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Homeotopy groups of surfaces whose boundary is the union of 1-spheres

by

Richard J. Tondra (Ames, Iowa)

Abstract. Let M be a surface (connected, separable, metrizable 2-manifold) with non empty boundary bd$M$ which is the union of a collection of 1-spheres $\{S_j\}_{j=1}^{\alpha(M)}$. Let D_j be the 2-cell obtained by taking the cone over S_j and let $\hat{M}$ denote the surface $M\bigcup_f(+\sum D_j)$ where $f\colon(+\sum S_j)\to M$ is defined by $f(x)=x$. As usual, identify D_j and M with the appropriate subsets of $\hat{M}$ and for each j let $p_j\in\operatorname{int}D_j\subset\hat{M}$. The principal result of this paper is that the homeotopy group $H(M)$ of M is isomorphic to the group $H(\hat{M},F)$ where $F=\{p_j\}_{j=1}^{\alpha(M)}$. This generalizes the results of D. J. Sprows concerning $H(M)$ where M is compact surface, bd$M\neq\emptyset$.

1. Introduction. Let M be an n-manifold (connected, separable, metrizable), bd$M\neq\emptyset$, $n\geqslant 2$. M will be called *sphere bounded* if each component of bd$M$ is an $(n-1)$-sphere. If M is a sphere bounded n-manifold, the *capping* of M, denoted by $\hat{M}$, is the n-manifold defined by

$$\hat{M}=M\bigcup_f\Big(+\sum_{j=1}^{\alpha(M)}D_j\Big)$$

where for each j, D_j is the cone over the jth boundary component S_j of M and

$$f\colon\Big(+\sum_{j=1}^{\alpha(M)}S_j\Big)\to M$$

is defined by $f(x)=x$. As usual, we will identify S_j, D_j and M with the appropriate subspaces of $\hat{M}$.

Let M be an n-manifold, F a subset of int$M$. As usual, the homeotopy group $H(M)$ of M is the quotient group $G(M)/G_0(M)$ where $G(M)$ is the group of all homeomorphisms of M onto itself and $G_0(M)$ is the normal subgroup of $G(M)$ consisting of those homeomorphisms g which are isotopic to the identity (denoted $\simeq 1_M$). Also $H(M,F)$ denotes the quotient group $G(M,F)/G_0(M,F)$ where $G(M,F)$ is the subgroup of $G(M)$ consisting of those g which map F onto F and $G_0(M,F)$ is the normal subgroup of $G(M,F)$ consisting of those homeomorphisms h in $G(M,F)$ which are isotopic to the identity by an isotopy which keeps F pointwise fixed (denoted $h\simeq 1_M$ (rel$F$)).

The main results of this paper are the following:

THEOREM 1.1. *Let M be a sphere bounded n-manifold, $n\geqslant 2$ and $n\neq 4$, $\hat{M}$ the capping of M. For each j such that S_j is a component of* $\operatorname{bd}M$, *let* $p_j\in\operatorname{int}D_j\subset\hat{M}$ *and let* $F=\{p_j\}_{j=1}^{\alpha(M)}$. *Then there is an epimorphism* $\psi\colon H(M)\to H(\hat{M},F)$.

THEOREM 1.2. *Let M be a sphere bounded 2-manifold, $\hat{M}$ the capping of M. Then* $H(M)\cong H(\hat{M},F)$.

THEOREM 1.3. *Let Q be a 2-manifold,* $\operatorname{bd}Q=\emptyset$ *and let F be a non-empty closed discrete subset of Q. Then*

(i) $H(Q,F)\simeq H(M)$ *for some sphere bounded 2-manifold M.*

(ii) *Let F' be a non-empty, closed discrete subset of Q such that F is homeomorphic to F'. If F and F' determine the same subspace of the space of ends $e(Q)$ of Q, then there is a homeomorphism h of the pair (Q,F) onto (Q,F') and hence*

$$H(Q,F)\cong H(Q,F').$$

2. Proof of the results. We will follow the procedure of Sprows in [6] with certain crucial modifications.

Proof of Theorem 1.1. We first note that we may assume that p_j is the vertex of the cone D_j over S_j. For each j, let e_j be a fixed homeomorphism of D_j onto the closed unit ball in $\mathscr{R}^n$ which carries p_j to the origin. If $h\in G(M)$, then we extend h to $h_c\in G(\hat{M},F)$ by coning; i.e., $h_c(x)=h(x)$ for all $x\in M$ and if $h(S_i)=S_j$, then any element y of D_i is of the form $y=e_i^{-1}(te_i(x)+(1-t)e_i(p_i))$ where $x\in S_i$ and $t\in[0,1]$ and $h_c(y)=e_i^{-1}(te_j(h(x))+(1-t)e_j(p_j))$. Since M together with the collection $\{D_j\}$ is a closed, nbd-finite family $\mathscr{F}$, $h_c\in G(\hat{M},F)$.

For $f\in G(M)$ and $g\in G(\hat{M},F)$, let $\tilde{f}$, and $\tilde{g}$ denote the equivalence classes in $H(M)$ and $H(\hat{M},F)$ respectively. We define $\psi\colon H(M)\to H(\hat{M},F)$ by $\psi(\tilde{f})=\tilde{f}_c$.

1) ψ is well defined. Suppose that $f,f'\in G(M)$ such that $\tilde{f}=\tilde{f}'$. Then $f'f^{-1}\in G_0(M)$ and thus $f'f^{-1}$ is isotopic to 1_M by some isotopy H_t. Since $H_t(\operatorname{bd}D_i)=\operatorname{bd}D_i$ for all i and all $t\in[0,1]$, if we let $H_t^c=(H_t)_c$, then since $\mathscr{F}$ is a closed, nbd finite family, it follows that H_t^c yields an isotopy $(f'f^{-1})_c\simeq 1_{\hat{M}}(\operatorname{rel}F)$. Since $(f'f^{-1})_c=f'_c(f^{-1})_c=f'_c(f_c)^{-1}$, it follows that $\tilde{f}_c=\tilde{f}'_c$.

2) ψ is a homomorphism. This follows since $(ff')_c=f_cf'_c$ for any $f,f'\in G(M)$.

3) ψ is an epimorphism. In order to show this, we first establish the following lemma:

LEMMA 2.1. *Suppose that $g\in G(\hat{M},F)$. Then there exists a $k\in G(\hat{M},F)$ such that $\tilde{g}=\tilde{k}$ and $k|_M=h\in G(M)$.*

Proof. By taking an appropriate collar of $\operatorname{bd}M$, for each i we get a closed annulus $A_i\subset M$ such that $\{E_i=A_i\cup D_i\}$ is a disjoint collection of closed n-cells with bicollared boundary and the collection $\mathscr{G}$ consisting of $\hat{M}-(\bigcup\operatorname{int}E_i)$ together with all the E_i is a closed, nbd-finite family. Now let $g\in G(\hat{M},F)$. For each i, let $g(i)$ denote the index such that $g(p_i)=p_{g(i)}$. Then for each i,

$$g(p_i)=p_{g(i)}\in\operatorname{int}D_{g(i)}\cap\operatorname{int}g(D_i)=\operatorname{int}D_{g(i)}\cap g(\operatorname{int}D_i).$$

For each i, there exists a closed n-cell $F_{g(i)}$ with bicollared boundary such that $p_{g(i)}\in\operatorname{int}F_{g(i)}\subset F_{g(i)}\subset\operatorname{int}D_{g(i)}\cap\operatorname{int}g(D_i)$. Since $n\geqslant 2$, $n\neq 4$, it follows from [4] that $g(E_i)-\operatorname{int}F_{g(i)}$ and $g(E_i)-\operatorname{int}g(D_i)$ are annuli and we can construct a homeomorphism α_i of $g(E_i)$ onto itself such that α_i restricted to $(\operatorname{bd}g(E_i)\cup\{p_{g(i)}\})$ is the identity and $\alpha_i(g(D_i))=F_{g(i)}$. It follows from the use of Alexander's technique [1] that α_i is isotopic to $1_{g(E_i)}$ relative to $(\operatorname{bd}g(E_i)\cup\{p_{g(i)}\})$.

Since $\mathscr{G}$ is a closed nbd-finite collection and $g\in G(\hat{M},F)$, it follows that the α_i can be extended to a homeomorphism $\alpha\in G(\hat{M},F)$ by setting

$$\alpha(x)=\begin{cases}x & \text{for}\quad x\in\hat{M}-(\bigcup\operatorname{int}g(E_i)),\\ \alpha_i(x) & \text{for}\quad x\in g(E_i).\end{cases}$$

Also, since each $\alpha_i\simeq\operatorname{id}_{g(E_i)}\operatorname{rel}(\operatorname{bd}g(E_i)\cup\{p_{g(i)}\})$, there is an isotopy $\alpha\simeq 1_{\hat{M}}(\operatorname{rel}F)$.

In a similar manner, we can construct a homeomorphism $\beta\in G(\hat{M},F)$ such that $\beta\simeq 1_{\hat{M}}(\operatorname{rel}F)$ and for all i, $\beta(F_{g(i)})=D_{g(i)}$ and $\beta|_{E_{g(i)}}$ is a homeomorphism of $E_{g(i)}$ onto itself which is the identity on $\operatorname{bd}E_{g(i)}\cup\{p_{g(i)}\}$. Now consider $k=\beta\alpha g$. Then clearly $k\simeq g\operatorname{rel}F$. Furthermore $k(p_i)=g(p_i)$ for all i and $k(D_i)=\beta(\alpha(g(D_i)))=\beta(F_{g(i)})=D_{g(i)}$. Therefore, since $g|_F$ is a homeomorphism of F onto F, it follows that $h=k|_M\in G(M)$.

We are ready to establish 3). Let $\tilde{g}\in H(M,F)$. Then it follows from the above lemma that there exists an element $k\in G(M,F)$ such that $k\simeq g$ $(\operatorname{rel}F)$ and $h=k|_M\in G(M)$. Since $\tilde{k}=\tilde{g}$, it suffices to show that there exists an $f\in H(M)$ such that $\psi(\tilde{f})=k$. Now let $\sigma=(h^{-1})_ck$. Now $\sigma|_{M\cup F}=1_{M\cup F}$. It thus follows that for each i there exists an isotopy $\{H_t^i\}$ of D_i fixed on $(\operatorname{bd}D_i\cup\{p_i\})$ between $\sigma|_{D_i}$ and 1_{D_i}. Again, we can extend to obtain an isotopy $\{H_t\}$ between σ and $1_{\hat{M}}\operatorname{rel}(M\cup F)$. Thus $\tilde{\sigma}=\tilde{1}_{\hat{M}}=(\tilde{1}_M)_c=\varphi(\tilde{1}_M)$. Therefore

$$\varphi(\tilde{h})=\varphi(\widetilde{h1_M})=\tilde{h}_c\tilde{\sigma}=\tilde{h}_c\widetilde{(h^{-1})}_c\tilde{k}=\tilde{k}.$$

Proof of Theorem 1.2. In view of Theorem 1.1, it suffices to establish that ψ is a monomorphism when $n=2$. We need only note that the construction used by Sprows in [6] can be used in the present case. Thus suppose $\tilde{f}\in H(M)$ and $\tilde{f}_c\simeq 1_{\hat{M}}(\operatorname{rel}F)$, i.e., $\tilde{f}\in\operatorname{kernal}\psi$. As in [6], let $r\colon\hat{M}-F$ be defined by setting $r(x)=x$, $x\in M$ and letting $r|_{D_i-\{p_i\}}$ be the retraction of $D_i-\{p_i\}$ onto $\operatorname{bd}D_i$. If $f_c\simeq 1_M(\operatorname{rel}F)$ by $\{H_t\}$, let $H'_t=H_t|_M$ and $G'_t=rH'_t$. Then G'_t is a homotopy on M with $G'_0=f$ and $G'_1=1_M$. Arguing as in [6], it follows that $f\simeq 1_M$.

Proof of Theorem 1.3. Let F be a non-empty closed discrete subset of Q. Then $F=\{q_i\}_{i=1}^{\alpha(F)}$ is at most countably infinite.

In order to prove (i), we need only observe that we can construct a disjoint, nbd-finite family of closed 2-cells $\{E_i\}$ such that for all i, $q_i\in\operatorname{int}E_i$ and $M=Q-(\bigcup_{i=1}^{\alpha(F)}\operatorname{int}E_i)$ is a sphere-bounded 2-manifold. Then since there is a homeomorphism $h\colon\hat{M}\to Q$ such that $h(M)=M$ and h carries (D_i,p_i) onto (E_i,q_i) for all i, it follows that $H(Q,F)\cong H(M)$.

In order to prove (ii) we will make use of results on the classification of sphere bounded surfaces. For the sake of completeness, the statements of the results and an outline of the proofs are included in the Appendix. These results were proven by the author independently but subsequent to classification results obtained by Barros [2].

Suppose that Q is a 2-manifold, $\mathrm{bd}Q \neq \emptyset$. Assume that Q is represented as in Theorem 1.1 of [7]. The closed totally disconnected set X referred to in the above theorem corresponds to the space of ends $e(Q)$ of Q. To say that $F = \{q_i\}$ and $F' = \{q_i'\}$ determine the same subspace K of $e(Q)$ means that the closure of F in $Q \cup e(Q)$ is $K \cup F$ and that the closure of F' in $Q \cup e(Q)$ is $K \cup F'$. Now as in the proof of (i) above, we can construct disjoint nbd-finite families of closed 2-cells $\{E_i\}$ and $\{E_i'\}$ such that for all i, $q_i \in \mathrm{int}E_i$, $q_i' \in \mathrm{int}E_i'$ and $M = Q - \bigcup_i \mathrm{int}E_i$, $M' = Q - \bigcup_i \mathrm{int}E_i'$ are sphere-bounded 2-manifolds. We may also assume that $\bigcup_i E_i$ and $\bigcup_i E_i'$ determine K. However, this implies that $\mathrm{bd}M$ and $\mathrm{bd}M'$ are homeomorphic; M and M' are of the same genus and orientability class; and that there is a homeomorphism k of the end-space tuple of M onto the end-space tuple of M' (see Definition A.6 of Appendix). It thus follows from Theorem A.7 of Appendix that there is a homeomorphism $\bar{h}$ of M onto M'. It is easily seen that $\bar{h}$ can be extended to a homeomorphism h of the pair (Q, F) onto (Q, F') and the result follows.

Appendix. Classification results concerning sphere bounded 2-manifolds have been obtained by Barros [2]. These results are obtained by considering $M' = M - \mathrm{bd}M$, where M is a sphere bounded 2-manifold. The author has obtained classification results for such manifolds by considering $\hat{M}$, the capping of M. Since the techniques used in the main part of this paper depended upon capping $\hat{M}$, it seems appropriate to give statements of these results together with an outline of their proofs.

Definition A.1. Let M be a non-compact 2-manifold such that either $\mathrm{bd}M = \emptyset$ or M is sphere bounded. A *boundary component* of M is a derceasing sequence $P_1 \supset P_2 \supset \ldots P_n \supset \ldots$ of open connected subsets of M such that:

(i) $\mathrm{Cl}P_k$, the closure of P_k in M, is not compact for all k;

(ii) $\mathrm{Fr}P_k$, the frontier of P_k in M, is compact for all k and $\mathrm{Fr}P_k \cap \mathrm{bd}M = \emptyset$;

(iii) P_k is a sphere bounded 2-manifold for all k;

(iv) for any compact subset $A \subset M$, $P_k \cap A = \emptyset$ for k sufficiently large.

Definition A.2. Let M be a non-compact 2-manifold such that either $\mathrm{bd}M = \emptyset$ or M is sphere-bounded. The space of ends of M, denoted by $e(M)$, is the topological space (X, T) where $X = \{[p]\} \mid p$ is a boundary component of M and $[p]$ denotes the equivalence class of p under equivalence for boundary components as defined in [5]} and T has basis B determined as follows:

For each open set $U \subset M$ such that

(i) each component of U is a sphere-bounded surface;

(ii) $\mathrm{Fr}\,U$ in M is compact; and

(iii) $\mathrm{Fr}\,U \cap \mathrm{bd}M = \emptyset$, define

$$[U] = \{[p] \mid \text{ if } p = P_1 \supset P_2 \supset \ldots \supset P_k \supset \ldots \text{ is a boundary component, then } P_N \subset U \text{ for } N \text{ sufficiently large}\}.$$

Then it is easily established that the collection B of all such $[U]$ is a basis for a topology T.

As in [5], a point $[p] \in e(M)$, where $p = P_1 \supset \ldots \supset P_n \supset \ldots$, is called planar $\Leftrightarrow P_n$ is planar for all n sufficiently large and $[p]$ is called orientable $\Leftrightarrow P_n$ is orientable for all n sufficiently large. We set

$$e'(M) = e(M) - \{[p] \mid [p] \text{ is planar}\}$$

and

$$e''(M) = e'(M) - \{[p] \mid [p] \text{ is orientable}\}.$$

It is easily seen that $e'(M)$ and $e''(M)$ are closed subsets of $e(M)$. The following proposition establishes an important relationship between $e(M)$ and $e(\hat{M})$.

Proposition A.3. *Let M be a sphere bounded 2-manifold. Then there is a homeomorphism h of the triple $(e(M), e'(M), e''(M))$ onto the triple $(e(\hat{M}), e'(\hat{M}), e''(\hat{M}))$ where $\hat{M}$ is the capping of M.*

Proof. We first note that in Definition 2 of [5], one can make the further requirement that U be open and the same topology will be generated by the basis obtained.

Let $X = +\sum D_j$ and let $q: X + M \to \hat{M} = X \bigcup_f M$ be the quotient map. Let U be a domain (open connected subset) of M which is a sphere bounded 2-manifold. Then $U \cap S_j \neq \emptyset \Leftrightarrow S_j \subset U$ for any component S_j of $\mathrm{bd}M$. Then $\tilde{U} = q(D(U))$ is a domain of $\hat{M}$ where $D(U) = \{+\sum D_j \mid U \cap S_j \neq \emptyset\} + U$. If $p = P_1 \supset P_2 \supset \ldots P_n \supset \ldots$ is a boundary component of M, then clearly $\tilde{p} = \tilde{P}_1 \supset \tilde{P}_2 \supset \ldots \tilde{P}_n \supset \ldots$ is a boundary component of $\hat{M}$. We define $h: e(M) \to e(\hat{M})$ by $h([p]) = [\tilde{p}]$. Then h is well defined. Straightforward arguments can now be given to show that h carries the triple $(e(M), e'(M), e''(M))$ homeomorphically onto the triple $(e(\hat{M}), e'(\hat{M}), e''(\hat{M}))$.

Definition A.4. Let M be a sphere bounded surface. A point $[p] \in e(M)$ will be called a *rim point of* $M \Leftrightarrow p = P_1 \supset P_2 \supset \ldots P_n \supset \ldots$ has the property that $P_n \cap \mathrm{bd}M$ has an infinite number of components for all $n \geq 1$. Clearly, this definition is independent of the choice of the boundary component representing $[p]$. The rim of $e(M)$, denoted by $\mathrm{rim}\,e(M)$, is defined by

$$\mathrm{rim}\,e(M) = \{[p] \in e(M) \mid [p] \text{ is a rim point of } M\}.$$

Proposition A.5. *Let M be a sphere bounded 2-manifold. Then $\mathrm{rim}\,e(M)$ is a closed subset of $e(M)$.*

Proof. This follows easily from the definition.

DEFINITION A.6. Let M be a sphere bounded 2-manifold. The end tuple of M is the ordered 4-tuple of spaces $(e(M), e'(M), e''(M), \mathrm{rim}\, e(M))$.

THEOREM A.7 (Kerekjarto's Theorem). *Suppose that M, M' are sphere bounded 2-manifolds such that M, M' are of the same genus and orientability class; i.e., $\hat{M}$, $\hat{M}'$ satisfy this condition. Then M and M' are homeomorphic $\Leftrightarrow$ $\mathrm{bd}M$ and $\mathrm{bd}M'$ are homeomorphic and there is a homeomorphism k of the end tuple of M onto the end tuple of M'.*

Proof. Necessity is clear so we only consider the problem of establishing sufficiency.

If $\mathrm{bd}M$ and $\mathrm{bd}M'$ have only a finite number of components, then $\mathrm{rim}\, e(M) = \mathrm{rim}\, e(M') = \emptyset$ and it follows from Proposition A.3 and the main result of [5] that $\hat{M}$ and $\hat{M}'$ are homeomorphic. Since $\mathrm{bd}M$ and $\mathrm{bd}\hat{M}$ have the same finite number of boundary components, a homeomorphism of M onto M' is easily obtained.

Now suppose that $\mathrm{bd}M$ and $\mathrm{bd}M'$ have an infinite number of components. We outline how the proof of Theorem 1 of [5] can be modified to obtain the result. As in [5], assume that M and M' are both infinitely non-orientable, the proof in other cases being similar.

Let k be a homeomorphism of the end tuple of M onto the end tuple of M'. The existence of a homeomorphism f of M onto M' will follow provided we can represent M and M' as the union of compact 2-manifolds $\{A_n\}_{n=1}^{\infty}$ and $\{A'_n\}_{n=1}^{\infty}$ respectively such that for all $n \geqslant 1$

(i) $A_n \cap \mathrm{bd}M$ is either empty or the union of a finite number of components of $\mathrm{bd}M$; similarly for $A'_n \cap \mathrm{bd}M'$.

(ii) $A_n \subset i_M A_{n+1}$ = interior of A_{n+1} relative to M; similarly $A'_n \subset i_{M'} A'_{n+1}$.

(iii) there is a homeomorphism f_n of the pair $(A_n, A_n \cap \mathrm{bd}M)$ onto the pair $(A'_n, A'_n \cap \mathrm{bd}M')$.

(iv) $f_{n+1} | A_n = f_n$.

(v) Each component U of $M - A_n$ has non-compact closure and simple closed curve $d(U)$ comprises $\mathrm{Fr}\, U$. Furthermore, either $U \cap \mathrm{bd}M = \emptyset$ or contains an infinite number of components of $\mathrm{bd}M$. Similarly for $M' - A'_n$.

(vi) If $f_n(d(U)) = d(U')$, then $k([U]) = [U']$.

To do this, we note that we can first construct sequences $\{B_n\}$ and $\{B'_n\}$ of compact 2-manifolds contained in M and M' which satisfy for all $n \geqslant 1$ the conditions

(vii) $B_n \cap \mathrm{bd}M$ is either empty or consists of a finite number of components of $\mathrm{bd}M$, $B_n \subset i_M B_{n+1}$ and $M = \bigcup_{n=1}^{\infty} B_n$. Likewise for $\{B'_n\}$.

(viii) Every component U of $M - B_n$ has non-compact closure, is either of genus zero or infinite genus, is either orientable or infinitely non-orientable, and either $U \cap \mathrm{bd}M = \emptyset$ or $U \cap \mathrm{bd}M$ consists of an infinite number of components of $\mathrm{bd}M$. Furthermore, $\mathrm{Fr}\, U$ in M is precisely one simple closed curve. Similarly for each component of $M' - B'_n$.

One now need only appropriately modify the techniques used in the proof of Theorem 1 of [5] to obtain the result.

A constructive procedure for obtaining a model of any sphere bounded 2-manifold is given by the following.

COROLLARY A.8. *Let $S^2 = \{x = (x_1, x_2, x_3) \in R^3 |\ \|x\| = 1\}$, $E^1 = \{x \in S^2 | x_3 \geqslant 0\}$ and $E^2 = \{x \in S^2 | x_3 \leqslant 0\}$ and let $S^1 = E^1 \cap E^2$. Every sphere bounded 2-manifold M is homeomorphic to a surface formed from S^2 by first removing a closed totally disconnected set $X \subset S^1$ from S^2 and then removing the interiors of a finite or infinite sequence $C_1, C_2, \ldots$, of pairwise disjoint 2-cells in $E^2 - S^1$ and then removing the interiors of a finite or infinite sequence of pairwise disjoint 2-cells $D_1, D_2, \ldots$, in $E^1 - S^1$. The boundaries of each C_k are identified to produce either a handle or crosscap as the case requires. The sequence $C_1, C_2, \ldots$, "approaches" X in the sense that if U is an open subset of S^2, $X \subset U$, then all but a finite number of C_i are contained in U. The sequence $D_1, D_2, \ldots$, "approaches" a closed subset $B \subset X$ in the sense that if V is an open set in S^2 and $B \subset V$, then all but a finite number of the D_j are contained in V. The pair (X, B) is homeomorphic to the pair $(e(M), \mathrm{rim}\, e(M))$.*

Proof. This follows easily from Theorem 1.1 of [7], the techniques used in [5], Proposition A.3 and Theorem A.7.

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