On function spaces of compact subspaces of Σ-products of the real line

by

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Abstract. The main result of this paper is that if K is a compact subspace of the Σ-product of $\mathfrak{m}$ copies of the real line for a cardinal $\mathfrak{m}$ and M is a metrizable separable space, then the function space $C(K, M)$ endowed with the pointwise topology is a Lindelöf space. This extends a recent theorem of Talagrand concerning Eberlein compacts.

1. Introduction. A general question is for which compact spaces K the function space $C(K, M)$ endowed with the pointwise topology is a Lindelöf space, when M is an arbitrary metrizable separable space, or simply the real line R. This problem was first deeply investigated by Corson [6] (see also Corson and Lindenstrauss [8], where the "dual" question about $C(M, K)$ was considered).

Recently, Talagrand [19] proved that $C(K, R)$ is Lindelöf for every Eberlein compact K ([1]), answering an old question of Corson (cf. [6] and [11], Problem 6′); in fact Talagrand showed even that $C(K, R)$ is K-analytic which yields that $C(K, R)$ is Lindelöf.

The main goal of this paper is to show that $C(K, M)$ is Lindelöf, when K is a compact subspace of the Σ-product of $\mathfrak{m}$ copies of the real line for a cardinal $\mathfrak{m}$ and M is metrizable and separable. This extends the Talagrand result, as the class of compact spaces we consider is essential wider than the class of Eberlein compacts (see Sec. 2). However, in our case the function space $C(K, R)$ need not be K-analytic (see Sec. 7).

2. Terminology and notation. Our topological terminology follows [9]; we refer to [16] and [11] for the notions related to functional analysis and to [5] for the notion of *K-analycity*. The symbol $|A|$ stands for the cardinality of a set A.

Given two spaces X and Y we denote by $C(X, Y)$ the space of continuous functions from X to Y endowed with the pointwise topology. The symbol R stands for the real line, N is the set of natural numbers and $D = \{0, 1\}$ is the two-point

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([1]) More precisely, Talagrand proved a much more general theorem that WCG Banach spaces are K-analytic in the weak topology.

discrete space. We consider D as the ring and thus we can consider also every function space $C(X, D)$ as the ring with the continuous pointwise operations.

The Σ-product of $\mathfrak{m}$ copies of R is the subspace

$$\Sigma(\mathfrak{m}) = \{x \in R^S\colon\ |\{s\colon\ x(s) \neq 0\}| \leqslant \aleph_0\}$$

of the product R^S, where S is a set of cardinality $\mathfrak{m}$ [7]. Since $\Sigma(\mathfrak{m})$ contains the space $c_0(\mathfrak{m})$ of real sequences "of length $\mathfrak{m}$" tending to 0, every Eberlein compact is a subspace of $\Sigma(\mathfrak{m})$ for some $\mathfrak{m}$, by a theorem of Amir and Lindenstrauss [2]; however there exist compact subspaces of $\Sigma(\aleph_1)$ which are not Eberlein (Talagrand, Theoreme 8 in [20]; see also [4], [1], [21] and Sec. 7).

Let $L(\mathfrak{m})$ stand for the Lindelöf space of cardinality $\mathfrak{m}$ with the unique non-isolated point p. Notice that $\Sigma(\mathfrak{m})$ is naturally homeomorphic to $C(L(\mathfrak{m}), R)$ ([7]; cf. also 8.2, Lemma A).

A space X is concentrated around a set $A \subset X$ if the complement of any neighborhood of A in X is at most countable ([10], § 40, VII); observe that any space concentrated around a point is a continuous image of some $L(\mathfrak{m})$.

3. Main result.

THEOREM. *If K is a compact subspace of the Σ-product of $\mathfrak{m}$ copies of the real line for a cardinal $\mathfrak{m}$ and M is a metrizable separable space, then the function space $C(K, M)$ is Lindelöf in the pointwise topology.*

Notice that there exists a compact X such that $C(X, R)$ is Lindelöf but X can not be embedded in any $\Sigma(\mathfrak{m})$ [14] (²).

We split the proof of Theorem into two parts. The first one (Sec. 4) contains a result about the space $L(\mathfrak{m})^{\aleph_0}$, which is necessary to apply some general reasoning about function spaces, given in the second part (Sec. 5); in Section 6 we summarize these facts, obtaining the proof.

4. The space $L(\mathfrak{m})^{\aleph_0}$. The aim of this section is to prove the following

PROPOSITION 1. *For an arbitrary metrizable separable space M and every cardinal $\mathfrak{m}$ the product $M \times L(\mathfrak{m})^{\aleph_0}$ is a Lindelöf space* (³).

Let us fix an infinite cardinal $\mathfrak{m}$, put $X = L(\mathfrak{m})$ and let p be the unique non-isolated point of X; let M be a separable metrizable space. Given $x = (x_1, \ldots, x_n)$, or $x = (x_1, x_2, \ldots)$, and $i \leqslant n$ we write $x|i = (x_1, \ldots, x_i)$. Finally, we put $E_0 = M$, $E_n = M \times X^n$, $E = M \times X^N$ and $p_n\colon E \to E_n$ is the projection.

We begin with the following observation

(1) *every open family $\mathcal{U}$ in E has an open point-countable refinement $\mathcal{V}$ with $\bigcup \mathcal{V} = \bigcup \mathcal{U}$.*

(²) It would be interesting to explain whether the space $C(X, M)$ is Lindelöf for every metrizable, separable M (cf. 8.1); we conjecture that this is the case.

(³) Notice that for the proof that $C(K, R)$ is Lindelöf we need only to verify that $(L(m))^{\aleph_0}$ is Lindelöf. The last fact is simple, and it is a particular case of a Noble's result (see [21], Corollary 42); we wish to thank to T. Przymusiński for this reference.

At first prove (1) with E_n instead of E; this follows easily by induction, as $E_{n+1} = E_n \times (X \setminus \{p\}) \cup E_n \times \{p\}$, the space $X \setminus \{p\}$ is discrete and the set $E_n \times \{p\}$ is a retract of E_{n+1}. Now, given an open family $\mathcal{U}$ in E, we can assume that $\mathcal{U}$ consists of basic open sets and so we can write $\mathcal{U} = \bigcup_n \mathcal{U}_n$ where $\mathcal{U}_n \subset \{p_n^{-1}(U)\colon\ U \subset E_n\}$. Using for each $\mathcal{U}_n$ the property of E_n we have just proved, we conclude (1).

By virtue of (1) it is now sufficient to prove that

(2) *every uncountable $A \subset E$ has an accumulation point in E*, as it is well known that given an open cover $\mathcal{U}$ of a space S one can choose a discrete in S set A with

$$S = \bigcup \{U \in \mathcal{U}\colon\ U \cap A \neq \emptyset\}.$$

So let A be an uncountable subset of E.

If there exists $t \in M$ such that $p_0^{-1}(t) \cap A$ is uncountable, then it is enough to verify that every uncountable subset of X^N has an accumulation point in X^N and this can be proved by means of some standard reasoning (see [14], Lemma 2 for a somewhat more general fact).

It remains to prove (2) in the case when p_0 restricted to A is *countable-to-one*; this is in fact the key point of our proof. Without loss of generality we can assume that p_0 restricted to A is one-to-one and onto M, i.e. we have

(3) $A = \{(t, a_t)\colon\ t \in M,\ a_t \in X^N$ and $a_t \neq a_s$ for $t \neq s$.

For every $n \in N$ and $x \in X^n$ define

(4)
$$W_x^i = \begin{cases} X & \text{if } i > n \text{ or } x(i) = p \\ \{x(i)\} & \text{in the opposite case,} \end{cases} \quad \text{and} \quad W_x = \mathop{\mathbf{P}}_{i=1}^{\infty} W_x^i$$

and put

(5) $A_x = \{t \in M\colon\ a_t \in W_x$ *and* $(t, x) \in E_n$ *is an accumulation point of the set* $p_n(A)\}$.

Observe that

(6) *if* $i \leqslant n \leqslant m$, $x \in X^n$, $y \in X^m$, $A_x \cap A_y \neq \emptyset$ *and* $x(i) \neq p \neq y(i)$, *then* $x(i) = y(i)$,

as in the opposite case we would have $W_x \cap W_y = \emptyset$ and so $A_x \cap A_y = \emptyset$, by (3).

We prove that

(7) *if $T \subset M$ is uncountable and $n \in N$, then there exists $x \in X^n$ with $A_x \cap T \neq \emptyset$.*

Let $H = \{a_t|n\colon\ t \in T\} \subset X^n$.

At first consider the case when $|H| \leqslant \aleph_0$. Then there exists an uncountable set $S \subset T$ and a point $x \in X^n$ with $a_t|n = x$ for $t \in S$. It is easy to verify that the uncountable set of accumulation points of the space S is contained in $A_x \cap T$.

Now assume that H is uncountable. As we have observed, the space X^n is

Lindelöf and therefore there exists an accumulation point $x \in X^n$ of the set H. Put $S = \{t \in T\colon a_t \in W_x\}$ and for every neighbourhood U of the point x in X^n define $S_U = (\overline{\{t \in S\colon a_t|n \in U\} \cap S})^d$. Every S_U is a closed nonempty subset of S and, since every countable intersection of neighbourhoods of x is again a neighbourhood of x, the family $\{S_U\}_U$ has the countable intersection property; thus there exists $t \in \bigcap_U S_U$. It is easy to see that $t \in A_x \cap T$.

Since (7) means that for every $n \in N$ the complement $M \setminus \bigcup \{A_x\colon x \in X^n\}$ is at most countable and since M is uncountable, there exists $t \in \bigcap_n (\bigcup \{A_x\colon x \in X^n\})$. So we have

(8) $t \in A_{x_1} \cap A_{x_2} \cap \ldots$, *where* $x_n \in X^n$ *for* $n = 1, 2, \ldots$

Applying (6) we can choose for every $i = 1, 2, \ldots$ a point $c_i \in X$ such that for every $n \in N$

(9) *if* $i \leqslant n$ *then either* $x_n(i) = p$ *or* $x_n(i) = c_i$.

The space

(10) $C = (\{p, c_1\} \times \{p, c_2\} \times \ldots) \subset X^N$ *is compact.*

Letting $\bar{x}_n(i) = x_n(i)$ for $i \leqslant n$ and $\bar{x}_n(i) = p$ for $i > n$ we define the point $\bar{x}_n$ such that (cf. (9))

(11) $\bar{x}_n \in C$ *and* $\bar{x}_n|n = x_n$.

By (10) and (11) there exists $c \in \mathfrak{C}$ such that

(12) c *is an accumulation point of the sequence* $(\bar{x}_n)_{n=1}^{\infty}$.

We claim that

(13) $a = (t, c)$ *is an accumulation point of the set* A.

Indeed, take a neighbourhood U of the point a; we can assume that U is basic, i.e. for some $i \in N$ we can write $U = p_n^{-1}(U_n)$, where $U_n = p_n(U)$ and $n \geqslant i$. By (12) there exists $n \geqslant i$ with $(t, \bar{x}_n) \in U$, by (11) $(t, x_n) \in U_n$ and by (8) and (5) the set $U_n \cap p_n(A)$ is infinite. Thus the set $U \cap A$ is infinite and this completes the proof of Proposition 1.

5. Auxiliary results on function spaces. The following proposition slightly improves Lemmas 1 and 3 in [14].

PROPOSITION 2. *Let K be a zero-dimensional space and let E be a subspace of $C(K, D)$ which separates the points of K. Then the function space $C(K, R)$ is a continuous image of a closed subspace of the product $E^N \times N^N$* ([4]).

([4]) It is not necessary that $E \subset C(K, D)$; the general case can be reduced to this case by means of the Stone–Weierstrass theorem. In the case of K not zero-dimensional one can prove, exploiting the idea of Talagrand [19], that $C(K, R)$ is a continuous image of a closed subspace of $E^N \times N^N \times I^{\mathfrak{m}}$ with $\mathfrak{m} = |K|$.

Proof. One can consider only infinite K and one can assume that the unity element of $C(K, D)$ belongs to E. The reasoning given in [14], the proof of Lemma 1, shows that

(1) $C(K, R)$ *is a continuous image of a closed subspace of* $C(K, D)^N$.

It remains to verify that

(2) $C(K, D)$ *is a continuous image of the space* $\bigoplus_{n=1}^{\infty} (E^n \times N)$.

To this end define for every $\sigma \in N^n$, where $n \in N$, the continuous mapping $\varphi_\sigma\colon E^{\sigma_1 + \ldots + \sigma_n} \to C(K, D)$ by the formula

$$\varphi_\sigma\colon (f_{ij})_{1 \leqslant j \leqslant \sigma_i}^{1 \leqslant i \leqslant n} \to \sum_{i=1}^{n} f_{i1} \cdot \ldots \cdot f_{i\sigma_i}$$

and observe that the mapping $\varphi = \bigoplus_\sigma \varphi_\sigma\colon \bigoplus_\sigma E^{\sigma_1 + \ldots + \sigma_n} \to C(K, D)$ is onto, as E generates algebraically the ring $C(K, D)$ (this becomes evident if we recall that the ring $C(K, D)$ can be identified with the Boolean ring of clopen subsets of K, see [17], § 17).

LEMMA 1. *Let K be a compact zero-dimensional space. If for every metrizable separable space Y the product $Y \times C(K, R)$ is Lindelöf, then the space $C(K, M)$ is Lindelöf for every metrizable separable M.*

Proof. We can assume that $M \subset R^N$. Let $\mathcal{K}(M)$ be the space of compact subsets of M endowed with the Vietoris topology; the space $\mathcal{K}(M)$ is metrizable and separable. The product

(3) $\mathcal{K}(M) \times C(K, R^N)$ *is Lindelöf.*

Indeed, since $D^N \underset{\text{top}}{\subset} R$ and $C(K, D^N) = C(K, D)^N$, the product $\mathcal{K}(M) \times N^N \times C(K, D)^N$ is Lindelöf by the assumption, and (3) follows by Proposition 2, as $C(K, R^N) = C(K, R)^N$.

Put

(4) $$F = \{(Z, f) \in \mathcal{K}(M) \times C(K, R^N)\colon f(K) \subset Z\}.$$

Standard arguments show (cf. [10], § 44, II) that F is closed in the space $\mathcal{K}(M) \times C(K, R^N)$ and hence it is a Lindelöf space, by (3). It is enough now to notice that the projection onto the second axis maps F onto the space $C(K, M)$.

6. Proof of Theorem. Let S be an infinite set of cardinality $\mathfrak{m}$ and let K be a compact subspace of the space $\Sigma(\mathfrak{m}) \subset R^S$ (see Sec. 2).

At first assume in addition that

(1) $$K \subset D^S.$$

Let $p_s(x) = x(s)$ for $x \in K$ and $s \in S$, let p be the null element of the ring $C(K, D)$ and put

$$E = \{p_s\colon s \in S\} \cup \{p\} \subset C(K, D).$$

Since E separates the points of K we infer from Proposition 2 that

(2) $C(K, R)$ is a continuous image of a closed subspace of $E^N \times N^N$.

Observe that the space E is concentrated around the point p. Indeed, if

$$U = \{f \in C(K, R)\colon |f(x_i)| < \varepsilon,\ i \leqslant n\},$$

where $x_i \in K$, is a basic neighbourhood of f and $S_0 = \bigcup_{i=1}^{n} \{s\colon x_i(s) \neq 0\}$, then $E \setminus U \subset \{p_s\colon s \in S_0\}$ (as for $s \notin S_0$ we have $p_s(x_i) = x_i(s) = 0$), but $|S_0| \leqslant \aleph_0$. Thus E is a continuous image of the space $L(\mathfrak{m})$ and hence by (2) the space $C(K, R)$ is a continuous image of a closed subspace of the product $L(\mathfrak{m})^{\aleph_0}$. The desired conclusion follows now from Proposition 1 and Lemma 1.

The general case of compact $X \subset \Sigma(\mathfrak{m})$ can be easily reduced to the case just considered as follows. Let us take a continuous mapping $\varphi\colon D^S \to R^S$ such that $\varphi(D^S) \supset X$ and $\varphi^{-1}(\Sigma(\mathfrak{m})) \subset \Sigma(\mathfrak{m})$, put $K = \varphi^{-1}(X)$ and $\Phi(f) = f \circ \varphi$ for $f \in C(X, M)$. Now K satisfies (1) and Φ embeds $C(X, M)$ into $C(K, M)$ as a closed subspace.

7. Example. *There exists a compact subspace K of $\Sigma(\aleph_1)$ such that the function space $C(K, R)$ is not K-analytic.*

To obtain such compact K we shall apply a general construction due to Talagrand [20] and [21].

Let T be a set of reals of cardinality $\aleph_1$, let $<$ be the usual order of reals and let $\prec$ be a well order of the type ω_1 on T. Define a family $\mathscr{A}$ of subsets of the set T letting: $A \in \mathscr{A}$ iff both order $<$ and $\prec$ coincide on A. Now put (cf. [21] observe that $\mathscr{A}$ is "adequate" in the terminology of Talagrand)

$$K = \{x \in D^T\colon x^{-1}(1) \in \mathscr{A}\}$$

and give the set $T^* = T \cup \{p\}$ $(p \notin T)$ the topology consisting of all subsets of T and of all complements of finite unions of members of $\mathscr{A}$.

It is well known that $\mathscr{A}$ consists of at most countable sets and therefore $K \subset \Sigma(\aleph_1)$.

By [20], the space T^* embeds in $C(K, R)$ as a closed subspace and so it is enough to verify that T^* is not K-analytic. We shall show that given sets $A_{i_1 \ldots i_k} \subset T^*$, where $(i_1, \ldots, i_k)$ runs over finite sequences of natural numbers, such that $T^* = \bigcup_i A_i$ and $A_{i_1 \ldots i_k} = \bigcup_i A_{i_1 \ldots i_k i}$, one can choose a sequence $(i_n) \in N^N$ and distinct points $t_k \in A_{i_1 \ldots i_k}$ such that the set $\{t_1, t_2, \ldots\}$ is discrete in T^*; this will complete the proof (see [15], Lemma B).

We shall proceed by induction. At first choose an uncountable set A_{i_1} and a point $t_1 \in A_{i_1}$ such that the set $B_1 = \{t \in A_{i_1}\colon t_1 < t\}$ is uncountable. Then choose i_2 such that $B_1 \cap A_{i_1 i_2}$ is uncountable and take $t_2 \in B_1 \cap A_{i_1 i_2}$ such that $t_1 \prec t_2$ and the set $B_2 = \{t \in B_1 \cap A_{i_1 i_2}\colon t_2 < t\}$ is uncountable. Continuing this process we obtain the points $t_k \in A_{i_1 \ldots i_k}$ such that $t_1 < t_2 < \ldots$ and $t_1 \prec t_2 \ldots$ The set $\{t_1, t_2, \ldots\}$ belongs to $\mathscr{A}$ and thus it is discrete in T^*.

8. Comments.

8.1. By Proposition 1, if X is a regular space concentrated around a point and M is metrizable and separable, then the product $M \times X^N$ is Lindelöf, whereas, as was shown by Michael [12] under CH, a product of a regular space concentrated around a countable set with a metrizable separable space need not be Lindelöf.

It seems interesting in this context to explain how one can extend the result of Section 4; for example, is it still true if $L(\mathfrak{m})$ is replaced by a regular space X of cardinality $\aleph_1$ with the property that if $A \subset X$ is uncountable, then there exists an uncountable $C \subset A$ concentrated around a point $c \in X$ (see [14], the question is related to footnote (2)).

8.2. The zero-dimensional Eberlein compacts are exactly the zero-dimensional compacts K with $C(K, D)$ σ-compact, whereas from a result of Talagrand [20] it follows that Eberlein compacts K (even zero-dimensional) can not be characterized by any topological property of the function space $C(K, R)$ which is closed hereditary and a continuous invariant.

In the case of the class of compact subspaces of $\Sigma(\mathfrak{m})$ the situation seems to be fairly unclear. The following is a result in this direction.

Proposition. *For a compact zero-dimensional space K the following conditions are equivalent:*

(a) *K can be embedded in $\Sigma(\aleph_1)$,*

(b) *$C(K, D)$ is a continuous image of a closed subspace of the space $\bigoplus_{n=1}^{\infty} L(\aleph_1)^n$.*

(c) *$C(K, D)$ is a continuous image of the space $L(\aleph_1)^{\aleph_0}$.*

Notice that the property of $C(K, D)$ formulated in (b) is closed hereditary and a continuous invariant.

The main facts on which the proof of Proposition bases are the following.

The implication (a) $\to$ (b) (with arbitrary $\mathfrak{m}$ instead of $\aleph_1$) follows easily by the reasonings of Section 6.

The implication (c) $\to$ (a) (again with arbitrary $\mathfrak{m}$ instead of $\aleph_1$) follows from

Lemma A. *If K is a compact subspace of the function space $C(L(\mathfrak{m})^{\aleph_0}, R)$, where $\mathfrak{m}$ is a cardinal, then K can be embedded in $\Sigma(\mathfrak{m})$.*

We do not know, whether $L(\mathfrak{m})^{\aleph_0}$ can be replaced here by its arbitrary closed subspace, even in the case of $\mathfrak{m} = \aleph_1$ (cf. (6), (5)).

Finally, the implication (b) → (c) follows from

LEMMA *B. Every closed subset of* $L(\aleph_1)^n$ *is a retract of* $L(\aleph_1)^n$ *for arbitrary* $n \in N$ (6).

8.3. As we have mentioned in (1) Talagrand [19] showed that for every Eberlein compact K the function space $C(K)$ is Lindelöf under the weak topology. Combining the Talagrand's reasoning (cf. (4)) with our approach one can show that it is consistent with the usual axioms of set theory that this is true for every compact $K \subset \Sigma(\mathfrak{m})$. The additional axioms for set theory are needed to show that every compact $K \subset \Sigma(\mathfrak{m})$ has the following property (M), every Radon measure on K has a separable support it follows immediately from a result of Arhangel'skiĭ [3] that there is a model of set theory such that every sequential compact with the Souslin property is separable (7).

Added in proof. The result stated in Theorem 1 was obtained independently by S. P. Gulko, *On properties of subsets of Σ-products*, DAN SSR 237 (3) (1977). More precisely Gulko proved (by the methods different from ours) that $C(X, M)$ is Lindelöf under the pointwise topology, when X is a closed subset of a Σ-product of metrizable separable spaces and M is metrizable and separable; however, our approach yields that if X is in addition compact, their $C(X, M)\times L$ is Lindelöf for each hereditarily Lindelöf L and we do not know whether this is also true in the case of non-compact X.

In the paper *On properties of some function spaces*, DAN SSR 243 (4) (1978), Gulko developed an extremely interesting general approach which yields many results about Σ-products and its function spaces. It follows in particular from the results of Gulko that $L(m)^{\aleph_0}$ in Lemma A in Section 8 can be replaced by its arbitrary closed subset; this means that the characterization of compact subspaces of Σ-products conjectured in footnote (5) is true.

A recent example of R. Hadon, *On dual L^1-spaces and injective bidual Banach spaces*, Israel J. Math. shows, by the results of Šapirowskiĭ [18], that the statements considered in Section 8.3 are from the usual set theory.

The first of the authors obtained in the paper, *A class of spaces whose Cartesian product with every hereditarily Lindelöf space is Lindelöf*, Fund. Math. (to appear) a far-reaching extension of Proposition 1 from Section 4.

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(5) The positive answer could characterize compact subspaces of $\Sigma(\mathfrak{m})$ as those compacts K whose function space $C(K, R)$ is a continuous image of a closed subspace of $L(\mathfrak{m})^{\aleph_0}$.

(6) This is not true neither for $L(\aleph_2)^2$ nor for $L(\aleph_1)^{\aleph_0}$.

(7) By a result of Šapirovskiĭ [17], in this model (M) holds for every compact K with the countable tightness; in particular this is the case when $C(K, R)$ is Lindelöf (see [6]; cf. also [20], Theorem 16).

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