(β) *si* $\varepsilon_1 = -\varepsilon_2$, $n = 7k$ *et* $m = 5k$, *on a*

$$g(x) = (x^{3k}+\varepsilon_2 x^{2k}+\varepsilon_2)(x^{3k}-x^k+\varepsilon_1);$$

et hors des cas (α) *et* (β) *le polynôme* $g(x)$ *est irréductible.*

Notons enfin que toutes les racines de $f(x)$ qui sont en même temps celles de l'unité (s'il en existe)

(a) sont simples à l'exception du cas où $n = 2m$ et $\varepsilon_2 = 1$, dans lequel elles sont doubles,

(b) satisfont à l'équation

$$x^d = \begin{cases} 1 & \text{si } \varepsilon_1 = -1 \quad \text{et } \varepsilon_2 = 1, \\ -1 & \text{si } \varepsilon_1 = (-1)^{n_1+m_1+1} \text{ et } \varepsilon_2 = (-1)^{n_1}, \end{cases}$$

dans laquelle $d = (n, m)$, $n = dn_1$ et $m = dm_1$.

Cela élargit partiellement les résultats de Ljunggren (voir [1]), qui a étudié la réductibilité des polynômes

$$f(x) = x^n+\varepsilon_1 x^m+\varepsilon_2 x^p+\varepsilon_3 \quad \text{où} \quad n > m > p > 0 \quad \text{et} \quad \varepsilon_1, \varepsilon_2, \varepsilon_3 = \pm 1.$$

TRAVAUX CITÉS

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[2] W. Sierpiński, *Remarques sur les progressions arithmétiques*, Colloquium Mathematicum 3 (1954), p. 44-49.

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THREE CONSECUTIVE INTEGERS CANNOT BE POWERS

BY

A. MĄKOWSKI (WARSAW)

In book [3], p. 154, LeVeque has stated: "It has not even been shown that no three consecutive integers are powers..." Sierpiński [4], p. 135, has raised the following question: "Does there exist three successive naturals each of which is a power with a natural exponent >1 of a natural number?"

The purpose of this paper is to show that the answer to this question is in the negative.

It may be supposed without loss of generality that the exponents of the powers mentioned in the problem are prime numbers. We show that *the system of equations*

$$x^p - y^q = 1,$$
$$y^q - z^r = 1$$

has no solution in positive integers x, y, z *and prime numbers* p, q *and* r.

Let x, y, z, p, q, r satisfy this system. By the theorem of Cassels [1], p. 98, $q \mid x$, $q \mid z$. Hence $q \mid x^p - z^r = 2$ and $q = 2$. The first equation becomes $x^p = y^2+1$, but this equation, as V. A. Lebesgue proved in [2], has no solution in integers with $y > 0$. This answers the question of Sierpiński.

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UNIVERSITY OF WARSAW

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