PROBLEMS

1. Does there exist a "real" example of a space X such that X^2 is perfect (or, better, perfectly normal) but X^3 is not? Can it have cardinality ω_1?

2. Does there exist a consistent example of a Moore space X such that X^2 is normal but X^3 is not?

3. Suppose X is generalized ordered and X^2 is perfect. Is X^ω perfect?

4. Suppose X^2 is perfectly normal. Is X submetrizable?

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Prolongationally stable discrete flows

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Abstract. Our objective in this paper is to obtain properties for certain homeomorphisms of Hausdorff spaces. In particular we extend the conclusions obtained by Knight in [7] for continuous flows of characteristic 0 on Hausdorff phase spaces to discrete flows.

1. Introduction and preliminaries. Flows of characteristic 0 from an important subclass of closed flows satisfying certain bilateral stability criteria. Ahmad introduced continuous flows of characteristics 0^+, 0^-, and $0^\pm$ in [1] and Knight introduced continuous flows of characteristic 0 in [5]. Classifications and characterizations of such flows were given in [1], [2], [5], [6], [7], and [8]. Lam made some observations about discrete and continuous flows containing subflows of characteristic $0^\pm$ in [9].

In this paper we extend the results of [7] to discrete flows. Much of the reasoning used in [7] yields similar discrete flow properties, however, several of the techniques employed either do not apply or directly apply here. Virtually all continuous flow results of [7] extend to discrete flows. In Section 2 we analyze discrete flows of characteristic 0 and in Section 3 we give characterizations of such flows.

The standard terminology, notations, and definitions used in the references cited above are used in this paper. The sets of integers, nonnegative integers, and nonpositive integers are denoted by Z, Z^+, and Z^-, respectively. By $C(x)$, $K(x)$, $L(x)$, $D(x)$, and $J(x)$ we mean the orbit, orbit closure, limit set, prolongation, and prolongational limit set of x, respectively. The corresponding positive and negative concepts carry the appropriate + or − superscript. The statements $y \in J^+(x)$ implies $x \in J^-(y)$, $y \in L^+(x)$ implies $x \in J^+(y)$, and $y \in K(x)$ implies $J^+(x) \subset J^+(y)$ are well known tools for continuous flows which are easily demonstrated for discrete flows.

A discrete or continuous flow (X, π) is said to be of *characteristic* 0^+ (0^-) if $D^+(x) = K^+(x)$ $(D^-(x) = K^-(x))$ for each point x in X, or equivalently, if $J^+(x) = L^+(x)$ $(J^-(x) = L^-(x))$ for each point x in X. A flow of characteristics 0^+ and 0^- is of characteristic $0^\pm$. We say that (X, π) is of *characteristic* 0 provided $D(x) = K(x)$ for each point x in X.

2. Discrete flows of characteristic 0. Throughout this section we shall consider a discrete flow (X, π) on a Hausdorff space X. The next three propositions are a result of applying the proofs for the corresponding propositions given in [7] for continuous flows to discrete flows.

PROPOSITION 1. *If (X, π) is of characteristic 0, then any restriction to an invariant subset of X is of characteristic 0.*

PROPOSITION 2. *The following statements are equivalent.*

(a) *(X, π) is of characteristic 0.*

(b) *$J^+(x) = J^-(x) = \emptyset$ or $K(x)$ for each $x \in X$.*

(c) *$J^+(x) = J^-(x) = C(x)$ or $L(x)$ for each $x \in X$.*

PROPOSITION 3. *Let (X, π) be of characteristic 0. If $L^+(x) \neq \emptyset$ $(L^-(x) \neq \emptyset)$ for some $x \in X$, then $J^+(x) = J^-(x) = L^+(x)$ $(J^+(x) = J^-(x) = L^-(x))$.*

COROLLARY 3.1. *Let (X, π) be of characteristic 0. If $J^+(x) \neq \emptyset$ for some $x \in X$, then*

$$J(x) = J^+(x) = J^-(x) = D(x) = D^+(x) = D^-(x) = K(x)\,,$$
$$K(x) = K^+(x) = L(x) = L^+(x) \text{ if } L^+(x) \neq \emptyset, \text{ and}$$
$$K(x) = K^-(x) = L(x) = L^-(x) \text{ if } L^-(x) \neq \emptyset.$$

PROPOSITION 4. *If (X, π) is of characteristic 0, then $L^+(x)$ $(L^-(x))$ is minimal for each $x \in X$.*

Proof. For $L^+(x) = \emptyset$ the result is trivial. Let $y \in L^+(x)$ for some $x \in X$. Then

$$K(y) \subset L^+(x) \subset J^+(x) \subset J^+(y) \subset D(y) = K(y)\,,$$

and hence, $L^+(x) = K(y)$. Thus, $L^+(x)$ contains no nonempty closed invariant proper subset.

COROLLARY 4.1. *Let (X, π) be of characteristic 0 and X be locally compact. If $L^+(x)$ $(L^-(x))$ is compact for some x in X, then $L^+(x)$ $(L^-(x))$ is positively and negatively minimal and each point of $L^+(x)$ $(L^-(x))$ is recurrent.*

Proof. The minimality properties follow easily. Recurrence follows from VI. 5 and VI. 6 p. 90 of [10].

PROPOSITION 5. *For a flow (X, π) of characteristic 0 with locally compact phase space the following statements are equivalent for $x \in X$.*

(a) *$L^+(x)$ is compact minimal.*

(b) *$L^+(x)$ consists of almost periodic points.*

(c) *$L^+(y) \neq \emptyset$ for each y in $L^+(x)$.*

(d) *$L^+(y) = L^+(x)$ for each y in $L^+(x)$.*

Proof. The equivalence of the statements follows trivially whenever $L^+(x) = \emptyset$ for $x \in X$. Let $L^+(x) \neq \emptyset$ for some $x \in X$. According to Proposition 2.5 of [4], (a) and (b) are equivalent. It is easy to see that (a) implies (c). If (c) holds, then for $y \in L^+(x)$ we have $x \in J^-(y)$. By Proposition 3, $x \in L^+(y)$, and hence, $L^+(y) = L^+(x)$.

Finally, if (d) holds, then $x \in L^+(y)$ for each $y \in L^+(x)$. Hence, $L^+(x) \subset A_W^+(x)$ (the region of weak attraction for x) and $L^+(x)$ is minimal. The compactness of $L^+(x)$ is all that remains to be shown.

In order to show that $L^+(x)$ is compact we consider the restriction (Y, σ) of (X, π) to the locally compact subspace $L^+(x)$. If Y is the only compact neighborhood of x in Y, we are done. Let $V \subset Y$ be an open neighborhood of x in Y with compact closure $\bar{V} \neq Y$. Each orbit in Y is frequently in V and $Y - \bar{V}$ in view of condition (d). For each $y \in V$ we define $t_y = \min\{t \in Z^+ \colon yt \in V \text{ and } y(t-k) \notin \bar{V} \text{ for some } k, 0<k<t\}$. For $y \notin V$ define $t_y = t_z$ where z is the first point of $C^+(y)$ in V. By the continuity of σ we have for each $y \in V$ an open neighborhood $V(y)$ of y in V and an integer $t \in Z^+$ such that $0<t<t_y$, $V(y)t \subset Y - \bar{V}$, and $V(y)t_y \subset V$. Thus, for each $z \in V(y)$, $t_z \leqslant t_y$. For any $y \in \partial V$ there is a least integer $\tau(y) \in Z^+$ such that $y\tau(y) \in V$. Choose an open neighborhood $V(y)$ of y such that $V(y)\tau(y) \subset V(y\tau(y))$. Then for each $z \in V(y)$ with $y \in \partial V$, $z\tau(y) \in V(y\tau(y))$ so that $t_z < t_{y\tau(y)}$. Owing to its compactness there is a finite cover $\{V(y_1), \ldots, V(y_n)\}$ for $\bar{V}$. Now for $y_k \in \partial V$ define $Q_k = \tau(y_k) + t_{y_k\tau(y_k)}$ and for $y_k \in V$ define $Q_k = t_{y_k}$, $1 \leqslant k \leqslant n$. For any $y \in \bar{V}$ we have $t_y \leqslant Q = \max\{Q_k \colon 1 \leqslant k \leqslant n\}$. For each $p \in \bar{V}Z^+ - \bar{V}$ there are minimum and maximum integers $\tau(p) \in Z^+$ and $T(p) \in Z^-$, respectively, such that $p\tau(p) \in V$ and $pT(p) \in V$. Now

$$[pT(p)][\tau(p) - T(p)] = p\tau(p) \in V$$

implies that $\tau(p) - T(p) \leqslant Q$. Thus, $p \in \bar{V}[0, Q]$, and we have, $\bar{V}Z^+ \subset \bar{V}[0, Q]$. Since $C^+(x)$ is a subset of the compact set $\bar{V}[0, Q]$, $L^+(x) \subset \bar{V}[0, Q]$, whence, $L^+(x)$ is compact. This completes the proof.

COROLLARY 5.1. *If (X, π) is of characteristic 0 with locally compact phase space and $L^+(x)$ is a nonempty compact minimal set for some $x \in X$, then $L(x) = L^\pm(x) = K(x) = K^\pm(x) = J(x) = J^\pm(x) = D(x) = D^\pm(x)$.*

COROLLARY 5.2. *Let (X, π) be of characteristic 0 with locally compact phase space and let $L^+(x)$ be a nonempty compact minimal set for each $x \in X$. Then (X, π) is of characteristics 0^+, 0^-, and $0^\pm$.*

We introduce the following notation for convenience. For a flow (X, π) we let

$$M_1 = \{x \colon L^+(x) \neq \emptyset \text{ and } L^-(x) = \emptyset\}\,,$$
$$M_2 = \{x \colon L^+(x) = \emptyset \text{ and } L^-(x) \neq \emptyset\}\,,$$
$$M_3 = \{x \colon L(x) = \emptyset \text{ and } J^+(x) \neq \emptyset\}, \text{ and}$$
$$M_4 = \{x \colon L^+(x) = L^-(x) = J^+(x)\}\,.$$

The next theorem follows by the reasoning given for the proof of Theorem 6 of [7].

THEOREM 6. *Let (X, π) be of characteristic 0. The collection*

$$\{M_i \colon 1 \leqslant i \leqslant 4 \text{ and } M_i \neq \emptyset\}$$

is a partition of X. The restriction of the flow to

(a) $M_1 \cup M_4$ *is of characteristics* 0 *and* 0^+, *and is of characteristics* 0^- *and* $0^\pm$ *if and only if* $M_1 = \emptyset$;

(b) $M_2 \cup M_4$ *is of characteristics* 0 *and* 0^-, *and is of characteristics* 0^+ *and* $0^\pm$ *if and only if* $M_2 = \emptyset$;

(c) M_4 *is of characteristics* 0 *and* $0^\pm$; *and*

(d) M_3 *is only of characteristic* 0 *provided it is not dispersive.*

COROLLARY 6.1. *Let* (X, π) *be of characteristic* 0. *Then* (X, π) *is of characteristic* 0^+ (0^-) *if and only if* $M_2 = M_3 = \emptyset$ $(M_1 = M_3 = \emptyset)$. *Furthermore,* (X, π) *is of characteristic* $0^\pm$ *if and only if* $X = M_4$.

COROLLARY 6.2. *A flow* (X, π) *of characteristic* 0 *with locally compact phase space is of characteristic* $0^\pm$ *if and only if each nonwandering point is Lagrange stable.*

A flow (X, π) of characteristic 0 is of characteristic $0^\pm$ provided $X = M_4$. Lam noted in [9] that for X a locally compact metric space M_4 can be decomposed into the open and closed sets $\{x: J^+(x) = \emptyset\}$ and $\{x: x \text{ is almost periodic}\}$, respectively. In view of Proposition 5 this decomposition holds for X a locally compact Hausdorff space. The set $F = M_1 \cup M_2 \cup M_3$ consists of the points which do not satisfy the characteristic $0^\pm$ condition. The following theorem indicates that $F \subset \overline{M}_4$ when X is a locally compact or a complete metric space. The results obtained by Lam in [9] apply here only when $F = \emptyset$, that is, only to flows of characteristic 0 which are also of characteristic $0^\pm$.

THEOREM 7. *Let* (X, π) *have a compact phase space. Then the characteristic* 0^+, 0^-, $0^\pm$, *and* 0 *properties are equivalent. In this case* $X = M_4$ *and* $K(x)$ *is compact minimal for each* $x \in X$.

Proof. A flow of characteristic $0^\pm$ is of all three other characteristics. Let (X, π) be of characteristic $0^\pm$. Then $L^\pm(x)$ is nonempty compact for each $x \in X$. For $y \in L^+(x)$ we have $L^+(y) \neq \emptyset$ so that $x \in J^+(y) = L^+(y) \subset L^+(x)$. Thus, $L^+(x) = L^+(y)$ and $L^+(x)$ is compact minimal and Poisson stable. The compact minimality of $L^+(x)$ implies $L^+(x) = L^-(x)$. If $p \in J^-(x)$, then $x \in J^+(p) = L^+(p)$. Also, if $L^+(p) \cap L^+(x) \neq \emptyset$, the minimality of $L^+(x)$ implies $L^+(p) = L^+(x)$. The Poisson stability of p follows as did that of x so that $p \in L^+(x) = L^-(x)$ implying $J^-(x) \subset L^-(x)$. Hence, $J^-(x) = L^-(x)$ for each $x \in X$. Thus, (X, π) is of characteristic 0^-. A dual argument yields a flow of characteristic 0^- to be of characteristic 0^+. Finally, if (X, π) is of characteristic 0, then by Proposition 5, $L^+(x)$ is a nonempty compact minimal set for each $x \in X$. Corollary 5.2 yields the desired result. This completes the proof.

THEOREM 8. *Let* (X, π) *be of characteristic* 0 *where* X *is metric and either locally compact or complete. Then* $X = \overline{M}_4$.

Proof. Let $Y = \{x \in X: J^+(x) \neq \emptyset\}$. Choose a sequence (x_i) in Y converging to a point x. Suppose that

$$x \notin J^+(x) = \bigcap \{D^+(xt): t \in Z^+\}.$$

Then there is an integer $t_0 > 0$ and a neighborhood V of xt_0 such that $x \notin D^+(xt_0)$ and $x \notin \overline{VZ^+}$. The sequence $(x_i t_0)$ is ultimately in V, and hence,

$$x_i \in J^+(x_i) \subset D^+(x_i t_0) \subset \overline{VZ^+}$$

holds ultimately. But this means that $x_i \to x$ in $\overline{VZ^+}$ which is absurd. Thus, Y is closed. It is easy to show that Y is invariant.

Whenever $L(x) \neq \emptyset$ for some $x \in X$, $L(x) \subset Y$ because $y \in L(x)$ implies $x \in J(y)$, and hence, $y \in J^+(y)$. Thus, $L(x) = L(x) \cap Y = L_Y(x)$ (limit set of x relative to Y) for each $x \in Y$. Also $J(x) = K(x) = K_Y(x) = J_Y(x)$ for each $x \in Y$. Each point of Y is nonwandering.

We shall show that the set of bilaterally Poisson stable points $Y \cap M_4$ is dense in Y, and consequently, that $X = \overline{M_4}$ (note that $X - Y \subset M_4$). Let d be the given metric on X. For each positive integer n define

$$B(n) = B^+(n) \cup B^-(n)$$

where

$$B^+(n) = \{x \in Y: d(x, xt) \geq 1/n \text{ for each } t > n\}$$

and

$$B^-(n) = \{x \in Y: d(x, xt) \geq 1/n \text{ for each } t < -n\}.$$

For each y in $\overline{B^+(n)}$ there is a sequence (x_i) in $B^+(n)$ converging to y. For t large enough we have

$$1/n \leq d(x_i, x_i t) \leq d(x_i, y) + d(y, x_i t)$$

so that $1/n \leq d(y, yt)$. Thus, $B^+(n)$ is closed. Similarly, $B^-(n)$ is closed.

Next, suppose that for some n, $B^+(n)$ is not nowhere dense in Y. Let M be an open subset of $B^+(n)$ and $x \in M$. Since $x \in J^+(x)$ there is a sequence $x_i \to x$ and $t_i \to +\infty$ such that $x_i t_i \to x$. For some i_0, $x_i \in M$ and $t_i > n$ whenever $i \geq i_0$. But $x_i \in B^+(n)$ and $t_i > n$ imply that

$$1/n \leq d(x_i, x_i t_i) \leq d(x_i, x) + d(x, x_i t_i) \to 0 \quad \text{as } i \to +\infty$$

which is clearly impossible. Hence, each set $B^+(n)$ and similarly each set $B^-(n)$ is nowhere dense in Y. The Baire Theorem yields

$$\bigcap \{Y - B(n): n \in Z^+\} = Y - \bigcup \{B(n): n \in Z^+\}$$

dense in Y.

No point of $\bigcup B(n)$ is bilaterally Poisson stable since $y \in B(k)$ implies $d(y, yt) \geq 1/k$ for $|k| > n$. On the other hand, if $y \in Y - \bigcup B(n)$, then there is a sequence $t_n \to +\infty$ with $d(y, yt_n) < 1/n$ for each n, and hence, $y \in L^+(y) = L^-(y)$. Thus, $Y - \bigcup B_n$ is the set of bilaterally Poisson stable points of X. This completes the proof.

THEOREM 9. *Let* (X, π) *be a flow of characteristic* 0 *with locally compact phase space. A closed connected invariant set* M *with compact boundary is either a component of* X *or is not isolated from nonempty compact minimal sets.*

Proof. Suppose M is a closed connected invariant set with compact boundary which is not a component of X. Then $\emptyset \neq L^+(x) \subset \partial M$ for every $x \in \partial M$, and hence, $L^+(x)$ is compact minimal for each $x \in \partial M$. Let W be a compact neighborhood of ∂M. If W contains no invariant subneighborhood of ∂M, then there exists a sequence $x_n \to x$ in ∂M and $(t_n) \subset Z$ such that $x_n t_n \to y$ in $\overline{W1} - W^0$. This is absurd since $D(x) \subset \partial M$. Thus, ∂M has a compact invariant neighborhood V each point of which is contained in a compact minimal set. The proof is complete.

Letting $A^+(x)$ $(A^+_W(x))$ denote the region of (weak) attraction of x in X we have the following proposition.

PROPOSITION 10. *Let (X, π) be a flow of characteristic* 0. *Then for each $x \in X$,*

$$A^+_W(L^+(x)) = A^+(L^+(x)) = L^+(x),$$
$$A^+_W(L^-(x)) = A^+(L^-(x)) = L^-(x), \quad \text{and}$$
$$A^+_W(C(x)) = A^+(C(x)) = K(x).$$

Proof. The proof given for Proposition 9 of [7] suffices here. However, one reference is cited which should be demonstrated for discrete flows. The proof here is easy. We need to show that for any set $M \subset X$, $x \in A^+_W(M)$ implies $J^+(x) \subset J^+(M)$. Let $x \in A^+_W(M)$. Then $K^+(x) \cap M \neq \emptyset$. If $y \in K^+(x) \cap M$, then

$$J^+(x) \subset J^+(y) \subset J^+(M).$$

Remark. In view of the example of a continuous flow of characteristic 0 given in [7] it is obvious that such a flow need not generate a discrete flow of characteristic 0.

3. Characterizations of discrete flows of characteristic 0. The characterization given in Proposition 2 is the nearest statement to the $J^+(x) = L^+(x)$, $J^-(x) = L^-(x)$, and $J^\pm(x) = L^\pm(x)$ for each $x \in X$ characterizations of flows (discrete or continuous) of characteristics 0^+, 0^-, and $0^\pm$, respectively. The $J(x) = L(x)$ for each $x \in X$ property does not characterize flows of characteristic 0. Indeed, in view of Theorem 6 it is easy to construct flows of characteristic 0 for which $J(x) \neq L(x)$ for some $x \in X$, i.e. $M_3 \neq \emptyset$.

Throughout this section (X, π) is a discrete flow on a Hausdorff phase space X.

THEOREM 11. *Let X be locally compact. Then (X, π) is of characteristic* 0 *if and only if*

(a) *each compact minimal set is bilaterally stable and*

(b) *$J(x) \subset K(x)$ for each x not is a compact minimal set.*

Proof. Let (X, π) be of characteristic 0 and let H be compact minimal. Then

$$D(H) = \bigcup \{D(x) \colon x \in H\} = \bigcup \{K(x) \colon x \in H\} = H.$$

Suppose that H is not bilaterally stable. Then some compact neighborhood V of H does not contain an invariant subneighborhood of H. Any neighborhood W of H in V has a component W_0 such that for some $t \in Z^+$, $W_0 t \cap V \neq \emptyset$ and $W_0 t \cap (X - V) \neq \emptyset$. Hence, $Wt \cap (\overline{V1} - V^0) \neq \emptyset$. Thus, there are sequences $x_i \to x$ in H and $t_i \geqslant 0$ such that $x_i t_i \to y$ in ∂V. But $y \in D(x) \subset D(H) = H$ which is absurd, and hence, H is bilaterally stable. Condition (b) follows immediately from Proposition 2.

On the other hand, let $x \in H$ where H is compact minimal. Then $H = K(x)$. Let $y \in D(K(x))$ and let V be a compact invariant neighborhood of $K(x)$. Then $D(K(x)) \subset V$. Evidently, local compactness yields $y \in K(x)$, and hence, $D(K(x)) \subset K(x)$. Thus, $D(x) = K(x)$. That $D(x) = K(x)$ for each x not in a compact minimal set follows trivially. The proof is complete.

COROLLARY 11.1. *Let (X, π) be of characteristic* 0 *with locally compact phase space. Each compact minimal subset of X has a neighborhood of Poisson stable points.*

COROLLARY 11.2. *A compact flow is of characteristic* 0 *if and only if each compact minimal set is bilaterally stable.*

The proof of the following theorem is the same as the one given in [7] for continuous flows.

THEOREM 12. *A necessary and sufficient condition for a flow (X, π) to be of characteristic* 0 *is that $A^+(C(x)) = D(x)$ for each $x \in X$.*

COROLLARY 12.1. *A flow (X, π) is of characteristic* 0 *if and only if $A^+(M) = D(M)$ for each invariant set $M \subset X$.*

COROLLARY 12.2. *Let (X, π) be of characteristic* 0. *Then a compact (closed with X regular) invariant set is asymptotically stable if and only if it is open.*

Proof. An open invariant set is trivially asymptotically stable. Conversely, Corollary 12.1 yields $A^+(M) = D(M)$ open for any asymptotically stable set M. We need only show that $D(M) = M$. Let $x \notin M$ and W and V be disjoint open neighborhoods of x and M, respectively. Since M is stable we can select V positively invariant yielding $D^+(M) \subset \overline{V} \subset X - W$. Thus, $x \notin D^+(M)$ and $D^+(M) \subset M$. By Corollary 3.1, $D(M) \subset M$, and hence, $D(M) = M$.

COROLLARY 12.3. *Let (X, π) be of characteristic* 0. *Then a compact (closed with X regular) connected invariant set is asymptotically stable if and only if it is a component of X. Furthermore, if X is connected, there are no compact (closed) connected invariant asymptotically stable proper subsets of X.*

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