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On a certain prewellordering

by

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In [1], a certain prewellordering of functions from ${}^{\omega}2$ of $\aleph_1$ is defined under the Axiom of Determinateness, and is shown to have length at least $\aleph_2$. We will show that this length is in fact at least θ, the least cardinal onto which the continuum cannot be mapped. We use throughout the notation and techniques of [1], particularly those of Theorem 1.4.

THEOREM (AD). $T_{\aleph_1} \geq \theta$.

Proof. Given $\gamma < \theta$, let f map ${}^{\omega}2$ onto $P_{\aleph_1}(\gamma)$, and let $f_\alpha\colon {}^{\omega}2 \to \aleph_1$, $\alpha < \gamma$, be defined by: $f_\alpha(r) =$ the order type of $\bigcup_{n<\omega} f^*(r^n) \cap \alpha$. For any $\alpha < \beta < \gamma$, we can show $f_\alpha < f_\beta$ by describing a winning strategy for player II in G'_{f_α, f_β}. Such a strategy consists of playing the real s to player I's real r so that each s^n is identical and all the reals $\{(r^m)^{k^*}\}_{m,k}$ are included in $\{(s^n)^{k^*}\}_k$ as well as a real t such that $\alpha \in f^*(t)$.

This establishes the functions $\{f_\alpha\}_{\alpha<\gamma}$ as a sequence of length γ in the prewellordering. ■

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