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ON TUBES FOR BLOCKS OF WILD TYPE

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Abstract. We show that any block of a group algebra of some finite group which is of wild representation type has many families of stable tubes.

The Auslander–Reiten quiver is an important homological invariant of a finite-dimensional algebra. We are interested in the Auslander–Reiten quiver of a block B of a group algebra kG where G is a finite group, and k is any field of characteristic $p > 0$; especially when the block B has wild representation type. By [6] (and [10]), any component of the stable Auslander–Reiten quiver of B is either of the form $\mathbb{Z}A_\infty$ or a tube.

Here we are concerned with the existence of tubes; one can deduce from [3] that any block of infinite type has at least one tube. In the present paper we show that the stable Auslander–Reiten quiver of a block of wild type has a large number of tubes. We exhibit a family of tubes parametrized as $\mathcal{T}_\lambda$, where $\lambda \in k^s$ for $s = p-1$ if $p > 2$, and $s \geq 2$ if $p = 2$, consisting of absolutely indecomposable modules. We note that for blocks of tame type, there are only 1-parameter tube families.

This is in contrast with other classes of finite-dimensional algebras. For example, let A be a connected algebra which is hereditary of infinite type. Then the stable Auslander–Reiten quiver of A has tubes if and only if A is tame. Further results may be found in [12].

For general results on algebras, we refer to [2]; for general properties of group representations, see [7].

1. Preliminaries. Assume that G is a finite group and k is a field of characteristic p. We work with kG-modules. Recall that kG is a symmetric algebra and therefore the Auslander–Reiten translation τ is isomorphic to Ω^2. We denote the Auslander–Reiten sequence $0 \to \tau(X) \to Y \to X \to 0$ by $\mathcal{A}(X)$, and the *quasi-length* of a module is the number of the row of the component to which it belongs if this has tree class A_∞.

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Let H be a subgroup of G. If M is a kH-module then we denote the induced module $M \otimes_{kH} kG$ by M^G, and if W is a kG-module then the restriction of W to kH is written as $W{\downarrow}_H$. Recall that a G-module W is *H-projective* (G arbitrary, H any subgroup) if there is some kH-module X such that W is a direct summand of X^G. We also recall that a kG-module M is *absolutely indecomposable* if for all finite extension fields F of k, the FG-module $M \otimes_k F$ is indecomposable.

The following is now well known; see [11], Cor. 9.4, p. 155.

1.1. LEMMA. *Let W be an indecomposable non-projective kG-module, and let W have Auslander–Reiten sequence*

$$\mathcal{A}(W): \quad 0 \to \tau(W) \to X \to W \to 0.$$

If H is a subgroup of G then $\mathcal{A}(W){\downarrow}_H$ splits if and only if W is not H-projective.

1.2. If W is indecomposable then a *vertex* of W is defined to be a minimal subgroup H such that W is H-projective. It is well known that vertices of W are unique up to conjugation and that they are p-subgroups.

If B is a block of kG recall that a *defect group* of the block is a subgroup D of G which is minimal such that all modules in B are D-projective. Then D is unique up to conjugation and it is a p-group.

The block is of wild representation type if and only if a defect group D is not cyclic or dihedral, semidihedral, or quaternion [5]. We shall use the fact that then D has a subgroup which is either elementary abelian of order p^2 (if $p > 2$), or non-cyclic abelian of order 8 (if $p = 2$).

1.3. Assume that $H = \langle x, y\rangle$, elementary abelian of order p^2. We will construct a k^{p-1}-family of periodic modules of dimension p and τ-period one.

Take a p-dimensional vector space; we define a representation on this space by specifying two commuting matrices X, Y of size $p \times p$ such that $X^p = 0$ and $Y^p = 0$ (where X, Y represent $x-1, y-1$ respectively). For X we take the indecomposable Jordan block with eigenvalue 0. Then for Y we can take any polynomial in X with zero constant term. Take Y to be a polynomial in X of degree $\le p-1$ with constant term zero. We label this as M_λ where $\lambda = (\lambda_1, \ldots, \lambda_{p-1}) \in k^{p-1}$ if $Y = \sum \lambda_i X^i$. Then $M_\lambda \cong M_\mu$ if and only if $\lambda = \mu$. By considering the restriction to $\langle x\rangle$ it is clear that any such module is absolutely indecomposable.

We claim that $\tau(M_\lambda) \cong M_\lambda$. This can be seen by general theory of varieties (the support variety of the module is a line; and for group algebras of abelian groups, all periodic modules have Ω-period at most two).

Alternatively, it is easy to prove it directly. Let $\zeta \in kH$ be the element

$$\zeta = Y - \sum_{i=0}^{p-1} \lambda_i X^i$$

(with $\lambda_0 = 1$). One checks that ζ generates the annihilator of M_λ, i.e. we can identify $\Omega(M_\lambda)$ with ζkH. Moreover, $\zeta^{p-1}kH \cong M_\lambda$ and ζ^{p-1} generates the annihilator of ζkH. So $\Omega^2(M_\lambda) \cong M_\lambda$.

1.4. Now assume that $p = 2$ and $H = \langle x, y, z\rangle$ is elementary abelian of order 8. We will similarly construct a family of absolutely indecomposable modules with $\tau(M) \cong M$ of dimension 4.

We start off by ensuring that the action of $\langle x, y\rangle$ is free (then M is absolutely indecomposable and will have τ-period 1, similarly to 1.3). So let X, Y be the matrices

$$X = \begin{pmatrix} J & 0 \\ 0 & J \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & I \\ 0 & 0 \end{pmatrix}.$$

where $J = \left(\begin{smallmatrix} 0 & 1 \\ 0 & 0 \end{smallmatrix}\right)$ and I is the 2×2 identity matrix. Then to get a representation of H we need a matrix Z with square zero which commutes with X and Y. Take $Z = \lambda_1 X + \lambda_2 Y + \lambda_3 XY$ for $\lambda_i \in k$, and denote the module by M_λ where $\lambda = (\lambda_1, \lambda_2, \lambda_3)$. As in 1.3 the isomorphism type is parametrized by $\lambda \in k^3$. The module has Ω-period one and lies in a 1-tube and is absolutely indecomposable.

1.5. Now assume that $p = 2$ and H is a group isomorphic to $C_2 \times C_4$, say $H = \langle x, y\rangle$ with x of order 2 and y of order 4. We will construct a similar family of modules with τ-period one. Take a space of dimension four. We let y act freely, so we take for Y (representing $y - 1$) the indecomposable Jordan block of size 4 with eigenvalue 0. Then we let X act as $\lambda_1 Y^2 + \lambda_2 Y^3$, and X represents $x - 1$. This is a representation and the isomorphism types are parametrized by $(\lambda_1, \lambda_2) \in k^2$. The module has Ω-period one and lies therefore in a 1-tube, and it is absolutely indecomposable.

2. The p-group case. Assume that G is a p-group. Then the group algebra kG is indecomposable, hence is a block. We assume that it is of wild representation type, so G is not cyclic or dihedral, semidihedral, or generalized quaternion (since G is a defect group). In this situation the trivial module, the only simple module, is not periodic; this is well known for example from group cohomology. Hence all tubes are stable.

The advantage of working in this situation is that Green's Theorem is available. Namely, if H is a subgroup of G and M is an absolutely indecomposable kH-module then the induced module M^G is absolutely indecomposable (see [7], Ch. VII). Note that if some indecomposable module in a tube is absolutely indecomposable then so are all modules in this tube. Therefore we can assume in this section that the field is algebraically closed.

Recall that kG is free as a module over kH, which implies that inducing commutes with Ω and then also with τ.

2.1. For the rest of Section 2 we assume that $H \lhd G$, that is, H is a normal subgroup of G and $H \neq G$. If M is any kH-module and $g \in G \setminus H$ then the conjugate $M^g := M \otimes g \subset M^G$ is an H-module. Let $I(M)$ be the stabilizer of M, that is, $I(M) = \{g \in G : M^g \cong M\}$. Then we have

$$M^G{\downarrow}_H = (M_1 \oplus \ldots \oplus M_s)^a$$

where the M_i are the pairwise non-isomorphic conjugates, and where $sa = [G : H]$ and $s = [G : I(M)]$. (For any module X we denote by X^a the direct sum of a copies of X). We say that M is *G-invariant* if $I(M) = G$. The group also acts by conjugation on the Auslander–Reiten components of H, which induces a graph automorphism. Clearly, if M belongs to a tube then $I(M)$ is also the stabilizer of the component.

2.2. Lemma ([4], (1.7)). *Let k be algebraically closed, let H be a normal subgroup of a p-group G and assume that M is an indecomposable non-projective H-module which is G-invariant. Then*

$$\mathcal{A}(M^G){\downarrow}_H \cong \mathcal{A}(M) \oplus \mathcal{E}$$

where $\mathcal{E}$ is split. In particular, $\mathcal{A}(M^G) \neq \mathcal{A}(M)^G$.

Let M be an indecomposable kH-module such that $I(M) = H$, the other extreme. Then we have $\mathcal{A}(M^G) = \mathcal{A}(M)^G$, by [4], (2.1); and then the same holds for all modules in the component of M.

2.3. We shall frequently use the following properties of some indecomposable G-module X.

(1) If $X{\downarrow}_H$ is not a direct sum of $[G : H]$ conjugates of some module then X is not H-projective. For otherwise there is some indecomposable H-module Y such that $X = Y^G$ and then the restriction of X to H is a summand of $Y^G{\downarrow}_H$ which is a sum of $[G : H]$ conjugates.

By similar arguments one has:

(2) Suppose $X{\downarrow}_H = \oplus M$ where M is indecomposable. If X is H-projective then $X \cong M^G$.

Assume Θ is a component of H whose stable part consists of modules of τ-period one. If H is cyclic, let $U_s \in \Theta$ have dimension s, for $1 \le s \le |H|$. Then $U_{|H|}$ is projective. Otherwise Θ is a stable 1-tube. Let U_t have quasi-length t, for $t \ge 1$ (cf. [9]). (Note that if H is quaternion then the only unstable component is a 2-tube.)

Let $M = U_s$ be non-projective. Then M^G is indecomposable, with $\tau(M^G) \cong M^G$. Since G is not cyclic the component $\mathcal{T}$ of M^G is a 1-tube. So let E_t be the module in the component of M^G which has quasi-length t. If $M^G = E_r$ then the AR-sequence of M^G is of the form

$$(*) \qquad 0 \to M^G \to E_{r+1} \oplus E_{r-1} \to M^G \to 0$$

(with $E_{r-1} = 0$ if $r = 1$).

2.4. PROPOSITION. *Assume $H \lhd G$ and Θ is a component of H whose stable part consists of absolutely indecomposable modules with τ-period one. If $M \in \Theta$ then M^G lies in a 1-tube. Moreover, one of the following holds*:

(a) *The component of M^G contains all modules $(U_s)^G$ with $U_s \in \Theta$, and* $\mathrm{ql}(U_s)^G = rs$ *where* $1 < r \equiv 0 \pmod p$. *The component Θ is a tube, and H is not cyclic.*

(b) *For any M in Θ, M^G has quasi-length which properly divides the index $[G : H]$. No two modules M^G for M in Θ lie in the same component. No further module in any of these components is H-projective.*

Proof. We may assume that k is algebraically closed. Let $M = U_s$ and $M^G = E_r$, with the above notation. By 2.2 and $(*)$ we have $(E_{r-1} \oplus E_{r+1})\downarrow_H \cong U \oplus M^d$ and $d = 2[G : H] - 2 = 2p^m - 2$. We set

$$E_{r-1}\downarrow_H = U' \oplus M^a, \qquad E_{r+1}\downarrow_H = U'' \oplus M^{d-a}$$

with $U' \oplus U'' \cong U$ (where U' could be zero), and $0 \le a \le d$.

For each t define $c(t) \ge 0$ to be the integer such that $E_t\downarrow_H \cong W \oplus M^{c(t)}$ and that W does not have a summand isomorphic to M.

(i) By 2.3, E_{r+1} cannot be H-projective and hence $\mathcal{A}(E_{r+1})\downarrow_H$ splits. We deduce that

$$(E_{r+2} \oplus E_r)\downarrow_H = (U'')^2 \oplus M^{2(d-a)}$$

and since $E_r\downarrow_H = M^{p^m}$ we have

$$E_{r+2}\downarrow_H = (U'')^2 \oplus M^{2(d-a)-p^m},$$

that is, $c(r + 2) = 2(d - a) - p^m$. If $c(r + 2) \ne 0$ then E_{r+2} cannot be H-projective and by the same argument we get $E_{r+3}\downarrow_H = (U'')^3 \oplus M^{c(r+3)}$ where $c(r + 3) = 3(d - a) - 2p^m$, and so on, and inductively if E_{r+s} is not H-projective for $s = 1, \ldots, t - 1$ then

$$E_{r+t}\downarrow_H = (U'')^t \oplus M^{c(r+t)} \qquad \text{with } c(r + t) = t(d - a) - (t - 1)p^m.$$

(ii) Similarly, if E_{r-s} is not H-projective for $s = 1, \ldots, t - 1$ then

$$E_{r-t}\downarrow_H = (U')^t \oplus M^{c(r-t)}, \qquad c(r - t) = ta - (t - 1)p^m.$$

CASE 1. Assume first that $E_{r-1} = 0$, that is, $r = 1$. Then $a = 0$ and in (i) we have $c(1 + t) = t(p^m - 2) + p^m > 0$ for all $t \ge 1$ and E_t is not

H-projective for all $t > 1$, hence the component of M^G does not contain any other module from Θ, proving part of (b).

CASE 2. Assume $U' \neq 0$. Then E_{r-1} is not H-projective, and we use (ii) to study decreasing quasi-length; the process in (ii) must stop. So let t be the first integer such that E_{r-t} is H-projective. Then $E_{r-t} = (U')^G$, so U' is indecomposable, and hence $t = [G : H] = p^m$ and $c(r-t) = 0$. We get $a = p^m - 1$. By dimensions $U' = U_{s-1}$; note also that $E_{r-t+1}{\downarrow_H} = (U')^{p^m-1} \oplus M$.

So if we repeat the argument with U' instead of M we find, for $k = 1, 2, \ldots$ as long as $r - kp > 0$, that $E_{r-kp} = (U_{s-k})^G$. Consider the case $k = s-1$; then $E_{r-(s-1)p-1}{\downarrow} = U_1^{p^m-1}$ and hence we get $E_1{\downarrow_H} = U_1$.

Now if we study increasing quasi-length then since $d - a = a$ here, we get $c(r + p^m) = 0$ and then it follows that $E_{r+p^m} = (U'')^G$ and so on. We see that the component contains $(U_b)^G$ for all b. Note also that in this case Θ must be infinite, so H must be non-cyclic.

CASE 3. Let $E_{r-1} \neq 0$ but $U' = 0$. Then we deduce from (i), since $c(r-t) = 0$ for $t = r$, that

$$ra = (r-1)p^m$$

and $r = p^b > 1$, $a = p^{m-b}(p^b - 1)$.

If $b = m$ then we get $E_{r+t} \cong U^G$; in particular, U must be indecomposable, so $U = U_2$ and $M = U_1$. We are in Case 2 with U instead of M. We deduce that the component contains all $(U_s)^G$ for $U_s \in \Theta$.

On the other hand, suppose $b < m$; then $c(r+t) > 0$ for $t = 1, 2, \ldots$ and M^G is the only H-projective module in the component. So M satisfies part (b) of the statement, the quasi-length of M^G is $p^b \geq 1$ and is a proper divisor of $[G : H]$.

So we have proved that either the component of M^G contains all modules induced from Θ or just one; and if just one then it follows that for two different modules in Θ, their inductions to G lie in different components. This completes the proof of 2.4.

2.5. Consider $d_G : kG\text{-mod} \to \mathbb{N}$ defined by

$$d_G(X) = \dim \mathrm{Hom}_{kG}(k, X).$$

Then d_G induces an additive function on any 1-tube of G. If M is a kH-module then by Frobenius reciprocity we have $d_G(M^G) = d_H(M)$ and the following shows that any two of the modules constructed in 1.3 to 1.5 induce to different components of G, with all M^G of quasi-length one since for these we have $d_H(M) = 1$.

LEMMA. *Let Θ be as in* 2.4. *If Θ contains a module X such that $d_H(X)$ is not divisible by p then* 2.4(b) *holds for Θ and* $\mathrm{ql}(M^G) = 1$ *for all $M \in \Theta$.*

Proof. We have $d_G(E_r)=rd_G(E_1)$ if E_r has quasi-length r. Let $X^G = E_r$. Then $rd_G(E_1) = d_G(X^G) = d_H(X) \not\equiv 0 \pmod p$ and (a) in 2.4 is excluded; actually we must have (b) with $r = 1$.

The following will be used later to deal with arbitrary blocks.

2.6. Proposition. *Let Δ be a component of G which contains M^G with M an indecomposable H-module which is G-stable, of τ-period one, and such that no other module in Δ is H-projective. Assume that $\mathrm{ql}(M^G) = 1$, that $d_H(M)$ is not divisible by p and that M has vertex H. Then all other modules in Δ have vertex G.*

Proof. We may assume k is algebraically closed. Suppose the statement fails; let E_s in Δ for $s > 1$ be minimal with a smaller vertex. Then there is a maximal subgroup P of G (normal, of index p) such that $E_s = X^G$ where X is an indecomposable P-module.

Assume first that X is G-stable. We apply 2.4 with P, X. Since $s > 1$ and the index $[G : P]$ is p we must have case 2.4(a). Hence $s = p$, and then $E_1{\downarrow_P} = X$, from the proof of 2.4. But $E_1 = M^G$, so we get $X = M^G{\downarrow_P} \cong (M{\downarrow_{H\cap P}})^P$.

Suppose we have $H \le P$. Then $M^G{\downarrow_P} = M^P = X$ and

$$d_P(M^P) = d_P(X) = d_G(X^G) = sd_G(E_1) \equiv 0 \pmod p;$$

but $d_P(M^P) = d_H(M)$ and we have a contradiction to the hypothesis. So we can only have $H \not\subseteq P$ and $P \cap H$ is a proper subgroup of H.

From the proof of 2.4, if U is the middle term of the Auslander–Reiten sequence of M then M is a direct summand of $(E_s){\downarrow_H}$. But $E_s = X^G$ and hence $E_s{\downarrow_H} = X^G{\downarrow_H} = (X{\downarrow_{P\cap H}})^H$. It follows that $M = (M'{\downarrow_{P\cap H}})^H$ and M has vertex strictly contained in H, a contradiction.

So X is not G-stable. By 2.2, the component is induced from P, and in particular, M^G is P-projective. So $H \le P$ and $\mathcal{A}(M^G) = \mathcal{A}(M_0)^G$ where $M_0 = M^P$. Then M_0 is not G-stable. On the other hand, for $g \in G$ we have $M_0^g \cong (M^g)^P \cong M^P = M_0$, a contradiction.

2.7. Corollary. *Let D be a p-group with kD of wild type. Then there is a family $(\mathcal{T}_\lambda)_{\lambda\in\Lambda}$ of 1-tubes where for all λ, any M in $\mathcal{T}_\lambda$ of quasi-length > 1 has vertex D, and where*

(i) $\Lambda = k^s$ *with* $s = \max(p-1, 2)$ *if the centre $Z(D)$ of D is not cyclic or a Klein 4-group,*

(ii) $|\Lambda| = |Z(D)| - 1$ *if $Z(D)$ is cyclic,*

(iii) $\Lambda = k$ *if $Z(D)$ is a Klein 4-group.*

Proof. Note first that any $H \subset Z(D)$ is normal in G and any H-module is automatically D-stable.

If $Z(D)$ is not cyclic then $Z(D)$ contains H as in 1.3–1.5, and by 1.3–1.5 and 2.4, 2.6 we get (i) or (iii). (If in 1.3 or 1.4 some of the modules are induced, then replace H by the vertex.)

Now suppose $Z(D)$ is cyclic, and take $H = Z(D)$; this has one component with $|H| - 1$ indecomposable non-projective modules, and by 2.4 and 2.6 we get (ii).

3. The general block. Now let G be arbitrary, and let B be a block of kG which is of wild type. Let D be a defect group of B; this is a p-group and kD is of wild type. We now show that the 1-tubes constructed for kD in 2.7 give rise to finitely many tubes of B.

3.1. Theorem. *Let $\mathcal{T}_\lambda$ be a family of* 1*-tubes of kD such that any $M \in \mathcal{T}_\lambda$ of quasi-length > 1 has vertex D. Then for each λ there are finitely many tubes $\mathcal{T}_{\lambda,i}$ of B such that almost all modules of $\mathcal{T}_{\lambda,i}$ are induced from $\mathcal{T}_\lambda$. Moreover, for $\lambda \neq \mu$ the tubes $\mathcal{T}_{\lambda,i}$ and $\mathcal{T}_{\mu,j}$ are distinct.*

Proof. This is a standard reduction.

Assume first that D is normal in G. Let $C = DC_G(D)$; this is then a normal subgroup of G. It is well known (see for example [1], (2.9)) that there is a block b of C with defect group D having the following properties. If e is the block idempotent of b and $T = \{g \in G : e^g = e\}$ then e is also a block idempotent of T. Moreover, the block ekT of T is Morita-equivalent to B (see for example [5], V.2.12), and vertices are preserved.

So without loss of generality, $G = T$, that is, $B = ekG$. Note that if M is a b-module then M^G is a B-module since $M = Me$ and therefore $M^T e = Me^T = M^T$, as e is central in kT.

By [6], (4.2), we know that the block b is Morita-equivalent to kD and the equivalence described there is vertex-preserving. So the results in §2 give an appropriate family of tubes for b.

It remains to induce the modules in this tube from b to kT. It is important that the index of C in T is not divisible by p (cf. [1]). So vertices are preserved; and for any indecomposable non-projective kC-module M if $M^T = \bigoplus_i W_i$ with indecomposable summands W_i then the W_i are not projective (consider the restriction to C), and by [8] we deduce

$$\mathcal{A}(M)^T \cong \bigoplus_i \mathcal{A}(W_i).$$

Now, $\tau(M^T) \cong \tau(M)^T$, so if $\tau(M) \cong M$ then τ induces a permutation of the W_i, and each orbit gives rise to one tube of B. Call the tube containing the module in the jth orbit $\mathcal{T}_{\lambda,j}$.

Now let G be arbitrary and let $N = N_G(D)$. For any tube $\mathcal{T}$ in which all modules of quasi-length > 1 have vertex D, we denote by $\mathcal{T}'$ the infinite connected translation subquiver which is obtained by deleting the modules of quasi-length one. Then by [10] the Green correspondence induces a graph isomorphism between $\mathcal{T}'$ for a tube $\mathcal{T}$ in the family for N and some infinite part of a tube for G. Since only one τ-orbit is left out this induces a 1-1 correspondence between such tubes of N and a tube family of G. Moreover, it is well known that there is a unique block b of N such that $M \in b$ if and only if $gM \in B$, and b has defect group D and is therefore also of wild type. By the first part the statement holds for b and by this correspondence it follows for B as well.

3.2. Let $R = k[T_1, \ldots, T_s]$ be the polynomial ring in s variables. For $\lambda \in k^s$ let S_λ be the corresponding simple R-module.

THEOREM. *Let B be a block of wild type with defect group D such that $Z(D)$ is not cyclic or a Klein 4-group. Let $s = \max(p-1, 2)$. Then there is an R-kD-bimodule W which is finitely generated and free as an R-module such that*

(i) *$S_\lambda \otimes_R W$ lies in a 1-tube $\mathcal{T}_\lambda$, and*

(ii) *there is a family of 1-tubes $\mathcal{T}_{\lambda,1}$ of B such that for every M in $\mathcal{T}_\lambda$ of quasi-length > 1, the induced module M^G has a summand in $\mathcal{T}_{\lambda,1}$ and for $\lambda \neq \mu$ the tubes $\mathcal{T}_{\lambda,1}$ and $\mathcal{T}_{\mu,1}$ are distinct.*

Proof. By the hypothesis, $Z(D)$ contains a subgroup H as in 1.3–1.5. The modules defined there are of the form $S_\lambda \otimes_R M$ where M is an R-kH-bimodule. Take $W = M \otimes_{kH} kD$. Since H is central in D, M is D-invariant and $S_\lambda \otimes_R M = M_\lambda$ is absolutely indecomposable. Apply 2.4; this shows that the component of $S_\lambda \otimes_R W = (M_\lambda)^D$ contains only one H-projective module. So for $\lambda \neq \mu$ the modules $S_\lambda \otimes_R W$ and $S_\mu \otimes_R W$ lie in different tubes. Take for $\mathcal{T}_\lambda$ the component of $S_\lambda \otimes_R W$. Then apply 3.1, and take $\mathcal{T}_{\lambda,1}$ as in 3.1.

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