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On normality and countable paracompactness

by

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Abstract. The purpose of this paper is to present certain construction techniques developed by the author in the study of the relationship between normality and countable paracompactness. Based on these techniques, the following results are obtained: (1) (MA+¬CH) There exists a countably paracompact, separable, non-normal Moore space. (2) (MA+¬CH) There exists a countably paracompact, screenable, non-normal Moore space. (3) There exists a pseudo-normal, separable Moore space which is not countably paracompact. (4) There exists a perfect, continuous map from a screenable Moore space onto a non-screenable Moore space. (5) (V = L) Each normal, first countable T_2-space in which each subset is an F_σ-set is σ-discrete. These results were obtained by the author in the period 1974–75, and all have been previously announced in the literature and presented at various mathematical meetings (see [R_5], [R_6], [R_7], [WFR], and [Ru_2]).

Motivation. It was established in 1951 by Dowker [D] that in perfect spaces (i.e. spaces in which closed sets are G_δ-sets), normality implies countable paracompactness. The extreme usefullness of this result led quite naturally to the search for (i) a Dowker space, a normal T_2-space which is not countably paracompact, and (ii) a countably paracompact, perfect, non-normal T_3-space. In particular, the relationships between normality, countable paracompactness, and pseudo-normality in Moore spaces have been of considerable interest ([Y], [Z], [P], [R_3], [T_2], and [K], for examples). Dowker spaces were finally constructed by M. E. Rudin in [Ru_1]. More recently, Wage in [W_1] developed an elegant construction technique which produced an example of a countably paracompact, perfect, non-normal T_3-space, and, under (Ma+¬CH), an example of a countably paracompact, non-normal Moore space. Furthermore, in [F_2], Fleissner showed that, under (CH), each coun-countably paracompact, separable Moore space is metrizable. In this paper, the author presents a basic construction technique that was derived from the considera-tion of Wage's in [W_1], but which is remarkable for its simplicity. Results (1)–(4) above are based on this technique. Note that (1) together with Fleissner's result from [F_2] establishes the consistency and independence w.r.t. ZFC of the existence of a countably paracompact, separable, non-normal Moore space. Result (2) answers a question often raised by F. D. Tall and contradicts an incorrect argument to the contrary in [G]. Results (3) and (4) answer questions raised by Proctor in [P] and Hodel in [Ho], respectively. Result (5) is based on Fleissner's theorem from [F_1] that, under (V = L), each normal T_2-space of character $\leqslant c$ is collectionwise Hausdorff. By use of this theorem and a construction technique similar to those developed in [R_4], the author establishes the consistency and independence w.r.t. ZFC of the existence of a normal, first countable T_2-space which is not σ-discrete but in which each subset is an F_σ-set. The consistency of such spaces, discovered by Silver and Tall in [T_1], has been crucial to recent work on normality and countable paracompactness.

Preliminaries. The following notation will remain consistent throughout the paper. Let I denote the non-negative integers, N the positive integers, X the x-axis, U the upper plane, and K a countable, dense subset of U. For each $p \in X$ and $n \in N$, let $R_n(p)$ denote the set consisting of p together with the interior of a circle of radius $1/n$ lying in U and tangent to X at p. If $H \subset X$, let T_H denote the tangent disk space over H, i.e. the usual topology on U and neighborhoods in the form of $R_n(p)$ for each $p \in H$ and $n \in N$. From [J] and [Ku], there exists an uncountable subset L of X such that each countable subset of L is a G_δ-set relative to L, and from [T_1] under (MA+$\neg$CH), there exists an uncountable subset Q of X such that each subset of Q is a G_δ-set relative to Q. It was noted in [B] that T_Q is a normal, separable, non-metrizable Moore space. Finally, if $\mathscr{C} = \{H_\alpha\}_{\alpha \in A}$ is a collection of copies of the set H, let π denote the function which assigns to each element in the union of $\mathscr{C}$ the corresponding element of H, and, for each α, let π_α denote π restricted to H_α.

Example 1. (MA+$\neg$CH) There exists a countably paracompact, separable Moore space which is not normal.

Construction. Let Q_1 and Q_2 denote disjoint copies of Q such that for each $q = (x, 0) \in Q$, $\pi_1^{-1}(q) = q_1 = (x_1, 0) \in Q_1$ and $\pi_1^{-1}(q) = q_2 = (x_2, 0) \in Q_2$. Let $S_1 = Q_1 \cup Q_2 \cup K$ and define a base $\mathscr{B}$ for the desired topology on S_1 as follows:

(1) if $k \in K$, let $\{k\} \in \mathscr{B}$; and

(2) if $q_i = (x_i, 0) \in Q$; for $i = 1$ or $i = 2$ and $n \in N$, let

$$\{q_i\} \cup (\{(a, b) \in K\colon x \neq a \text{ and } i|x-a|<b<(i+1)|x-a|\} \cap R_n(q)) \in \mathscr{B}.$$

(Basic open sets for $q_1 \in Q_1$ and $q_2 \in Q_2$ in S_1 are non-intersecting "butterfly neighborhoods" which, except for q_1 and q_2, are contained in the tangent disk neighborhoods for q in T_Q). By comparison with the normal, separable Moore space T_Q, it follows that S_1 is a countably paracompact, separable Moore space which is not normal. To see that S_1 is countably paracompact, suppose that $\{H_n\}$ is a decreasing sequence of closed subsets of $Q_1 \cup Q_2$ with empty intersection. Since T_Q is countably paracompact, there exists a sequence $\{D_n\}$ of open sets in T_Q such that for each n, $\pi(H_n) \subset D_n$ and $\bigcap \overline{D}_n = \emptyset$. By the construction of S_1, $\{\pi^{-1}(D_n)\}$ is a sequence of open sets in S_1 such that for each n, $H_n \subset \pi^{-1}(D_n)$ and $\bigcap \overline{\{\pi^{-1}(D_n)\}} = \emptyset$ in S_1. Hence S_1 is countably paracompact. To see that S_1 is not normal, it suffices to note that, since each uncountable subset of Q contains a limit point of itself w.r.t. the topology of the real line, the union of a collection of $n+h$ basic open sets for an uncountable subset of Q_1, must always contain a limit point in Q_2. Hence, Q_1 and Q_2 can not be separated by open sets in S_1.

Example 2. (MA+$\neg$CH) There exists a countably paracompact, screenable Moore space which is not normal.

Construction. Let Q_1 and Q_2 be as in Example 1. Let $S_2 = Q_1 \cup Q_2 \cup U$ and define a base $\mathscr{B}$ for the desired topology on S_2 as follows:

(1) if $u \in U$, let $\{u\} \in \mathscr{B}$;

(2) if $q_1 = (x_1, 0) \in Q_1$ and $n \in N$, let

$$\{q_1\} \cup (\{(a, b) \in U\colon x<a \text{ and } b = a-x\} \cap R_n(q)) \in \mathscr{B};$$

and

(3) if $q_2 = (x_2, 0) \in Q_2$, let

$$\{q_2\} \cup (\{(a, b) \in U\colon a<x \text{ and } b = x-a\} \cap R_n(q)) \in \mathscr{B}.$$

(The author thinks of the space S_2 as a splitting of Heath's "V-space" in [H].) As in Example 1, by comparison with the normal Moore space T_Q, S_2 is seen to be a countably paracompact, non-normal Moore space. Obviously, each open covering of S_2 can be refined by two collections of pairwise disjoint open sets whose union covers S_2. Hence S_2 is screenable.

Pseudo-normality. Proctor in [P] defined a space to be pseudo-normal provided each two disjoint closed sets, one of which is countable, can be separable by open sets. Proctor also noted that countably paracompact T_3-spaces are pseudo-normal and gave an example of a pseudo-normal, separable, non-metrizable Moore space. Another such space, in fact T_L, was given by Tall in [T_2]. However, to show the non-countable paracompactness of these spaces, one must assume the Continuum Hypothesis. In particular, under (MA+$\neg$CH), T_L is perfectly normal. An absolute example of a pseudo-normal, non-countably paracompact, non-separable Moore space of card. $= \aleph_1$ was given by the author in [R_3]. The space given in Example 3 below is the first such separable space to be constructed, and it answers Proctor's question from [P]. Recently, van Douwen in [vD] has obtained the unexpected result that, under (MA), each Moore space of card. $<c$ is pseudo-normal.

Example 3. There exists a pseudo-normal, separable Moore space that is not countably paracompact.

Construction. Let L be as mentioned above. For each $i \in N$, denote by L_i a copy of L such that if $i \neq j$, $L_i \cap L_j = \emptyset$ and such that for each $p = (x, 0) \in L$, $\pi_i^{-1}(p) = p_i = (x_i, 0) \in L_i$. Let $S_3 = (\bigcup_{i=1}^{\infty} L_i) \cup K$ and define a base $\mathscr{B}$ for the desired topology on S_3 as follows:

(1) if $k \in K$, let $\{k\} \in \mathscr{B}$; and

(2) if $p_i = (x_i, 0) \in L_i$ and $n \in N$, let

$$\{p_i\} \cup (\{(a, b) \in K\colon x \neq a \text{ and } i|x-a|<b<(i+1)|x-a|\} \cap R_n(p)) \in \mathscr{B}.$$

(Note that S_3 is similar to S_1 of Example 1, except here the subset of the x-axis is split infinitely many times.) By comparison with the pseudo-normal, separable

Moore space T_L, S_3 is easily seen to be a separable Moore space. To see that S_3 is also pseudo-normal, suppose that H and M are disjoint closed subsets of $\bigcup_{i=1}^{\infty} L_i$ and and that M is countable. Let $H' = \{h \in H| \ \pi(h) \notin \pi(M)\}$. Since T_L is pseudo-normal, there exist disjoint open sets in T_L separating $\pi(H')$ and $\pi(M)$. Hence, by the construction of S_3, there exist disjoint open sets in S_3 separating H' and M. Furthermore, since H/H' is countable, there must also exist disjoint open sets in S_3 separating H and M. To see that S_3 is not countably paracompact, consider the decreasing sequence $\{H_n\}$ of closed sets in S_3 with empty intersection such that for each n, $H_n = \bigcup_{i=n}^{\infty} L_i$. Suppose that $\{D_n\}$ is a sequence of open sets in S_3 such that for each n, $H_n \subset D_n$. Since each uncountable subset of L contains a limit point of itself w.r.t. the topology of the real line, if follows from the construction of S_3 that, for each n, $L_1 \setminus (\bar{D}_n \cap L_1)$ is countable. Thus $\bigcap_{n=1}^{\infty} \bar{D}_n \neq \emptyset$ and S_3 is not countably para-compact.

EXAMPLE 4. There exists a perfect, map from a screenable Moore space Z onto a non-screenable Moore space V.

Construction. Let V denote Heath's "V-space" defined on the x-axis X and the upper plane U, i.e., $V = X \cup U$ and a base $\mathcal{B}_v$ for V is defined as follows:

(1) if $u \in U$, the $\{u\} \in \mathcal{B}_v$; and

(2) if $x = (x, 0) \in X$, then

$$\{x\} \cup \{(a, b) \in U\colon (x-1/n<a<x \text{ and } b = x-a) \text{ or } (x<a<x+1/n \text{ and } b = a-x)\} \in \mathcal{B}_v .$$

The space V was given in [H] as an example of a metacompact, non-screenable Moore space. Let X_1 and X_2 denote disjoint copies of X such that if $(x, 0) \in X$ then $(x_1, 0)$ and $(x_2, 0)$ denote the corresponding elements in X_1 and X_2, respectively. Let $Z = X_1 \cup X_2 \cup U$ and define a base $\mathcal{B}_Z$ for Z as follows:

(1) if $u \in U$, then $\{u\} \in \mathcal{B}_Z$; and

(2) if $x_1 = (x_1, 0) \in X$, and $n \in N$, then

$$\{x_1\} \cup \{(a, b) \in U\colon x-1/n<a<x \text{ and } b = x-a\} \in \mathcal{B}_Z;$$

and

(3) if $x_2 = (x_2, 0) \in X_2$ and $n \in N$, then

$$\{x_2\} \cup \{(a, b) \in U\colon x<a<x+1/n \text{ and } b = a-x\} \in \mathcal{B}_Z.$$

(Note that Z is simply the screenable space of Example 2 defined over the entire x-axis.) It follows immediately that the map which assigns to each element of Z the corresponding element in V is perfect.

Remarks on Example 4. In perfect spaces, screenability lies properly between paracompactness and metacompactness, both of which are preserved by closed

mappings. Hence, it is somewhat surprising to the author that screenability in perfect spaces fails to be preserved even under perfect mappings. Observe that, under (MA + ¬CH), the mapping construction in Example 4 defined on Q, as in Example 2, yields a perfect map from a countably paracompact, screenable Moore space onto a non-screenable Moore space. As noted above, Example 4 answers a question raised by Hodel in [Ho].

THEOREM 5. (V = L) *Each normal, first countable T_2-space Y in which each subset is an F_σ-set is σ-discrete.*

Proof. The outline of the proof is

(1) to construct a normal T_2-space S of character $\leqslant c$ over Y by a technique similar to those developed by the author in [R_4],

(2) to use Fleissner's result from [F_1] that, under (V = L), such spaces are collectionwise Hausdorff, and

(3) to show that S being collectionwise Hausdorff implies that Y is σ-discrete.

Construction of S. For each $i \in I$, Let Y_i denote a copy of Y such that, if $i \neq j$, $Y_i \cap Y_j = \emptyset$. Let $S = \bigcup_{i=0}^{\infty} Y_i$, and define a base $\mathcal{B}$ for the desired topology on S as follows:

(1) if $p \in Y_i$ and $i \neq 0$, let $\{p\} \in \mathcal{B}$; and

(2) if $p \in Y_0$, $n \in N$, and $\{G_i(\pi(p))\}_{i=n}^{\infty}$ is a sequence of open sets in Y each containing $\pi(p)$, then let $\{p\} \cup \bigcup_{i=n}^{\infty} \pi_i^{-1}(G_i(\pi(p))) \in \mathcal{B}$.

It follows immediately that S is a T_2-space of character $\leqslant c$. To see that S is normal, suppose that H and M are disjoint closed subsets of Y_0. Since each subset of Y is and F_σ-set in Y, $\pi(H) = \bigcup_{n=1}^{\infty} H_n$ and $\pi(M) = \bigcup_{n=1}^{\infty} M_n$ where for each n, $H_n \subset H_{n+1}$, $M_n \subset M_{n+1}$, and each of H_n and M_n is closed in Y. Now, since Y is normal, for each n there exist disjoint open sets $D(H_n)$ and $D(M_n)$ in Y containing H_n and M_n, respectively. By the construction of S, $D(H) = H \cup \bigcup_{n=1}^{\infty} \pi_n^{-1}(D(H_n))$ and $D(M) = M \cup \bigcup_{n=1}^{\infty} \pi_n^{-1}(D(M_n))$ are disjoint open sets in S containing H and M, respectively. Hence, S is normal.

Y is σ-discrete. From [F_1], under (V = L), S is collectionwise Hausdorff. Thus, there exists a collection $\mathcal{G}$ of pairwise disjoint open sets in S covering Y_0 such that each element of $\mathcal{G}$ contains only one point of Y_0. For each $n \in N$, let

$$\mathcal{G}_n = \{G \in \mathcal{G}\colon G \cap Y_n \neq \emptyset\}, \quad \mathcal{D}_n = \{\pi(G \cap Y_n)\colon G \in \mathcal{G}_n\},$$

and

$$Y(n) = \{y \in Y\colon \pi_0^{-1}(y) \text{ is contained in an element of } \mathcal{G}_n\}.$$

Observe that $Y = \bigcup_{n=1}^{\infty} Y(n)$ and, for each n, $\mathcal{D}_n$ is a collection of pairwise disjoint open sets in Y covering $Y(n)$ such that each element of $\mathcal{D}_n$ contains only one point of $Y(n)$. Hence, Y is σ-discrete.

COROLLARY. *Suppose that S is a normal, first countable T_2-space in which each subset is an F_σ-set, then it is independent of an consistent with* ZFC *that S is metrizable.*

Proof. Assume (V = L). From [F_1] and Theorem 5, S is collectionwise Hausdorff and σ-discrete. Hence from [R_1], S is metrizable.

Assume (MA + $\neg$CH). T_Q is a non-metrizable space with the desired properties. In fact, the author proved in [R_8] that, under (MA + $\neg$CH), each normal, non-metrizable Moore space of card. $< c$ would suffice.

QUESTIONS.

(1) Does there exist a countably paracompact, non-normal Moore space in ZFC? From Wage's construction in [W_1], it follows that the existence of a normal, non-metrizable Moore space implies the existence of a countably paracompact, non-normal Moore space. Is the converse true?

Under (CH), is each countably paracompact, screenable Moore space metrizable?

Does there exist a countably paracompact, perfect, first countable, non-normal T_3-space in ZFC?

See [W_2] for a discussion of these and related questions.

(2) In a lecture on normality and countable paracompactness which he gave at the 1975 Memphis State University Topology Conference, the author defined the concepts of a Δ-set and star-paracompactness and raised the following still unanswered questions. A *Δ-set* is an uncountable subset D of the real line such that for every decreasing sequence $\{H_n\}$ of its subsets with empty intersection, there exists a sequence $\{V_n\}$ of G_δ-sets w.r.t. D with empty intersection such that for each n, $H_n \subset V_n$. The author noted that T_H is countably paracompact if and only if H is a Δ-set. Does there exist a model of set theory in which there exists a Δ-set but not a Q-set? That is, does there exist a tangent disk space that is countably paracompact but not normal? An answer to these questions would be helpful in deciding if (CH) in Fleissner's theorem [F_2] can be replaced with ($2^{\aleph_0} < 2^{\aleph_1}$), as is the case with the metrizability of normal, separable Moore spaces [J]. E. K. van Douwen pointed out to the author that in the definition of a Δ-set, the term "G_δ-sets" can be replaced with "open sets". Furthermore, Przymusiński in [Prz_1] has improved the author's original observation in the context of tangent disk spaces by showing that the existence of a Δ-set is actually equivalent to the existence of a countably paracompact, separable, non-normal Moore space. A space S is said to be *star-paracompact* provided if $\mathcal{G}$ is a collection of open sets covering S, then there exists a locally finite collection $\mathcal{F}$ of open sets covering S such that $\mathcal{F}$ refines $\{\mathrm{st}(p, \mathcal{G}) \mid p \in S\}$. The author has previously considered the similarly defined concepts of star-screenability and star-strong screenability in [R_2]. Observe that a countably paracompact, separable space is star-paracompact. Suppose S is a star-paracompact Moore space, must S be either countably paracompact or star-strongly screenable? Does there exist a star-paracompact, non-normal Moore space in ZFC. The author finds these questions interesting in view of the following easily verified results: (1) there exists a star-strongly screenable Moore space that is not star-paracompact; and (2) under (MA + $\neg$CH), there exist (i) a normal, star-paracompact, non-metrizable Moore space, (ii) a normal, not star-paracompact Moore space, and (iii) a star-paracompact, non-normal Moore space. The apparent difficulty of the above questions on star-paracompactness is witnessed by the example given by Przymusiński in [Prz_2] of a Moore space with a locally finite open cover which can not be refined by a σ-discrete open cover.

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