottom filter*".

Proof of Lemma 1.3. In N, let $F = \{A \subseteq \lambda\colon \exists n \forall m \geqslant n[\mu_m(A \cap \varkappa_m) = 1]\}$. As each μ_n is a measure on $\varkappa_n$ in V, F is clearly a filter. To show that F is a Rowbottom filter over $\lambda(=\aleph_\omega)$ in N, let $\delta<\lambda$, and let $g \in N$ be so that $g\colon [\lambda]^{<\omega} \to \delta$. As g can easily be coded by a set of ordinals, by Lemma 1.1 $g \in V[G \restriction f]$ for some $f \in K$. Thus, as $V[G \restriction f] \subseteq N$, we will be done if we can find $A \subseteq \lambda$, $A \in V[G \restriction f]$ so that $|g''[A]^{<\omega}| \leqslant \omega$ and so that $\exists m \forall n \geqslant m[\mu_m(A \cap \varkappa_n) = 1]$.

To show this, we first note that in $V[G \restriction f]$, each $\varkappa_n$ is a measurable cardinal with normal measure $\mu_n^* = \{A \subseteq \varkappa_n\colon \exists B \subseteq A[\mu_n(B) = 1]\}$. This is since, using the notation of the previous lemma, we can write P as $\prod_{i\geqslant n} Q_i \times \prod_{i<n} Q_i$. $\prod_{i\geqslant n} Q_i$ is $\varkappa_{n-1}$ closed as we have already observed, so forcing with it will add no new subsets to $\varkappa_{n-1}$ and hence preserve the fact that $\varkappa_{n-1}$ is measurable with normal measure μ_{n-1}. $|\prod_{i<n} Q_i|<\varkappa_{n-1}$; this is since $f(0) \in (\aleph_{n_0+1}, \varkappa_0)$ and for $n \geqslant 1$, $f(n) \in (\varkappa_{n-1}, \varkappa_n)$, so as each $\varkappa_n$ is strongly inaccessible, $|Q_i|<\varkappa_i$, so $|\prod_{i<n} Q_i|<\varkappa_{n-1}$. Hence, by the results of Levy–Solovay [5], forcing with $\prod_{i<n} Q_i$ leaves $\varkappa_{n-1}$ a measurable cardinal with normal measure μ_{n-1}^* as defined above. Since by the product lemma, forcing with $P \restriction f$ can be viewed as first forcing with $\prod_{i\geqslant n} Q_i$ and then forcing with $\prod_{i<n} Q_i$, and since n above is arbitrary, each $\varkappa_n$ will in $V[G \restriction f]$ be a measurable cardinal with normal measure μ_n^*.

By a theorem of Prikry [6], in $V[G \restriction f]$ $F' = \{A \subseteq \lambda\colon \exists n \forall m \geqslant n[\mu_m^*(A \cap \varkappa_m) = 1]\}$ is a Rowbottom filter on λ, so in this model let $A \subseteq \lambda$ be such that $|g''[A]^{<\omega}| \leqslant \omega$ and $\exists n \forall m \geqslant n[\mu_m^*(A \cap \varkappa_m) = 1]$. For such an n and for any $m \geqslant n$, there is a set $B_m \subseteq A \cap \varkappa_m$ so that $\mu_m(B_m) = 1$; use AC in $V[G \restriction f]$ to pick such sets B_m. $B = \bigcup_{m\geqslant n} B_m$ will then be so that $|g''[A]^{<\omega}| \leqslant \omega$, $\exists n \forall m \geqslant n[\mu_m^*(B \cap \varkappa_m) = 1]$, and $\exists n \forall m \geqslant n[\mu_m(B \cap \varkappa_m) = 1]$. This proves Lemma 1.3. ■

LEMMA 1.4. $N \models DC_{\aleph_{n_0}}$.

Proof of Lemma 1.4. First, note that, by arguments as in Lemma 1.2, since each P_i is at least $\aleph_{n_0}$ closed, and since a condition $p \in P$ can be non-trivial infinitely often, P is $\aleph_{n_0}$ closed, so $\aleph_{n_0}$ is the same in V, N, $V[G]$, or $V[G \restriction f]$ for any $f \in K$. Now, let $p_0 \Vdash$ "$X \in N$ is a set and $R \in N$ is a binary relation on X so that for $\langle x_\alpha\colon \alpha<\beta<\aleph_{n_0}\rangle$ a sequence (in N) of elements of X, there is some $x \in X$ so that $\langle x_\alpha\colon \alpha<\beta<\aleph_{n_0}\rangle Rx$", and assume that $p_0 \Vdash$ "$\tau_0 \in X$". Define (in V) inductively a sequence $\langle \tau_\alpha\colon \alpha<\aleph_{n_0}\rangle$ of terms and a sequence $\langle p_\alpha\colon \alpha<\aleph_{n_0}\rangle$ of forcing conditions as follows: $p_{\alpha+1}$ is a condition which extends p_α so that $p_{\alpha+1} \Vdash$ "$\langle \tau_\beta\colon \beta \leqslant \alpha\rangle R\tau$", and $\tau_{\alpha+1}$ is such a τ. For δ a limit ordinal, p_δ is a condition which extends each p_α for $\alpha<\delta$ and τ_δ is the sequence $\langle \tau_\alpha\colon \alpha<\delta\rangle$; note that the closure properties of P ensure that p_δ exists.

Finally, let p extend each p_α. As above, the closure properties of P imply the existence of p. By the fact that each P_i is a Levy collapse ordering, $p \in P \restriction f$ for some $f \in K$ (each p_i where $p = \langle p_i\colon i<\omega\rangle$ is an element of $\mathrm{Col}(\aleph_{n_0+1}, \alpha)$ for $\alpha \in (\aleph_{n_0}, \varkappa_0)$ or $\mathrm{Col}(\varkappa_{i-1}, \alpha)$ for $\alpha \in (\varkappa_{i-1}, \varkappa_i)$). Thus, $p \Vdash$ "$\langle \tau_\alpha\colon \alpha<\aleph_{n_0}\rangle$ is a sequence in N so that for $\beta<\aleph_{n_0}$, $\langle \tau_\alpha\colon \alpha<\beta\rangle R\tau_\beta$". This proves Lemma 1.4. ■

Note that in the above lemma, we assumed that X and R could be evaluated in N. However, as $X, R \in N$, an $f \in K$ such that $p \in P \restriction f$ and X and R may be evaluated in N certainly exists.

Lemmas 1.2–1.4 complete the proof of Theorem 1. ■

In conclusion, we note that $DC_{\aleph_{n_0+1}}$ is false in N. This is since any set of ordinals must be in $V[G \restriction f]$ for some $f \in K$, but G_0 (which can be coded by a set of ordinals) lies in no such $V[G \restriction f]$.

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On a shape characterization of some two-polyhedra

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Abstract. The main purpose of this paper is to give a shape characterization of surfaces.

Introduction. K. Borsuk has formulated the following problem: give a shape characterization of manifolds. We solve this problem in a very special case: for surfaces (i.e. closed 2-dimensional manifolds). We will prove (Corollary (2.11) and Theorem (3.23)) the following:

THEOREM. *A continuum (metric) X has the shape of a surface if and only if X is pointed movable, the shape dimension* $\mathrm{Fd}\,X$ *is* 2, *the second Čech homology group* $\check{H}_2(X, Z_2) \cong Z_2$ *and the first shape group* $\check{\pi}_1(X)$ *is isomorphic to a fundamental group of a surface.*

In fact, in § 2, we give a shape characterization of two polyhedra of a class which contains all surfaces with the trivial second homotopy group (Theorem (2.9)).

In § 3 we give a shape characterization of a bouquet (one point union) of the projective plane and 2-spheres.

If X is a connected compact FANR with vanishing "Wall obstruction", then X has the shape of a pointed finite simplicial complex with dimension $\max(3, \mathrm{Fd}\,X)$, see [8]. There is no shape characterization of the class of all (finite) two-polyhedra. If X is a connected compact FANR with vanishing "Wall obstruction", $\mathrm{Fd}\,X = 2$ and $\check{\pi}_1(X) \cong Z_p$, then X has the shape of a (finite) 2-polyhedron; X has the shape of a bouquet of the pseudoprojective plane of order p and 2-spheres.

We assume that the reader is familiar with the basic notions of shape theory for metric compacta (see [2], [4] or [17]).

1. Shape of pointed movable continua with fundamental dimension 2 and with finitely presented 1-shape group. J. Krasinkiewicz has proved ([12], Theorem 3.1, p. 151 and Theorem 4.2, p. 152) that if (X, x) is a pointed 1-movable continuum then there exists a pointed ANR-sequence $(\boldsymbol{X}, \boldsymbol{x}) = \{(X_n, x_n), p_n^m\}$ associated with (X, x) (i.e. $\varprojlim(\boldsymbol{X}, \boldsymbol{x}) = (X, x)$) such that the corresponding sequence of fundamental groups $\pi_1(\boldsymbol{X}, \boldsymbol{x})$ is an epi-sequence; if $(\boldsymbol{X}', \boldsymbol{x}') = \{(X', x_n'), p_n'^m\}$ is any ANR-sequence associated with (X, x) then X_n can be obtained from X_n' by attaching to X_n' a finite number of 2-cells. It is easy to see ([3], proof of Theorem 2, p. 616) that if $\boldsymbol{G} = \{G_n, q_n^m\}$ is an epi-sequence of groups such that the inverse limit $\varprojlim \boldsymbol{G}$ is