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ANZELM IWANIK (1946–1998)

BY

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Anzelm Iwanik was born on April 21, 1946, in Tomaszów Mazowiecki, a town in central Poland. He was the youngest of three children of Hipolit Iwanik, a chemical engineer, and Ludwika (née Lechowska), a dentist. In 1963, after completing his school education, he enrolled at the Technical University of Wrocław. In 1969 he received a Master of Science degree in Electronics and became an assistant at the Institute of Electric Metrology of the Technical University. Simultaneously, in the years 1968–1972, he was an extramural student of mathematics at Wrocław University. He completed his studies with a master thesis entitled *Complete algebras with infinite support* (whose main results are contained in [1a,b]) and obtained a diploma with distinction. Encouraged by Edward Marczewski, the supervisor of his thesis, he moved to the Institute of Mathematics of the Technical University of Wrocław in 1972 and remained on the staff there until the end of his life.

Due to his outstanding abilities, great industry and research passion, Iwanik quickly progressed in his career. In 1974 he obtained his doctoral degree for a dissertation entitled *Point realizations of transformation semigroups* (published, in part, as [11]) prepared under the supervision of Czesław Ryll-Nardzewski at the Technical University of Wrocław. In 1978 the Scientific Council of the Institute of Mathematics of the Polish Academy of Sciences conferred on him a second degree (habilitation) for a dissertation entitled *Extreme operators on classical Banach spaces* (comprising the papers [10], [12], [13] and [14]). He obtained a professorship in 1990 and was appointed full professor in 1996. He died on September 28, 1998, in Wrocław.

The scientific output of Anzelm Iwanik consists of 60 papers (20 of which were written jointly with other authors), published mostly in renowned mathematical journals. His first papers concerned abstract algebra, espe-

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cially the theory of semigroups. Next, he dealt with semigroups of measurable transformations and semigroups of operators on function spaces, and with the general theory of operators on such spaces. Finally, he concentrated on topological dynamics and ergodic theory, where he obtained his most important results. We present below a survey of his work.

A. Universal algebra and semigroups. Papers [1a,b], [2], [3] and [4], [5], [8a,b], [9], respectively, are devoted to these domains. Inspired by work of L. Bieberbach (1931) and W. Sierpiński (1931), Iwanik established in [1a,b] some results concerning an optimal choice of fundamental operations in a complete algebra (i.e., such that every operation on its support is algebraic). Those results were cited later by J. C. Oxtoby, P. Erdős and R. D. Mauldin. In [8a,b], it is proved, using a composition theorem of Sierpiński (1935), that every countable semigroup can be embedded in a semigroup generated by three idempotents.

Paper [11], inspired by work of R. Sikorski (1949) and G. W. Mackey (1962), lies in the boundary between the theory of semigroups and that of Boolean algebras. It contains a partial solution to the problem of representing a semigroup of σ-endomorphisms of a measurable space modulo a σ-ideal by a semigroup of pointwise transformations. A similar representation problem is also studied in [9].

B. General theory of operators on function spaces. In a series of papers [7], [12], [13], [15], [20], [21] (related problems are also discussed in [9], [11] and [16]) Iwanik dealt with various representations of (continuous linear) operators on $L^p(\mu)$ and on $C(X)$, where $1 \le p \le \infty$, μ is a positive measure, and X is a compact space, with geometric and topological properties of the unit ball and some of its subsets in operator spaces, and with norm attaining operators.

In [13] he established a necessary and sufficient condition that the unit ball $\mathcal{U}$ in $L(L^1(\mu), L^1(\nu))$ satisfy the assertion of the Krein–Milman theorem with respect to the weak (equivalently, strong) operator topology. In [12] he found, under some assumptions on the measures μ and ν, a description of the extreme points of $\mathcal{U}$. (Extremality in spaces of operators was further studied by his student R. Grząślewicz.)

The main theorem of [15] asserts that the norm attaining operators are (norm) dense in $L(L^1(\mu), L^1(\nu))$. This theorem refuted a conjecture of J. J. Uhl, Jr. (1976) that for a Banach space E the norm attaining operators are dense in $L(L^1[0,1], E)$ if and only if E has the Radon–Nikodym property. Norm attainment is still an active subject of research and many authors, e.g., Y. S. Choi (1997) and M. D. Acosta (1999), refer to [15].

C. Stochastic operators, Markov operators, and multiple recurrence. An important theorem proved in [16] asserts that a typical (in the sense of category) stochastic operator is ergodic and conservative. This theorem is discussed in a survey article by J. R. Choksi and V. S. Prasad (1983), and subsequently applied by the same authors jointly with S. J. Eigen (1987) in their investigations on when a stochastic operator preserves a given measure. Moreover, various generalizations of Iwanik's theorem were obtained by his students W. Bartoszek and R. Rębowski. In [39] and [42] the following question is considered: In which operator spaces (and topologies) is mixing a typical property of a stochastic operator? A positive result is obtained for operators on $L^1(m)$, where m is a σ-finite measure, the property of complete mixing and both the norm and strong operator topologies. For the latter topology a similar result also holds in the set of operators preserving an arbitrary σ-finite measure λ equivalent to m.

In [24] the existence and properties of roots of stochastic operators on L^1-spaces are considered. A necessary condition for the existence of nth roots of an ergodic and conservative operator is formulated in terms of eigenvalues, extending the corresponding part of a classical theorem of P. R. Halmos (1956) on pointwise induced (i.e., deterministic) operators. Convolution operators, doubly stochastic operators, and stochastic matrices are studied more closely in this context. In particular, in the class of convolution operators an example is provided to show that the other direction of Halmos' theorem fails for non-pointwise induced operators.

Many important results of Iwanik in the theory of Markov operators are related to uniqueness of an invariant measure, nowadays called monoergodicity. For instance, in [18] he proved that every weak* mean ergodic Markov operator T on $C(X)$, where X is a compact metric space, restricted to every minimal invariant subset of X is monoergodic. (Weak* mean ergodicity is the property that the adjoints to the Cesàro means of T converge in the weak* operator topology on the dual of $C(X)$.) For pointwise induced T this theorem is due to J. C. Oxtoby (1952). Iwanik's theorem was further extended by W. Bartoszek (1987) to arbitrary compact Hausdorff spaces and applied to solving a problem of H. P. Lotz. It is also mentioned in the monograph *Ergodic Theorems* by U. Krengel (Walter de Gruyter & Co., 1985). Some other conditions sufficient for monoergodicity are given in [22]. In [23] an interesting example of an irreducible pointwise induced Markov operator (in other words, a minimal dynamical system) is constructed, whose set of ergodic measures is homeomorphic to a multidimensional torus. Independently and simultaneously, S. Williams (1984) proved that every compact metric space can be realized in this way. Inspired by these results, Iwanik raised the question of whether the set of ergodic measures of a minimal dynamical system is always compact. This question was answered in the negative by another student of Iwanik, T. Downarowicz (1988, 1991).

A continuation of the earlier investigations on representations of operators and extremality (see Section B) can be found in [25], where it is proved that every stochastic operator on L^1 can be represented as the barycenter of a measure supported by the set of pointwise induced operators. The extreme points in the set of Markov operators are those induced by continuous transformations, but then, as proved by Iwanik, an analogous representation theorem fails, in general. However, he also obtained some positive results in that direction.

Another important subject of Iwanik's research on Markov operators is the phenomenon of multiple recurrence (or multi-recurrence, in more modern terminology), i.e., recurrence of a point to its neighborhood simultaneously under the action of several operators. A classical theorem of Furstenberg asserts that multi-recurrent points exist for finitely many commuting continuous transformations. Even though the notion of recurrence is weaker for Markov operators than for transformations, there is no obvious way of adapting the proofs of either Furstenberg's theorem or of its measure-theoretic analog (known as the Furstenberg–Szemerédi theorem) to establish the existence of multi-recurrent points in the case of operators. Nevertheless, in [26] and [28] the above-mentioned deterministic theorems are successfully applied for two commuting Markov operators. The method relies on constructing a measure on the set of trajectories (with two-dimensional time) invariant under both vertical and horizontal shifts and compatible with these operators (i.e., yielding the same stationary N^2-process). This method fails in higher dimensions; in [28] an example is given of three commuting stochastic matrices admitting no trajectories at all. The multiple recurrence problem for three (or more) commuting Markov operators still remains open.

D. Topological dynamics. In the second half of the 80s, Iwanik devoted a large part of his research to this area (some earlier examples of pointwise induced operators, already mentioned in Section C, also belong here). Papers [27] and [32] are about the set of minimal periods of pointwise periodic transformations of compact Hausdorff spaces. For instance, it is proved in [27] that, under the additional assumption of equicontinuity, this set consists of multiples of finitely many natural numbers. This subject matter is closely related to the notion of independence in topological dynamics discussed in [31], [32] and [38]. The results of these papers concern mainly the existence of large independent sets (e.g., uncountable, nonmeasurable, or of full measure).

Among the main achievements of Iwanik in topological dynamics is an application of the Weyl pseudometric to the study of certain properties of dynamical systems, especially to the study of Toeplitz flows. Both notions were introduced in a 1969 paper by K. Jacobs and M. Keane. Iwanik generalized some results of that paper by proving that every almost periodic point in the sense of the Weyl pseudometric generates a monoergodic system with discrete spectrum ([29]). In [30] it is shown that the number of minimal invariant sets, the number of ergodic measures and the topological entropy of the orbit-closure of a point are lower semi-continuous functions with respect to the Weyl pseudometric. As an application, some questions concerning possible realizations of these three parameters in the class of Toeplitz flows have been answered. The methods introduced in [30] were applied later by Iwanik and are still being applied by his students and collaborators in constructions of minimal almost 1-1 extensions of group rotations (or minimal flows) with a prescribed set of invariant measures or with prescribed values of certain topological and measure-theoretic parameters such as spectral type and multiplicity, rank, entropy, etc.

Iwanik returned to Toeplitz flows in [47] and [51], where monoergodicity of such flows is studied, and where examples are given with interesting spectral and approximation properties. For instance, he constructed a Toeplitz flow with an irrational eigenvalue. This construction was later generalized by his collaborators.

In [55] Iwanik gave a complete answer to the following general question: When does an abstract set X with a permutation T admit a compact Hausdorff topology in which T is a homeomorphism? Namely, it is always so for $|X| \geq c$, while in the opposite case some conditions on the orbits are needed. This generalizes and improves a theorem of H. de Vries (1957) established, under the continuum hypothesis, for $|X| = c$.

E. Spectral theory of measure-preserving transformations. Starting from the late 80s, this subject was dominant in Iwanik's research. Papers [36], [37] and [40] deal with spectral multiplicity in L^p-spaces. Inspired by the still open question of J.-P. Thouvenot of whether a Bernoulli automorphism can have an L^1-simple spectrum (in L^2 such a spectrum is always of infinite multiplicity), he proved that in the class of group automorphisms for every finite $p > 1$ the multiplicity in L^p is always infinite. Moreover, for Gaussian automorphisms he proved that the spectral multiplicity does not depend on p (as long as $p > 1$).

In the 90s Iwanik published many papers (some of them jointly with other authors) about the so-called Anzai skew products ([43]–[46], [48], [50], [52], [53], [56], [58]). In the totality of Iwanik's published work, his contribution to the spectral theory of Anzai skew products is undoubtedly the most significant and has been recognized by other specialists in Poland and abroad.

An Anzai skew product is a map $T : [0,1) \times [0,1) \to [0,1) \times [0,1)$ given by

$$T(x, y) := (x + \alpha, y + \phi(x)),$$

where addition is taken modulo 1, the rotation number α is irrational, and $\phi : [0,1) \to [0,1)$ is a measurable function, called a cocycle. Iwanik's main results describe how spectral properties of T depend on the smoothness of ϕ and approximation properties of α. We formulate below some of his theorems:

(1) If ϕ is absolutely continuous, has nonzero topological degree and its derivative has bounded variation, then T has countable Lebesgue spectrum in the orthocomplement to the subspace spanned by the eigenfunctions ([43]; an analogous theorem was formerly known for C^2-cocycles). A similar result was obtained in [52] for some C^1-cocycles. An interesting construction is presented in [43] of a degree one continuous cocycle having bounded variation, which is a coboundary; this is done over the rotation about arbitrary α with unbounded partial quotients. A possibility of such a construction was anticipated by H. Furstenberg in the 60s. It is worth noting that in the slightly narrower class of absolutely continuous cocycles, a similar phenomenon does not occur. In a technically involved paper [58] singularity of the spectrum is proved for certain piecewise absolutely continuous cocycles.

(2) Let α admit a rational approximation with speed $o(1/n)$. Then the "majority" of cocycles (i.e., a residual set in L^1) generate a skew product of rank one (hence have simple spectrum) and provide a weakly mixing extension of the rotation by α ([44]). In a later paper [46] the former property has been proved for an arbitrary irrational α. Similar theorems hold for C^r-cocycles with topological degree zero ([48]) or for analytic cocycles ([50]). In [48] and [50] some estimates for the speed of cyclic approximation of Anzai skew products are given, as well as some bounds for the Hausdorff dimension of the maximal spectral type. In [46] we find an interesting theorem concerning the cyclic approximation of the irrational rotation itself: it is as good as the rational approximation of the rotation number α. This result refuted the opinion that irrational rotations are hardly distinguished in terms of ergodic theory.

(3) Some of the results discussed in (1) and (2) have been generalized in [53] to the case of an n-dimensional torus. The topological degree is then replaced by the rank of the rotation matrix. Generally speaking, if the rank is zero, then a typical cocycle generates a skew product admitting a fast cyclic approximation, hence having simple spectrum. If the rank equals n, then countable Lebesgue spectrum is typical. In intermediate cases we obtain mixed spectra—partly simple singular, partly countable Lebesgue. Such spectra appear very rarely (if ever) in explicit examples of ergodic

transformations. In [56] a description of the spectrum is given for certain real-valued cocycles (inducing skew products with an infinite measure).

F. Other topics. Papers [10], [14], [34], [35], [41], [54], [57], [59] and [60] are devoted to topics related to but not directly included in Iwanik's main areas of interest. We briefly discuss some of them below.

In [10] functionals and operators of on some *normed commutative Banach algebras are considered. Extremality in the positive part of the unit balls of the corresponding Banach spaces, multiplicativity, and preservation of the absolute value are proved to coincide. This topic originates with analogous results for algebras $C(X)$, where X is a compact Hausdorff space, due to A. and C. Ionescu Tulcea (1961) and A. J. Ellis (1964). Related material is also contained in [12].

Paper [14] is devoted to locally compact groups of operators on Banach lattices, including L^1-spaces. It establishes a generalization of the Blum–Hanson theorem on strong mixing. Some methods developed in [14] were subsequently applied and generalized by A. Tempel′man (*Ergodic Theorems on Groups*, Vilnius Mokslas Publishers, 1986; in Russian).

In [34] a positive answer to the two-dimensional case of a problem raised by T. M. Rassias (1990) is obtained. Namely, it is shown that, given a doubly stochastic measure μ on $I\times I$, where $I=[0,1]$, absolutely continuous with respect to the two-dimensional Lebesgue measure, and $C\subset I\times I$ with $\mu(C)=1$, there exists a Lebesgue measure preserving transformation T of I such that $(x,Tx)\in C$ for almost all $x\in I$. For an infinite-dimensional version of the problem a negative answer is established in [41].

In [57] submeasures on a finite set E are considered that are upper envelopes of a family of measures on E. In the case of symmetric submeasures, the minimal cardinality of such a family is estimated from above by $2^{|E|-1}$, and this estimate is shown to be sharp.

In 1977 M. Keane and M. Smorodinsky constructed a finitary code between arbitrary Bernoulli shifts B_1, B_2 such that B_2 has strictly smaller entropy than B_1. Their construction was then adapted by M. A. Akcoglu, A. del Junco and M. Rahe (1979) to obtain a finitary code between mixing Markov shifts with different entropies. J. Serafin (1996) proved that the expected length of the Keane–Smorodinsky code is finite. In [59], through highly nontrivial calculations, the length of the corresponding code between mixing Markov shifts is proved to have finite expectation as well. Moreover, it has finite moments of all orders $1\le p<4/3$.

Paper [60] (presented in this volume) is about odometers. A (generalized) odometer $\mathcal{K}_G$ is defined as the closure of the set of expansions of the natural numbers with respect to a general increasing base G, the action on $\mathcal{K}_G$ is an extension of the operation of adding 1 on the integers. Such an action need not be continuous and it may happen that the odometer carries no invariant measures. In [60] conditions for continuity of the action and minimality of $\mathcal{K}_G$ in terms of elementary properties of G are found. Moreover, odometers are classified by their topological and combinatorial properties.

Anzelm Iwanik was honored many times for his scientific, pedagogical and organizational achievements. He received the Polish Mathematical Society Award for Young Mathematicians (1974), an award of the Polish Academy of Sciences (1980), three ministerial awards and numerous awards of his university.

Four Ph.D. theses (see the list on p. 12) and nine M.Sc. theses were written under his supervision. Three of his Ph.D. students subsequently obtained a second degree; one of them is now a full professor.

For more than ten years Iwanik conducted, jointly with Zbigniew S. Kowalski, a research seminar on ergodic theory at the Institute of Mathematics of the Technical University. In addition to his students and close collaborators, seminar speakers included numerous guests from many Polish and foreign centers. Iwanik also co-organized two international conferences on ergodic theory (1989, 1997) and participated in more than 60 conferences devoted to various fields of mathematics. In the 70s and 80s he held several visiting positions in the USA (Carbondale, Illinois; Fullerton, California) and Canada (Montreal). In the 90s he was a frequent guest at French universities (Aix-Marseille I, Aix-Marseille II, Brest, Paris XIII, Rouen).

Iwanik was a reviewer for Mathematical Reviews starting in 1985 and, in the last months of his life, he began to write reviews for the Zentralblatt für Mathematik. He refereed papers for numerous Polish and foreign mathematical journals. He served on the Editorial Committee of this journal from 1978 until his death. He played an important role in editing E. Marczewski's *Collected Mathematical Papers* published by the Institute of Mathematics of the Polish Academy of Sciences (Warsaw, 1996).

Iwanik managed to combine extremely intensive research work with numerous pedagogical and organizational activities. He was a talented teacher esteemed by his students, colleagues and university authorities. He held many courses of lectures and guided seminars on various subjects for undergraduate, graduate and postgraduate students. For two terms he was Vice-Director of the Institute, charged with organization of research and co-operation with industry. He was also active in the Polish Mathematical Society, serving, in particular, as President of the Wrocław Branch in the years 1987–1989. He did not seek posts and influence. However, having assumed a duty, he carried it out with his usual conscientiousness and integrity.

Professor Iwanik was a demanding teacher and researcher, setting high standards for his students and colleagues, but even higher for himself. He had broad mathematical interests; he read extensively and eagerly attended seminar talks on various topics. He was able to quickly absorb new ideas; he made apt remarks and asked penetrating questions. He was an authority not only on matters of research; his opinions on other matters of scientific life were also held in great esteem in the mathematical community. Iwanik was ambitious and, at the same time, modest; he enjoyed helping people and was popular with those who had the pleasure of knowing him. His family and friends called him Anek—a nickname he coined himself. He retained good spirits and great activity even in the last period of his life when struggling with a fatal illness.

Anek had many interests outside mathematics. He liked learning foreign languages; he was fluent in English and French, good at Russian, and well-read in the literature of those languages. He took pleasure in visiting museums, and was especially interested in paintings. He was a keen tourist; mountains and lakes attracted him most. Summer canoe wanderings were among his favorite ways of spending a holiday. He was also a good swimmer, and he enjoyed picking mushrooms and birdwatching.

In September 1998 Anek was planning to attend a conference on real functions and measure theory in Italy. He intended to go there by car, with the purpose of doing some sightseeing on the way. Unfortunately, a sudden worsening of his health destroyed those plans and, in the end, led to his untimely death.

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