Theorems 1 and 3 and from the lemma it follows that if M is a complete metrizable closed subspace of a collectionwise normal space X or M is a closed subset of a paracompact p-space, then the Dugundji Extensions Theorem is valid for the pair (X, M).

In [3] we give an example of a hereditarily paracompact space and a closed separable metric space $M \subset X$ such that the Dugundji Extension Theorem is not valid for the pair (X, M). This implies that the theorems on the factorization of a map $f\colon X \to Y$ through $X \xrightarrow{h} Z \to Y$, $uZ = uM \geqslant uY$, such that $h|M$ is a homeomorphism onto a closed subspace $hM \subset Z$, cannot be obtained without additional assumptions on embedding the set $M \subset X$, even if we assume that the space X is nice.

Another corollary which can be obtained from Theorem 3 by putting $M = \{\text{point}\} = Y$, is a result of Archangel'skiĭ [1] stating that each paracompact p-space has a perfect map onto a metrizable space.

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Uniform homotopy

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Abstract. For X a finite dimensional normal space and Y a compact space, let $\beta^*\colon [\beta X, Y] \to [X, Y]$ be the natural quotient map from the set of uniform homotopy classes of maps from X to Y to the set of homotopy classes of maps. It is shown that if there exists a topological group G with the structure of a CW-complex of finite type such that its classifying space dominates Y then $\ker\beta^* = 0$. Hence if β^* is a homomorphism, for suitable group structures on the sets, maps from X to Y are homotopic if and only if they are uniformly homotopic. The condition that Y be compact can be removed when considering bounded maps.

This paper is concerned with the problem of when are uniform homotopy and homotopy equivalent. More specifically, under what conditions on the spaces X and Y are a pair of maps $X \to Y$ homotopic if and only if they are uniformly homotopic? This is known to be the case when X is countably compact and Y is compact [5], [16]. On the otherhand, if Y is the circle S^1 and X is not pseudocompact then there is a map from X to Y which is homotopic to a constant map but not uniformly so [5], [8] [25, p. 225].

In fact the techniques used allow consideration of a slightly different problem, namely, "when does the embedding $\beta\colon X \to \beta X$ of a space into its Stone–Čech compactification induce a bijection β^* between the set $[\beta X, Y]$, of homotopy classes of maps from βX to Y, and the set $[X, Y]$ of homotopy classes of maps from X to Y". This is equivalent to the original problem for Y compact.

For X a finite dimensional (covering dimension) normal space and Y a space dominated by a CW-complex B, where B is the classifying space of a group G and G is a CW-complex of finite type it is shown that β^* is onto and has the null class as "kernel". Hence if β^* is a homomorphism then β^* is a bijection. If in addition to all this, Y is also compact then every pair of maps from X to Y are homotopic if and only if they are uniformly homotopic.

As corollaries we have that if X is a finite dimensional normal space then the Čech cohomology of X (based on locally finite covers) is isomorphic to that of βX in all dimensions over a finite coefficient group and in dimensions higher than 1 over a finitely generated abelian coefficient group, and that a map from X into an n-sphere ($n > 1$) is uniformly homotopic to a constant map if and only if it is null homotopic.

§ 1

(1.1) For convenience we shall work in the category of *pointed normal spaces and maps*. That is, every space is normal and has a distinguished point, *, and maps (continuous functions) take * to *.

(1.2) For spaces X and Y, a map $H\colon X\times I\to Y$ is called a *uniform homotopy* if it can be "extended" to a map $H\colon \beta X\times I\to Y$. Here I is the closed unit interval and $\beta(\ldots)$ denotes Stone–Čech compactification.

(1.3) If Y is a compact metric space with metric ϱ then $H\colon X\times I\to Y$ is a uniform homotopy if and only if for every $\varepsilon>0$ there exist a $\delta>0$ such that $|t-t'|<\delta$ implies that $\varrho\big(H(x,t),H(x,t')\big)<\varepsilon$ for all $x\in X$. This is the definition in [5].

(1.4) A map $H\colon X\times I\to Y$ is a uniform homotopy if and only if for every finite open cover $\mathscr{U}$ of Y there exist finite open covers $\mathscr{V}$ of X and $\mathscr{W}$ of I such that $\mathscr{V}\times\mathscr{W}$ refines $H^{-1}\mathscr{U}$, [9].

For other equivalent definitions see [16] and [17].

(1.5) Two maps $f,g\colon X\to Y$ are called *uniformly homotopic* if there is a uniform homotopy $H\colon X\times I\to Y$ such that $H(x,0)=f(x)$ and $H(x,1)=g(x)$ for all $x\in X$. Write $f\underset{\beta}{\sim}g$.

Clearly if Y is compact then $f\underset{\beta}{\sim}g$ if and only if the irextensions $\beta(f),\beta(g)\colon X\to Y$ are homotopic. It is not true for Y noncompact that $f\underset{\beta}{\sim}g$ if $\beta(f),\beta(g)\colon \beta X\to\beta Y$ are homotopic (see (5.4), cf [17, (8.1)]).

(1.6) A map which is uniformly homotopic to the constant map will be called *uniformly trivial.*

(1.7) A map is *bounded* if the closure of $f(X)$ is compact. A map $f\colon X\to Y$ extends to a map $\beta(f)\colon \beta X\to Y$ if and only if f is bounded.

(1.8) It follows easily from (1.4) that if Y is locally compact and $H\colon X\times I\to Y$ is a uniform homotopy then if $H|(X\times\{t\})$ is bounded for some $t\in I$ then it is bounded for all $t\in I$ and $H(\beta X\times I)\subset Y$.

Hence, if $f\underset{\beta}{\sim}g$ and f is bounded then so is g.

(1.9) Lemma. *If P is a finite complex and $f,g\colon X\to P$ are contiguous maps (for each $x\in X$, $f(x)$ and $g(x)$ are contained in a single simplex of P) then $f\underset{\beta}{\sim}g$.*

Proof. Let V be the vertex set of P. Let $\{\pi_v\colon P\to I\colon v\in V\}$ be barycentric coordinates on P. Since f and g are continguous we can define $H\colon X\times I\to Y$ by

$$\pi_v H(x,t)=(1-t)\pi_v f(x)+t\pi_v g(x)\,.$$

Then, H is a uniform homotopy by (1.3).

(1.10) Lemma. *Let X be a normal space, P a simplicial complex and $f\colon X\to P$ a bounded map. There exist a simplicial complex Q and maps $\pi\colon X\to Q$ and $g\colon Q\to P$ such that*

(1) *g is bounded,*

(2) $\dim Q\leqslant \dim X$,

(3) $g\pi\underset{\beta}{\sim}f$,

(4) *g is null homotopic if f is null homotopic.*

Proof. For terminology and general reference in this proof see [4, Appendix 2].

Since f is bounded, by restriction to the carrier of $f(X)$ we may assume P is a finite complex [14, p. 43, Thm 1.5]. So (1) will follow automatically.

In the notation of (1.9) let

$$\mathscr{U}=\{(f\pi_v)^{-1}(0,1]\colon v\in V\}\,.$$

Then $\mathscr{U}$ is a numerable cover of X. "Numerable" is equivalent to "locally finite" for normal spaces and so by [5] or [6], $\mathscr{U}$ has a numerable refinitement $\mathscr{U}$ of order $\leqslant\dim X+1$. That is, the nerve $\nu\mathscr{U}$ of $\mathscr{U}$ has dimension $\leqslant\dim X$. Let $\nu\mathscr{V}$ be the nerve of $\mathscr{V}$ and $\pi_v^{\mathscr{U}}\colon \nu\mathscr{U}\to\nu\mathscr{U}$ be a canonical map (projection in the terminology of [9]). Define $h\colon \nu\mathscr{V}\to P$ by

$$h\big((f\pi_v)^{-1}(0,1]\big)=v$$

for all $v\in V$ and extend simplicially. Let $g=h\pi_v^{\mathscr{U}}$ and let $\pi\colon X\to\nu\mathscr{U}$ be a canonical map defined using a numeration of $\mathscr{U}$. Then $g\pi$ is contiguous to f and so by (1.9) $g\pi\underset{\beta}{\sim}f$. Taking $Q=\nu\mathscr{U}$ gives (2) and (3).

Suppose f is homotopic to ($\sim$) the constant map to *. Then $g\pi\sim{*}$ and so by [18, (4.2)] there is a numerable refinement $\mathscr{W}$ of $\mathscr{U}$ such that $g'=g\pi_{\mathscr{U}}^{\mathscr{W}}\sim{*}$, where $\pi_{\mathscr{U}}^{\mathscr{W}}$ is a canonical map. As before $\mathscr{W}$ can be chosen so that $\dim\nu\mathscr{W}\leqslant\dim X$. Let $\pi'\colon X\to\nu\mathscr{W}$ be a canonical map. Then $g'\pi'$ is contiguous to f so taking $Q=\nu\mathscr{W}$ gives (2), (3), and (4).

§ 2

(2.1) A CW-complex is said to be of *finite type* if it has a finite number of cells in each dimension. In otherwords, a CW-complex with compact skeleta.

(2.2) Let $\beta\colon X\to\beta X$ be the embedding of a space X into its Stone–Čech compactification. For spaces X and Y, $[X,Y]$ denotes the set of homotopy classes of maps from X to Y. If $f\in\alpha\in[X,Y]$ then we denote α by $[f]$. The null homotopy class will be denoted by 0.

(2.3) Lemma. *If X is a finite dimensional normal space and Y is dominated* by a CW-complex of finite type, then $\beta\colon X\to\beta X$ induces a surjection*

$$\beta^*\colon [\beta X,Y]\to[X,Y]\,,$$

where $\beta^*[f]=[f\beta]$.

(**In fact, of course, Y then has the homotopy type of a CW-complex of finite type.*)

Proof. Let B be a CW-complex of finite type which dominates Y. That is, there exist maps $\varphi\colon Y\to B$ and $\psi\colon B\to Y$ such that $\psi\varphi\sim$ identity on Y.

By results of Barratt and Dowker there is a metric complex P homotopically equivalent to B, [14, p. 131]. Let $\sigma\colon B\to P$ be a homotopy equivalence and $\tau\colon P\to B$ a homotopy inverse to σ.

Let $f\colon X\to Y$ be a map. By (1.10) there is a finite dimensional simplical complex Q and maps $\pi\colon X\to Q$ and $g\colon Q\to P$ such that $g\pi$ is homotopic to $\sigma\varphi f$. By the cellular approximation theorem, there is a cellular map $h\colon Q\to B$ such that $h\sim\tau g$. Since B is of finite type and Q is finite dimensional, h is bounded. Hence $\psi h\pi\colon X\to Y$ is bounded and homotopic to f. Let $F\colon \beta X\to Y$ be the extension of $\psi h\pi$ then $\beta^*[F]=[f]$.

§ 3

(3.1) A space will be called "nice" if there is a CW-complex B which dominates it and B is the classifying space [3, (7.2)] of some topological group G and G is a CW-complex of finite type.

(3.2) LEMMA. *A "nice" space has the homotopy type of a* CW-*complex of finite type and finite fundamental group.*

Proof. Let G be a group of finite type and $\varrho\colon E\to B$ a universal G-bundle. Then B is a classifying space for G. From the homotopy sequence of the fibration ϱ, $\pi_1(B)\cong\pi_0(G)$ which is finite and $\pi_n(B)\cong\pi_{n-1}(G)$, which is finitely generated for $n>1$.

Now suppose Y is a space dominated by B then the identity on $\pi_n(Y)$ factors through $\pi_n(B)$ for all $n\geqslant 0$ and so $\pi_1(B)$ is finite and $\pi_n(B)$ is finitely generated. Because it is dominated by a CW-complex, Y must have the homotopy type of one [14, p. 137] and so by [23] Y must have the homotopy type of a CW-complex of finite type.

(3.3) In [2] the following lemma is proved.

LEMMA. *Let $p\colon E\to B$ be a principle G-bundle with G a topological group of finite type and B a compact space. Then for any finite dimensional* CW-*complex X and map $f\colon X\to E$ there is a bounded map $g\colon X\to E$ such that $pg=pf$.*

(3.5) THEOREM. *If X is a finite dimensional normal space and Y is a "nice" space then* $\ker\beta^*=(\beta^*)^{-1}\{0\}=\{0\}$.

In otherwords, a map $f\colon \beta X\to Y$ is null homotopic if and only if it is uniformly trivial.

Proof. Let Y be dominated by the CW-complex B and let B be the classifying space of a group G of finite type. We may assume that B is a metric complex since B is certainly homotopic to one and any space homotopic to B dominates Y and classifies G.

Let $p\colon E\to B$ be a universal G-bundle. Let $\varphi\colon Y\to B$ and $\psi\colon B\to Y$ be maps such that $\psi\varphi\colon Y\to Y$ is homotopic to the identity on Y.

Let $[f]\in\ker\beta^*$. Then $f\beta\colon X\to Y$ is bounded, and null homotopic and hence so is $\varphi f\beta\colon X\to B$. By (1.10) there is a finite dimensional simplicial complex Q and maps $\pi\colon X\to Q$ and $g\colon Q\to B$ such that

(A) $$\pi g\underset{\beta}{\sim}\varphi f\beta$$

and g is null homotopic and bounded.

So g lifts to E and so by (3.4) there is a bounded map $\hat{g}\colon Q\to E$ such that $p\hat{g}=g$. Now $\hat{g}\pi$ is bounded and so can be extended to a map $h\colon \beta X\to E$. But

$$ph\beta=p\hat{g}_Q=g\pi$$

and hence ph is the extension of $g\pi$ to βX. So $ph\sim df$ by (A). Thus since E is contractable [3, (7.2)],

$$*\sim\psi ph\sim\psi\varphi f\sim f.$$

(3.6) COROLLARY. *If X is a finite dimensional normal space and Y is a compact "nice" space then a map from X to Y is uniformly trivial if and only if it is null homotopic.*

(3.7) It follows from (2.3), (3.2) and (3.5) that if X is a finite dimensional normal space, Y is a "nice" space and $\beta^*\colon[\beta X, Y]\to[X, Y]$ is a homomorphism in some sense then β^* is a bijection. In particular this will be the case if Y is also an H-space because then $[-, Y]$ is a function from the category of pointed space to that of groups, [11, p. 2].

Another situation where β^* is a homomorphism is when, for some $n>1$, $\dim X\leqslant 2n$ (and hence $\dim\beta X\leqslant 2n$, [24]) and Y is n-connected [20]. The dimension condition on X is unnatural to homotopy considerations and can be replaced by the condition that X be $(2n+1)$-coconnected (the Čech cohomology groups $H^q(X)$ of X vanish for $q\geqslant 2n+1$) since then as we shall see in (4.3), βX is also $(2n+1)$-coconnected and one can use the standard methods of obstruction theory to define Borsuk–Spanier group structures on $[\beta X, Y]$ and $[X, Y]$ making β^* a homomorphism.

(3.8) THEOREM. *If X is a finite dimensional normal space, Y a compact "nice" space and either* (a) *Y is an H-space or* (b) *X is $(2n+1)$-coconnected and Y is n-connected for some $n>1$, then a pair of maps $f, g\colon X\to Y$ are homotopic if and only if they are uniformly homotopic.*

§ 4

(4.1) If π is a finitely generated abelian group then an Eilenberg–Maclane space $K(\pi, n)$ of type (π, n) is a "nice" space for $n>1$ and if π is a finite group $K(\pi, n)$ is "nice" for all n. This is because, for π finitely generated abelian finite $K(\pi, n)$ has finite type for $n>1$, $K(\pi, 0)=\pi$ with the discrete topology and $K(\pi, n)$ is the classifying space of $K(\pi, n-1)$.

The nth Čech cohomology functor $H^n(-;\pi)$ based on numerable cover and with coefficients group π is homotopy represented by $K(\pi, n)$ for all topological spaces [4, p. 366], i.e. $H^n(X;\pi)\cong[X, K(\pi, n)]$ and so from (3.7) and [2] we have the following.

(4.2) THEOREM. *If X is a finite dimensional normal space and π a finitely generated abelian group then $\beta\colon X\to\beta X$ induces an isomorphism*

$$\beta^*\colon H^n(\beta X;\pi)\to H^n(X;\pi)\quad \textit{for}\quad n>1$$

and

$$\beta^*\colon H^1(\beta X;\pi)/D \to H^1(X;\pi)$$

where D is a divisible subgroup of $H^1(\beta X;\pi)$.

If π is a finite group (not necessarily abelian) then

$$\beta^*\colon H^n(\beta X;\pi) \to H^n(X;\pi)$$

is an isomorphism for all n.

(4.3) COROLLARY. *A finite dimensional normal space X is n-coconnected $(n>1)$ if and only if βX is n-coconnected.*

(4.4) To obtain other examples of nice spaces we consider the following.

LEMMA. *If Z is a connected* CW*-complex of finite type then SZ, the suspension of Z, is a "nice" space.*

Proof. It can be deduced from the proof of Theorem 1 in [10] that the free (Greav) group FZ, on Z is a space of finite type.

Let $\mu\colon Z\times FZ\to Z$ be the restriction of the multiplication in FZ (Z is a subspace of FZ). Let E be the adjunction space

$$E = CZ\times FZ \cup_\mu FZ$$

where CZ is the cone on Z. Then $p\colon E\to SZ$ given by $p\langle\langle z,t\rangle, y\rangle = \langle z,t\rangle$ for all $z\in Z$, $t\in I$ and $y\in FZ$, is a principle FZ-bundle [13]. Here $\langle\ \rangle$ denotes "equivalence classes of" under the appropriate identifications.

I claim that p is a universal FZ-bundle. To see this let F^+ denote the free monoid (reduced product) on Z [12]. Then the natural inclusion $i\colon F^+\to FZ$ is a homotopy equivalence [15]. Let

$$E^+ = CZ\times F^+ \cup_\mu F^+,$$

then $p^+\colon E^+\to SZ$ given by $p^+\langle\langle z,t\rangle, y\rangle = \langle z,t\rangle$ is a quasifibration with E^+ contractable [13]. Now i and the identity on CZ induce an embedding $j\colon E^+\to E$. Applying "Five lemma" to the ladder of homotopy sequences given by p, p^+, i, j and the identity on SZ shows that $\pi_n(E) = 0$ for all n. Hence p is a universal FZ-bundle [4, (7.5)].

(4.5) COROLLARY. *If Y is dominated by a suspension of a connected* CW*-complex of finite type then Y is a "nice" space.*

Note. If Y is dominated by the suspension of a space then Y is an H'-space (co-H-space).

(4.6) THEOREM (cf. [1]). *For $n>1$, any map from a finite dimensional normal space into the n-sphere is uniformly trivial if and only if it is null homotopic.*

Proof. By (3.6) and (4.5).

§ 5

The condition that X by finite dimensional is essential in both (2.3) and (3.5) and hence for all the results of this paper, as the following examples show.

(5.1) EXAMPLE. Let $X = Y = K(Z,2)$. Then Y is nice. Let I_X denote the identity on X. Suppose $\beta^*\colon [\beta X, Y]\to[X,Y]$ is onto. Then there is a map $f\colon \beta X\to Y$ such that $f\beta = I_X$.

Since X is of finite type $f\beta(X)$ is contained in X^m the m-skeleton of X for some m. Let $i\colon X^m\to X$ be the inclusion map. Now $[if\beta] = [I_X]$ and so

$$H^q(X)\xrightarrow{i^*}H^q(X^m)\xrightarrow{f^*}H^q(\beta X)\xrightarrow{\beta^*}H^q(X)$$

is an isomorphism for all q.

Now $H^q(X^m) = 0$ for $q>m$, but $H^q(K(Z,2)) \neq 0$ for all even q [19, p. 84]. This is a contradiction so β^* is not onto.

(5.2) EXAMPLE. Let F be the Free (Greav) group on the $(n-1)$-sphere, $n>1$. Let $p\colon X\to S^n$ be the universal F-bundle constructed as in (4.4). Then X is contractable and so $[X, S^n] = 0$, but I claim that the extension $f\colon \beta X\to S^n$ of p is not null homotopic.

Suppose that $f\sim *$, then there exist a lift $g\colon \beta X\to X$ such that $pg = f$ and so $pg\beta = f\beta = p$. Now $\overline{g\beta(X)}$ is compact and $g\beta(p^{-1}\{x\}\subset p^{-1}\{x\}$ for all $x\in S^n$. Consider F as $p^{-1}\{*\}$ and the ladder of fiber homotopy sequences induced by the diagram

$$\begin{array}{ccc} X & \xrightarrow{g\beta} & X \\ {\scriptstyle p}\downarrow & & \downarrow{\scriptstyle p} \\ S^n & = & S^n \end{array}$$

Since X is contractable $g\beta|F\colon F\to F$ is a weak homotopy equivalence (by "Five lemma" and hence a homotopy equivalence by Whitehead's Theorem. Let $h\colon F\to F$ be a homotopy inverse to $g\beta|F$. Now $g\beta(F)$ is contained in F^m the m-skeleton of F for some m. Then

$$H^q(F)\xrightarrow{h^*}H^q(F^m)\xrightarrow{(g\beta)^*}H^q(F)$$

is an isomorphism for all q and so $H^q(F) = 0$ for $q>m$. But by the Wang sequence for p [22]

$$H^q(F)\neq 0 \quad \text{for all } q\equiv 0 \bmod (n-1).$$

This is a contradiction, so f is not null homotopic.

Hence if X is not finite dimensional, $\ker\beta^*$ need not be zero even for Y a compact "nice" space.

(5.3) It was shown in (3.2) that "nice" spaces have finite fundamental group. Assuming the Burnside conjecture for finitely presented groups ("Every infinite finitely presented group contains an element of infinite order") the next lemma shows that this requirement is essential to the problem.

LEMMA. *If X is non-pseudocompact and $\pi_1(Y)$ contains an element of infinite order then* $\ker(\beta^*: [\beta X, Y] \to [X, Y]) \neq 0$.

Proof. By [8, p. 68] there is a bijection between the homotopy classes of maps in $\ker(\beta^*: [\beta X, S^1] \to [X, S^1])$ and the quotient group of the additive group of real valued functions on X modulo the bounded ones. So unless X is pseudocompact (i.e. unless this group is 0) there exists a map $v: X \to S^1$ which is null homotopic but not uniformly trivial.

Let $w: \beta X \to S^1$ be the extension of v. Then w is essential and so induces an essential map $w^*: H^1(S^1) \to H^1(\beta X)$, Bruschlinsky's Theorem or [4, p. 366]. But then the map $w_*: H_1(\beta X) \to H_1(S^1)$ induced by w must be essential.

Let $w_\#: \pi_1(\beta X) \to \pi_1(S^1)$ be the map induced by w and let $q: \pi_1(\beta X) \to H_1(\beta X)$ be the quotient map obtained by considering H_1 as the abelianisation of π_1. Then $w_\# = w_* q$ and since q is an epimorphism, $w_\# \neq 0$.

Let $u: S^1 \to Y$ represent a element of infinite order in $\pi_1(Y)$ then $u_\#: \pi_1(S^1) \to \pi_1(Y)$ is a monomorphism and so $u_\# w_\# \neq 0$. Hence $uw: \beta X \to Y$ is an essential map.

(5.5) Remark. Throughout this paper the space X can be replaced by a pointed normal pair (X, A) (X a normal space, A a closed subspace of X and $* \in A$) by virtue of the fact that $[(X, A), (Y, *)] = [X/A, Y]$ and $\beta(X/A)$ is homeomorphic to $\beta X/\beta A$.

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