number of critical points of an $m$-function extending $f$. It should be possible to obtain similar results as in our case if one consider the chain complex induced by $f$ in place of homology model $R^0$ in the definition of $\mathcal{U}(M)$.

For non-simply connected manifolds the minimization problems are much more difficult and even for Morse functions no satisfactory calculations are known. However, Theorems 2 and 3 are ineffective enough to have straightforward generalizations to that case. Our arguments ought to work if the homology groups of $M$ and $\partial M$ are replaced by the homology groups of universal covering considered as modules over the integer group rings $\mathbb{Z}_1 M$ and $\mathbb{Z}_1 \partial M$, at least when $i_\ast: \pi_1 \partial M \to \pi_1 M$ is an isomorphism.

References


**Special bases for compact metrizable spaces**

by

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Abstract. Each compact metrizable space has a base $\mathcal{B}$ such that

1. for every finite $A \subseteq \mathcal{B}$, if any two members of $\{A: A \in \mathcal{A}\}$ intersect then $\bigcap A \neq \emptyset$; and
2. if $\mathcal{R}$ is the ring generated by $\mathcal{B}$, then $\mathcal{R}$ consists of regularly open sets and

$$\mathcal{J}(\mathcal{F})^\ast = \bigcap\{A: A \in \mathcal{F}\}$$

for every finite $\mathcal{F} \subseteq \mathcal{R}$. This implies that every compact metrizable space is regularly supercompact. The construction of $\mathcal{B}$ is complicated but elementary.

0. Conventions and definitions. As usual, if $X$ is a space, $\bar{\cdot}$, $^0$ and $^c$ denote the closure operator, the interior operator and the complementation operator in $X$; if $\mathcal{F}$ is a family of subsets of $X$ we write e.g. $\bar{\mathcal{F}}$ for $\{F: F \in \mathcal{F}\}$.

If $X$ is a space and $\mathcal{F}$ is a family of subsets, then $\mathcal{F}$ is called a *closed subbase* if it is a subbase for the closed sets, i.e. $\mathcal{F}^\ast$ is a subbase for the open sets, a ring if $F \cap G \in \mathcal{F}$ and $F \cup G \in \mathcal{F}$ for all $F, G \in \mathcal{F}$, linked if $F \cap G \neq \emptyset$ for any $F, G \in \mathcal{F}$ (not necessarily distinct), binary if every linked subfamily has nonempty intersection.

A space is called *supercompact* if it has a binary closed subbase, regularly *supercompact* if it has a binary closed subbase $\mathcal{F}$ such that the ring generated by $\mathcal{F}$ consists of regularly closed sets, regularly *Wallman* if it has a closed subbase which is a ring and which consists of regularly closed sets.

1. Introduction. The notion of supercompactness was introduced by de Groot in [dG]. It is a trivial consequence of Alexander's Subbase Lemma, [A], that every supercompact space is compact. An easy example of a compact $T_1$-space that is not supercompact was given by Verbeek, [V, II. 2.2(8)]. The question of whether all compact Hausdorff spaces are supercompact was settled in the negative by Bell, [B], this is a nontrivial result in spite of the fact that the answer was to be expected, [dG]. Subsequently van Douwen and van Mill showed that every infinite supercompact Hausdorff space has many nontrivial convergent sequences, [dVdM]; this gives a rich supply of compact Hausdorff spaces that are not supercompact.

This paper deals with de Groot's conjecture that all compact metrizable spaces are supercompact, [dG]. The first result is due to de Groot who proved that compact polyhedra are supercompact. An erroneous proof of de Groot’s conjecture was
We prove the theorem from the lemma by constructing a suitable subfamily $\mathcal{B}$ of $\mathcal{U}$.

Note that Corollary 2 follows from the lemma in the same way Corollary 1 follows from the theorem.

I am grateful to Jan van Mill for supplying me with some information, given above, that does not occur in print.

2. Proof of the lemma. We first show that if suffices to prove the following claim. $X$ has a base $\mathcal{V}$ consisting of regularly open sets, satisfying

(*) for any two disjoint finite $\mathcal{F}, \mathcal{B} \subseteq \mathcal{V}$, if $\bigcap (\mathcal{F} \cup \mathcal{B}^-) = \emptyset$ then $\bigcap (\mathcal{F} \cup \mathcal{B}^{--}) = \emptyset$.

Given $\mathcal{V}$ as in the claim, we will show that

$\mathcal{W} = \{\bigcup \mathcal{F} : \mathcal{B} \subseteq \mathcal{V} \text{ is finite}\}$

consists of regularly open sets and satisfies

(1) $\bigcap \mathcal{F} = (\bigcap \mathcal{F})^-$ for every finite $\mathcal{F} \subseteq \mathcal{W}$.

Since the intersection of finitely many regularly open sets again is regularly open, we then can define our base $\mathcal{U}$ by

$\mathcal{U} = \{\bigcap \mathcal{F} : \mathcal{F} \subseteq \mathcal{W} \text{ is finite}\}$.

In order to show that $\mathcal{W}$ is as stated, we need the following strengthening of (*):

(**) $\bigcap (\mathcal{F} \cup \mathcal{B}^{--}) = (\bigcap (\mathcal{F} \cup \mathcal{B}^{-}))^-$ for any two disjoint finite $\mathcal{F}, \mathcal{B} \subseteq \mathcal{V}$.

To see that (**) holds, let $\mathcal{F}, \mathcal{B} \subseteq \mathcal{V}$ be finite and disjoint. Clearly $(\bigcap (\mathcal{F} \cup \mathcal{B}^{-}))^- \subseteq \bigcap (\mathcal{F} \cup \mathcal{B}^{--})$. Let $x \notin (\bigcap (\mathcal{F} \cup \mathcal{B}^{-}))^-$. It is easy to see that $x \notin \bigcap (\mathcal{F} \cup \mathcal{B}^{--})$ by arbitrariness. Then there is a $x \in \bigcap (\mathcal{F} \cup \mathcal{B})^-$. Hence, (**) follows from (**) with $x \notin \mathcal{B}$ instead of $\mathcal{F}$ that $x \notin \bigcap (\mathcal{F} \cup \mathcal{B}^{--})$.

Evidently (1) is nothing but the special case $\mathcal{F} = \emptyset$ of (**).

Since the members of $\mathcal{V}$ are regularly open, we see from the special case $\mathcal{F} = \emptyset$ of (1) that for all finite $\mathcal{B} \subseteq \mathcal{V}$

$$(\bigcup \mathcal{B})^c = (\bigcup \mathcal{B})^{c-} = (\bigcup \mathcal{B}^-)^{c-} = (\bigcap \mathcal{B}^-)^c = (\bigcap \mathcal{B}^{--})$$

$$(\bigcup \mathcal{B}^-)^c = \bigcup \mathcal{B}^- = \bigcup \mathcal{B}$$.

Hence the members of $\mathcal{W}$ are regularly open.

It remains to prove the claim.

Proof of the claim. Let $\mathcal{A}$ be any countable base for $X$. Enumerate

$$\langle A, B \rangle \in \mathcal{A} \times \mathcal{A} : A \subseteq B \rangle \text{ as } \langle A(n, 0), B(n, 0) : n \in \omega \rangle,$$

and

$$\langle F, G \rangle : F, G \subseteq \omega; F \cap G = \emptyset, |F \cup G| < \omega \rangle \text{ as } \langle F_k, G_k : k \in \omega \rangle.$$
It is a straightforward exercise in normality to construct with recursion on \(1 \leq k < \omega\) open set \(A(n,k)\) and \(B(n,k)\) for all \(n \in \omega\) in such a way that for all \(n \in \omega\):

(a) \(A(n,k) \subseteq A(n,k+1), A(n,k+1)^c \subseteq B(n,k+1),\) and \(B(n,k+1)^c \subseteq B(n,k),\)

for \(k \in \omega;\) and

(b) if \(\bigcap \{A(i,k)^c: i \in F_k\} \cap \bigcap \{B(j,k)^c: j \in G_k\} = \emptyset\), then

\[
\bigcap \{B(i,k+1): i \in F_k\} \cap \bigcap \{A(j,k+1)^c: j \in G_k\} = \emptyset.
\]

For each \(n \in \omega\) define

\[
V_n = \bigcup_{k \leq n} (A(n,k))^{-c},
\]

and let \(\mathcal{V} = \{V_n: n \in \omega\}\).

It follows from (a) that

(c) \(A(n,k)^c \subseteq V_n\) and \(V_n \subseteq B(n,k)\) for \(k \in \omega, \ n \in \omega\).

Hence \(\mathcal{V}\) is a base for \(X\). Obviously \(\mathcal{V}\) consists of regularly open sets. We check (\(*\)). Let \(k \in \omega\) be arbitrary. Write \(\bigcap_{i} \) and \(\bigcap_{j}\) for the intersections with \(i \in F_k\)

and \(j \in G_k\), respectively. If

\[
\bigcap_{i} V_i \cap \bigcap_{j} V_j^{-c} = \emptyset,
\]

then using (c), (b) and (c) in turn, we see that

\[
\bigcap_{i} A(i,k)^c \cap \bigcap_{j} B(j,k)^c = \emptyset,
\]

hence

\[
\bigcap_{i} B(i,k+1)^c \cap \bigcap_{j} A(j,k+1)^c = \emptyset.
\]

(In the last step \(V_j^{-c} \subseteq A(j,k+1)^c\) since \(A(j,k+1)^c \subseteq V_j^{-c}\) and \(A(j,k+1)\) is open.)

Remark 1. Note that (\(*\)) implies that

\(\bigcap_{\mathcal{U}}\) for all \(U, V \in \mathcal{V}\), if \(U\) is a proper subset of \(V\), then \(U \subseteq V\).

(For \(U \subseteq V\), hence \(U \cap V^c = \emptyset\), hence \(U \cap V^c \subseteq V^c\), hence \(U \subseteq V^c = V_0 = V_\omega\).

It was shown in [G] that every metacompact Moore space has a basis satisfying (\(\bigcap\))

Remark 2. Since every metrizable space is perfectly normal and has a \(\sigma\)-discrete base, for every metrizable space \(X\) one can find for each \(n \in \omega\) a collection \(\langle A_{\gamma}, B_{\gamma} : \gamma \in \Gamma_n \rangle\), where \(\langle \Gamma_n : n \in \omega \rangle\) is a pairwise disjoint collection of index sets, such that

\(A_{\gamma}\) is closed, \(B_{\gamma}\) is open and \(A_{\gamma} \subseteq B_{\gamma}\) for \(\gamma \in \Gamma_n\);

\(\langle B_{\gamma} : \gamma \in \Gamma_n \rangle\) is a discrete family; and

if \(\mathcal{U} = \langle U_{\gamma} : \gamma \in \bigcup \Gamma_n \rangle\) is any open family such that \(A_{\gamma} \subseteq U_{\gamma} \subseteq B_{\gamma}\) for all \(\gamma \in \bigcup \Gamma_n\),

then \(\mathcal{U}\) is a base for \(X\).

One can probably use this to show that the lemma holds for all metrizable spaces. Since I do not know an application I did not bother to check this. (I recall that I once verified the special case that every metrizable space has a base \(\mathcal{V}\) satisfying both (\(\bigcap\)) and

\(\bigcap\) for all \(U, V \in \mathcal{V}\), if \(U \cap V = \emptyset\) then \(U \cap V = \emptyset\).

Remark 3. The technique of constructing a base by successive approximation is not new, the earliest reference I am aware of is [Os, Lemma 4]. Also, Aarts proves that every metrizable compactification is a Wallman compactification using a one-step approximation, in [Aa].

Remark 4. There is an easy example of a compact metrizable space which has a base \(\mathcal{U}\) which is a ring consisting of regularly open sets, yet \(\bigcap (\mathcal{F})^{-c} \neq \bigcap \mathcal{F}^{-c}\) for some finite \(\mathcal{F} \subseteq \mathcal{U}\). Indeed, let \(\omega + 1\), the ordinals \(\leq \omega\) have the order topology. Define

\[
U = \{3n : n \in \omega\}, \quad V = \{3n + 1 : n \in \omega\};
\]

and

\[
\mathcal{B} = \{\{n\} : n \in \omega\} \cup \{(n, \omega) : n \in \omega\} \cup \{U, V\}.
\]

Then \(\mathcal{B}\) is a base for \(\omega + 1\). One easily verifies that the ring generated by \(\mathcal{B}\) consists of regularly open sets. However, \((U \cap V)^{-c} \neq U \cap V^{-c}\).

3. Proof of the theorem. Let \(X\) be a (nonempty) compact metrizable space, let \(d\) be a compatible metric for \(X\), and for \(x \in X\) let \(S(x, \varepsilon)\) be the \(\varepsilon\)-sphere about \(x\). By the lemma there is a base \(\mathcal{U}\) for \(X\) such that

(A) \(\mathcal{U}\) is a ring consisting of regularly open sets; and

(B) \(\bigcap (\mathcal{F})^{-c} = \bigcap \mathcal{F}^{-c}\) for all finite \(\mathcal{F} \subseteq \mathcal{U}\).

We construct our base \(\mathcal{B}\) by finding for each \(n \in \omega\) a finite \(\mathcal{B}_n \subseteq \mathcal{U}\) such that

(C) \(U \subseteq \mathcal{B}_n\); and

(D) \(\text{diam}(B) < 2^{-n}\) for \(B \in \mathcal{B}_n\); and

(E) \(\bigcup \{A_k : k \leq n\}\) is binary;

for all \(n \in \omega\). Then \(\mathcal{B} = \bigcup \{\mathcal{B}_n : n \in \omega\}\) is a base for \(X\) by (C) and (D). Clearly (1) follows from (E), and (2) follows from (A) and (B) since \(\mathcal{U} \subseteq \mathcal{B}\).

We construct the \(\mathcal{B}_n\)'s with recursion on \(n \in \omega\), using the following claim which we prove later.

Claim. Let \(n \in \omega\). If \(\mathcal{A}\) is a finite binary subfamily of \(\mathcal{U}\), then there is a finite closed cover \(\mathcal{E}\) of \(X\) such that

\(\mathcal{A} \cup \mathcal{E}\) is binary;

for all \(E \in \mathcal{E}\) and \(A \in \mathcal{A}\) if \(E \cap A = \emptyset\) then \(E \cap \mathcal{A} = \emptyset\); and

\(\text{diam}(E) < 2^{-n}\) for \(E \in \mathcal{E}\).

Let \(n \in \omega\) and suppose \(\mathcal{B}_k\) to be constructed for \(k < n\). Then \(\mathcal{A} = \bigcup \{\mathcal{B}_k : 0 \leq k < n\}\) is a (possibly empty) finite subcollection of \(\mathcal{U}\). Let \(\mathcal{E}\) be as in the claim. Since \(\mathcal{U}\) is
closed under finite unions, \( \{ U \in \mathcal{U} : U \supseteq E \} \) is a neighborhood base for \( E \), for \( E \in \mathcal{E} \). Therefore we can find for each \( E \in \mathcal{E} \) a \( U(E) \in \mathcal{U} \) such that

\[
E \subseteq U(E) \text{ for } E \in \mathcal{E};
\]

for all \( E \in \mathcal{E} \) and \( A \in \mathcal{A} \), if \( E \cap A = \emptyset \) then \( U(E) \cap A = \emptyset \); and

for any two \( E, E' \in \mathcal{E} \), if \( E \cap E' = \emptyset \) then \( U(E) \cap U(E') = \emptyset \); and

\[
\text{diam}(U(E)) \leq 2^{-n} \text{ for } E \in \mathcal{E}.
\]

Then \( \mathcal{A}_n = \{ U(E) : E \in \mathcal{E} \} \) is as required, as one can easily check.

We assume that \( X \in \mathcal{A} \); alternatively accept the convention that \( \emptyset = X \).

Proof of the claim. The idea is to take a closed cover \( \mathcal{F} \), consisting of sufficiently small sets, and then to enlarge each \( F \in \mathcal{F} \) to \( E(F) \) in such a way that

for all \( \mathcal{A} \subseteq \mathcal{A} \) and \( \mathcal{A}' \subseteq \mathcal{F} \), if \( \mathcal{A} \cup \mathcal{A}' \) is linked then \( \mathcal{A} \cup \{ E(F) : F \in \mathcal{A}' \} \) has no nonempty intersection,

making sure that the enlarging does not cause more families to become linked. In other words, we also will require

for all \( A \in \mathcal{A} \) and \( F, F' \in \mathcal{F} \), if \( A \cap F = \emptyset \text{ then } A \cap E(F) = \emptyset \), and if \( F \cap F' = \emptyset \text{ then } E(F) \cap E(F') = \emptyset \).

For \( x \in X \) we define \( \mathcal{A}_x \subseteq \mathcal{A} \) and \( \delta(x) > 0 \) by

\[
\mathcal{A}_x = \{ A \in \mathcal{A} : x \notin \overline{A} \};
\]

\[
S(x) = d(x, \bigcup \mathcal{A}_x), \text{ where } d(x, \emptyset) = 1.
\]

Step 1. We construct a finite closed cover.

For each \( x \in X \) define a neighborhood \( N_x \) of \( x \) by

\[
N_x = \begin{cases} \{x\} \quad & \text{if } x \text{ is isolated}; \\ S(x, 2^{-n-2}) \cap S(x, \frac{1}{2}\delta(x)) \quad & \text{if } x \text{ is not isolated.} \end{cases}
\]

Since \( X \) is compact, there is a finite \( Y \subseteq X \) such that \( \bigcup \{ N_y : y \in Y \} = X \). For each \( y \in Y \) choose a neighborhood \( I_y \subseteq N_x \) such that \( I_y \cap I_{y'} = \emptyset \) for distinct \( y, y' \in Y \). For \( y \in Y \) define

\[
F_y = N_y - \bigcup \{ I_y' : y' \in Y, y' \neq y \}.
\]

One can easily check the following facts:

(a) \( \bigcup \{ F_y : y \in Y \} = X \);

(b) \( I_y \subseteq F_y \) for \( y \in Y \);

(c) \( I_y \cap I_{y'} = \emptyset \) for distinct \( y, y' \in Y \);

(d) \( F_y \supseteq S(y, 2^{-n-2}) \cap S(y, \frac{1}{2}\delta(x)) \) for \( y \in Y \) and

(e) \( F_y = \{ y \} \) if \( y \) is isolated.

Remark. The \( I_y \)'s will be used when we enlarge the \( F_y \)'s.

\begin{fact}
Fact 1. For all \( y, y' \in Y \), if \( F_y \cap F_{y'} \neq \emptyset \text{ then } \mathcal{A}_y \subseteq \mathcal{A}_{y'} \) or \( \mathcal{A}_{y'} \subseteq \mathcal{A}_y \).

Proof of Fact 1. Let \( y, y' \in Y \) be arbitrary, and suppose there are \( A \in \mathcal{A}_y - \mathcal{A}_{y'} \) and \( A' \in \mathcal{A}_{y'} - \mathcal{A}_y \). Then \( y \notin A \) but \( y' \in A \) hence \( d(y, y') > d(y, A) \geq d(y, \bigcup A_y) = \delta(y) \). Similarly, \( d(y, y') > d(y, A) \geq d(y, \bigcup A_y) = \delta(y) \). Therefore (d) implies that

\[
F_y \cap F_{y'} \subseteq S(y, \frac{1}{2}d(y, y')) \cap S(y, \frac{1}{2}d(y, y')) = \emptyset.
\]

Step 2. We enlarge the \( F_y \)'s.

Since any linked family that contains a singleton set has nonempty intersection, we only have to enlarge \( F_y \) if \( y \) is not isolated, because of (e). Define \( Z \) and \( \mathcal{L} \) by

\[
Z = \{ y \in Y : y \text{ is not isolated} \};
\]

\[
\mathcal{L} = \{ L \subseteq Z : \{ F_y : y \in L \} \text{ is linked} \}.
\]

Since each \( L \in \mathcal{L} \) is finite, there is by Fact 1 for each \( L \in \mathcal{L} \) an \( a(L) \in L \) with \( \mathcal{A}_y \subseteq \mathcal{A}_{a(L)} \) for all \( y \in L \).

Since \( \mathcal{A} \) is a finite subset of \( \mathcal{U} \), it follows from (B) that

\[
\{ a(L) : L \in \mathcal{L} \} \subseteq \{ A \subseteq X : A \text{ is a neighborhood of a single point} \}
\]

for all \( L \in \mathcal{L} \). Hence we can pick \( p_L \) for \( L \in \mathcal{L} \) in such a way that

(i) \( p_L \in (I \cup \mathcal{A}) \cap \{ A \in \mathcal{A} : L \subseteq \overline{A} \} \); \#\( \mathcal{A}(\mathcal{A}) \),

(g) \( p_L \neq p_{L'} \) for distinct \( L, L' \in \mathcal{L} \),

for \( Z \) is finite, and for each \( L \in \mathcal{L} \) the set \( I(L) \) is a neighborhood of the (non-isolated) point \( a(L) \), and \( \mathcal{A}(\mathcal{A}) \) is a finite collection of closed sets none of which contains \( a(L) \).

For each \( y \in Y \) define an enlargement \( E_y \) of \( F_y \) by

\[
E_y = \begin{cases} \{ y \} \quad & \text{if } y \text{ is isolated}; \\ F_y \cup \{ p_L : x \in L \} \quad & \text{if } y \text{ is not isolated.} \end{cases}
\]

Step 3. Proof that this works.

We first note that (b), (d) and the definition of \( \delta \) imply

\[
(\ast) \quad y \in A \text{ if and only if } F_y \cap A \neq \emptyset \text{ if } F_y \cap A \neq \emptyset \text{ for all } y \in Y \text{ and } A \in \mathcal{A}.
\]

\begin{fact}
Fact 2. For all \( \mathcal{A} \subseteq \mathcal{A} \) and \( L \subseteq Z \), if \( \mathcal{A} \cup \{ F_y : y \in L \} \) is linked, then \( \mathcal{A} \) is linked.

We may assume that \( \mathcal{A} \neq \emptyset \) since \( X \in \mathcal{A} \), and that \( L \neq \emptyset \) since our hypothesis about \( \mathcal{A} \) tells us that \( \mathcal{A} \neq \emptyset \). We also may assume that \( L \subseteq Z \), the set of nonisolated points in \( Y \), since \( F_y = E_y = \{ y \} \) for \( y \in Y \).

Hence \( L \in \mathcal{L} \), so our construction guarantees that \( p_L \in E_y \) for all \( y \in L \), and that \( p_L \in A \) for all \( A \in \mathcal{A} \) since (\ast) implies that \( L \subseteq \bigcup \mathcal{A} \).

So we did enlarge the \( F_y \)'s enough. It remains to show that we did not enlarge the \( F_y \)'s too much.
Fact 3. For all $A \in \mathcal{A}$ and $y \in Y$, if $A \cap F_y = \emptyset$ then $\bar{A} \cap E_y = \emptyset$.

First observe that $y \notin \bar{A}$ because of (a), hence $A \in \mathcal{A}_y$. So if $L \in \mathcal{L}$ contains $y$, then $\alpha(L) \notin \bar{A}$ since $\mathcal{A}_y \subseteq \mathcal{A}_{\alpha(L)}$, hence then $p_L \notin \bar{A}$. Since $F_y \cap \bar{A} = \emptyset$ because of (a), it follows that $\bar{A} \cap E_y = \emptyset$, both for isolated $y$ and nonisolated $y$.

Fact 4. For all $y, y' \in Y$, if $F_y \cap F_{y'} = \emptyset$ then $E_y \cap E_{y'} = \emptyset$.

If $z \in Z$ is isolated then $E_z = I_z = \{z\}$, and if $z \in Z$ is not isolated then $E_z \subseteq F_{z'} \cup \{I_z: z' \in Z \text{ not isolated}\}$, hence if one of $y$ and $y'$ is isolated, then $E_y \cap E_{y'} = \emptyset$. Next assume that neither $y$ nor $y'$ is isolated. If $L \in \mathcal{L}$ contains $y$ and $L' \in \mathcal{L}$ contains $y'$, then clearly

$$y \notin L' \quad \text{and} \quad y' \notin L.$$ 

Hence $p_L \neq p_{L'}$ by (g); and $\alpha(L) \neq y$ and $\alpha(L') \neq y$, hence $p_L \notin F_{y'}$ and $p_{L'} \notin F_y$ by (c) since $p_L \in I_{\alpha(L)}$ and $p_{L'} \in I_{\alpha(L')}$. Consequently $E_y \cap E_{y'} = \emptyset$.

Fact 5. $\text{diam}(E_y) \leq 2^{-n}$ for $y \in Y$.

If $y$ is isolated, $\text{diam}(E_y) = \text{diam}((y)) = 0$. If $y$ is not isolated, then for all $L \in \mathcal{L}$, if $y \in L$ then $p_L \notin F_{\alpha(L)}$ and $F_{\alpha(L)} \cap F_y = \emptyset$, so $d(\alpha(L), y) < 2^{-n-1}$ by (d), hence $E_y \subseteq S(y, 2^{-n-1})$, again by (d). Therefore $\text{diam}(E_y) < 2^{-n}$ since $E_y$ is compact.

It now follows from (a) and Facts 1 through 5 that $\mathcal{E} = \{E_y: y \in Y\}$ has all properties required.

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