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ON ADDITIVE FUNCTIONS FOR STABLE TRANSLATION QUIVERS

BY

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Abstract. The aim of this note is to give a complete description of the positive additive functions for the stable nonperiodic translation quivers with finitely many orbits. In particular, we show that all positive additive functions on the stable translation quivers of Euclidean type (respectively, of wild type) are periodic, and hence bounded (respectively, are unbounded, and hence nonperiodic).

1. Main results and related background. A *quiver* $\Delta=(\Delta_0,\Delta_1,s,e)$ is given by a set Δ_0 of *vertices*, a set Δ_1 of *arrows*, and two maps $s,e:\Delta_1\to\Delta_0$ which assign to each arrow α its *source* $s(\alpha)$ and its *end* $e(\alpha)$. We will usually write $\Delta=(\Delta_0,\Delta_1)$ and omit the maps s and e.

Let $\Delta=(\Delta_0,\Delta_1)$ be a quiver and $x,y\in\Delta_0$ be vertices. A *path* from x to y of length $l>0$ in Δ is a sequence of arrows $\alpha_l\ldots\alpha_1$ such that $s(\alpha_{i+1})=e(\alpha_i)$ for any $i=1,\ldots,l-1$, and $s(\alpha_1)=x$ and $e(\alpha_l)=y$. For each vertex $z\in\Delta_0$ we also introduce a path $(z|z)$ of length 0. A path of positive length from z to z is called an *oriented cycle*. A quiver Δ without oriented cycles is said to be *directed*. The vertex x will be called a *predecessor* of y provided there exists a path from x to y. In this case y is called a *successor* of x. For each vertex $x\in\Delta_0$ we denote by x^- the set of all direct predecessors of x in Δ, that is, the set of all vertices $y\in\Delta_0$ such that there exists an arrow $\alpha\in\Delta_1$ with $s(\alpha)=y$ and $e(\alpha)=x$. Similarly, by x^+ we denote the set of all direct successors of x. All quivers we deal with in the paper are supposed to be *locally finite*, which means that, for each vertex $x\in\Delta_0$, the sets x^- and x^+ are finite.

By a *translation quiver* $\Gamma=(\Gamma_0,\Gamma_1,\tau)$ we mean a quiver (Γ_0,Γ_1) with an injective map $\tau:\Gamma_0'\to\Gamma_0$, where $\Gamma_0'\subseteq\Gamma_0$, such that for any vertices $x\in\Gamma_0'$ and $y\in\Gamma_0$ the number of arrows from y to x is equal to the number of arrows from τx to y. The function τ is called a *translation*. The translation quiver Γ is said to be *stable* provided $\Gamma_0'=\Gamma_0$ and $\tau(\Gamma_0)=\Gamma_0$ (that is, $\tau:\Gamma_0\to\Gamma_0$ is a bijection). Important examples of translation quivers are

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provided by the connected components of the Auslander–Reiten quivers of finite-dimensional algebras.

Let $\Delta = (\Delta_0, \Delta_1)$ be a directed quiver. We define a stable translation quiver $\mathbb{Z}\Delta$. The set of vertices of $\mathbb{Z}\Delta$ is given by $\mathbb{Z} \times \Delta_0$, and given an arrow $\alpha : x \to y$ in Δ there are arrows $(n, \alpha) : (n, x) \to (n, y)$ and $(n, \alpha)' : (n, y) \to (n+1, x)$ in $\mathbb{Z}\Delta$, $n \in \mathbb{Z}$. Finally, we define a translation $\tau : \mathbb{Z} \times \Delta_0 \to \mathbb{Z} \times \Delta_0$ by $\tau(n, x) := (n-1, x)$ for any $x \in \Delta_0$ and $n \in \mathbb{Z}$.

Let $\Gamma = (\Gamma_0, \Gamma_1, \tau)$ be a stable translation quiver. A function $f : \Gamma_0 \to \mathbb{Z}$ is said to be an *additive function for* Γ provided

$$f(x) + f(\tau x) = \sum_{y \in x^-} d_{y,x} f(y)$$

for any vertex $x \in \Gamma_0$. Here $d_{y,x}$ denotes the number of arrows from y to x in Γ. The function f is said to be *positive* if $f(x) \geq 0$ for all $x \in \Gamma_0$ and there exists a vertex $x \in \Gamma_0$ such that $f(x) > 0$. Finally, the function f is said to be *periodic* with period $n > 0$ if $f(\tau^n x) = f(x)$ for any vertex $x \in \Gamma_0$.

Let $\Delta = (\Delta_0, \Delta_1)$ be a connected finite (Δ_0 and Δ_1 are finite sets) directed quiver. Consider the *Cartan matrix* $C = C_\Delta \in \mathbb{Z}^{\Delta_0 \times \Delta_0}$ whose x-y-entry is the number of paths from y to x in Δ. Then C is invertible over $\mathbb{Z}$ and we may consider the *Coxeter matrix* $\Phi = \Phi_\Delta := -C^{\mathrm{T}} C^{-1}$. For a function $f : \mathbb{Z} \times \Delta_0 \to \mathbb{Z}$ and $n \in \mathbb{Z}$, denote by $f(n)$ the vector in $\mathbb{Z}^{\Delta_0}$ such that $f(n)_x := f(n, x)$ for any $x \in \Delta_0$. Moreover, we denote by $\mathcal{D}_{\mathrm{r}}(\Delta)$ the set of the dimension-vectors of all regular finite-dimensional complex representations of the quiver Δ (see Section 2 for details). We note that $\mathcal{D}_{\mathrm{r}}(\Delta)$ is not empty if and only if Δ is not of Dynkin type ($\mathbb{A}_m$, $\mathbb{D}_n$, $\mathbb{E}_6$, $\mathbb{E}_7$, $\mathbb{E}_8$).

We are now able to formulate the main result of this note.

Theorem. *Let Δ be a finite connected directed quiver and $f : \mathbb{Z} \times \Delta_0 \to \mathbb{Z}$ a function. Then f is a positive additive function for $\mathbb{Z}\Delta$ if and only if there exists a vector $\mathbf{d} \in \mathcal{D}_{\mathrm{r}}(\Delta)$ such that $f(n)^{\mathrm{T}} = \Phi_\Delta^{-n} \mathbf{d}^{\mathrm{T}}$ for any $n \in \mathbb{Z}$.*

We get the following direct consequences of the above theorem and the known facts in the representation theory of quivers (see also [2], [3, 6.5], [4], [5], [7, p. 362]).

Corollary 1. *Let Δ be a finite connected directed quiver not of Dynkin type. The following conditions are equivalent*:

(i) *Δ is of Euclidean type ($\widetilde{\mathbb{A}}_m$, $\widetilde{\mathbb{D}}_n$, $\widetilde{\mathbb{E}}_6$, $\widetilde{\mathbb{E}}_7$, $\widetilde{\mathbb{E}}_8$).*
(ii) *Every positive additive function for $\mathbb{Z}\Delta$ is periodic.*
(iii) *There exists a positive additive periodic function for $\mathbb{Z}\Delta$.*
(iv) *Every positive additive function for $\mathbb{Z}\Delta$ is bounded.*
(v) *There exists a positive additive bounded function for $\mathbb{Z}\Delta$.*

COROLLARY 2. *Let Δ be a quiver of Dynkin type. Then there is no positive additive function for $\mathbb{Z}\Delta$.*

2. Preliminaries on representations of quivers. The crucial tool in our investigations is the theory of representations of quivers. Thus, throughout the paper K denotes an algebraically closed field (as we treat representations of quivers as a tool, we can even assume that K is the field of complex numbers). Throughout the note $\Delta = (\Delta_0, \Delta_1)$ denotes a fixed connected finite directed quiver.

By a (finite-dimensional) representation of Δ we mean a system $V = (V_x, V_\alpha)_{x\in\Delta_0,\alpha\in\Delta_1}$ of finite-dimensional K-linear spaces V_x, $x \in \Delta_0$, and K-linear maps $V_\alpha : V_{s(\alpha)} \to V_{e(\alpha)}$, $\alpha \in \Delta_1$. If $V = (V_x, V_\alpha)$ and $W = (W_x, W_\alpha)$ are two representations of Δ then $f = (f_x)_{x\in\Delta_0}$, with $f_x : V_x \to W_x$ a K-linear, is a map of representations provided $W_\alpha f_{s(\alpha)} = f_{e(\alpha)} V_\alpha$ for each arrow $\alpha \in \Delta_1$. In this way we obtain an abelian category rep_Δ of representations of the quiver Δ. If $V = (V_x, V_\alpha)$ is a representation of Δ then we assign to V its *dimension-vector* $\mathbf{dim}\, V \in \mathbb{Z}^{Q_0}$ in the following way: $(\mathbf{dim}\, V)_x := \dim_K V_x$ for any $x \in \Delta_0$.

There are two important endofunctors $\tau = \tau_\Delta = \mathrm{D}\,\mathrm{Tr}$, $\tau^- = \tau_\Delta^- = \mathrm{Tr}\,\mathrm{D} : \mathrm{rep}_\Delta \to \mathrm{rep}_\Delta$, called the *Auslander–Reiten translations* of rep_Δ. They are useful if we want to count the Ext-groups. Namely, we have the following Auslander–Reiten formulas:

$$\mathrm{Ext}(V, W) \simeq \mathrm{D}\,\mathrm{Hom}(W, \tau V) \simeq \mathrm{D}\,\mathrm{Hom}(\tau^- W, V)$$

for any representations V and W from rep_Δ. Here D denotes the standard duality with respect to the field K and we write Ext instead of Ext^1. In particular, we get $\dim \mathrm{Ext}(V, W) = \dim \mathrm{Ext}(\tau V, \tau W)$ if W is nonprojective and $\dim \mathrm{Ext}(V, W) = \dim \mathrm{Ext}(\tau^- V, \tau^- W)$ if V is noninjective.

The Auslander–Reiten translations allow us also to introduce three classes of indecomposable representations. The first class consists of *preprojective* representations, which are the indecomposable representations X for which there exists an integer $n \geq 0$ such that $\tau^n X$ is projective. Similarly, an indecomposable representation X is called *preinjective* provided there exists an integer $n \geq 0$ such that $\tau^{-n} X$ is injective. Finally, the indecomposable representation X is called *regular* if it is neither preprojective nor preinjective. In the case of Dynkin quivers all indecomposable representations are both preprojective and preinjective. If Δ is not of Dynkin type then there are disjoint classes of preprojective and preinjective representations. Moreover, in this case, the class of regular representations is not empty. If Δ is of Euclidean type then the class of regular representations consists of all indecomposable representations X for which there exists an integer $n > 0$ with $\tau^n X \simeq X$. For the remaining (wild) quivers Δ the indecomposable regular representations of Δ are never periodic with respect to τ. In general, by a regular representation of Δ we mean a (finite) direct sum of indecomposable regular representations. For more information about the structure of the category of representations of quivers we refer to [1], [6], [7].

Let x be a vertex of Δ. We define a representation $S(x) = (S(x)_y, S(x)_\alpha)$ putting $S(x)_y := \delta_{x,y}K$ for any $y \in \Delta_0$ and $S(x)_\alpha := 0$ for any $\alpha \in \Delta_1$. Here $\delta_{a,b}$ denotes the Kronecker delta. Let $P(x)$ be a projective cover of $S(x)$ and $Q(x)$ be an injective envelope of $S(x)$ in rep_Δ. It is known that the representations $P(x)$, $x \in \Delta_0$, form a complete set of pairwise nonisomorphic indecomposable projective representations of Δ. Similarly, the representations $Q(x)$, $x \in \Delta_0$, form a complete set of pairwise nonisomorphic indecomposable injective representations of Δ. We denote by $\mathbf{p}(x)$ the dimension-vector of $P(x)$ and by $\mathbf{q}(x)$ the dimension-vector of $Q(x)$. It is easily seen that

$$\mathbf{p}(x)_y = \#\{\text{paths from } x \text{ to } y\} \quad \text{and} \quad \mathbf{q}(x)_y = \#\{\text{paths from } y \text{ to } x\}.$$

In addition, for any representation V from rep_Δ we have

$$\dim_K \mathrm{Hom}(P(x), V) = (\mathbf{dim}\, V)_x = \dim_K \mathrm{Hom}(V, Q(x)).$$

Let $C = C_\Delta \in \mathbb{Z}^{\Delta_0 \times \Delta_0}$ be the Cartan martix of Δ. It follows from the above formulas that its yth column is given by $\mathbf{p}(y)^{\mathrm{T}}$ and its xth row is given by $\mathbf{q}(x)$, hence its x-y-entry is given by $\dim_K \mathrm{Hom}(P(x), P(y)) = \dim_K \mathrm{Hom}(Q(x), Q(y))$, for each $x, y \in \Delta_0$. The assumption that Δ is directed gives us that C is invertible (over $\mathbb{Z}$) and we may define the Coxeter matrix $\Phi = \Phi_\Delta$ as $\Phi := -C^{\mathrm{T}}C^{-1}$. It is an easy exercise to see that $\Phi\mathbf{p}(x)^{\mathrm{T}} = -\mathbf{q}(x)^{\mathrm{T}}$ for any $x \in \Delta_0$. Further (see [7, p. 75]),

$$(\mathbf{dim}\, \tau V)^{\mathrm{T}} = \Phi(\mathbf{dim}\, V)^{\mathrm{T}} \quad \text{and} \quad (\mathbf{dim}\, \tau^- W)^{\mathrm{T}} = \Phi^{-1}(\mathbf{dim}\, W)^{\mathrm{T}}$$

for any nonprojective indecomposable representation V and any noninjective indecomposable representation W from rep_Δ, which will play an important role in our proofs.

3. Additive functions for $\mathbb{Z}\Delta$. The aim of this section is to prove some facts on the structure of additive functions for the translation quiver $\mathbb{Z}\Delta$. For two vertices $x, y \in \Delta_0$, we denote by $d_{x,y}$ the number of arrows from x to y in Δ.

First we define a matrix $A = A_\Delta = (a_{x,y})_{x,y \in \Delta_0}$ by the following inductive formula for the number of predecessors of the vertex x:

$$a_{x,y} := \sum_{z \in x^-} d_{z,x} a_{z,y} + d_{x,y} - \delta_{x,y}$$

for any $x, y \in \Delta_0$. Notice that the lack of oriented cycles in Δ guarantees that the above definition is correct.

The origin of the above matrix becomes clear from the following lemma.

LEMMA 3.1. *If f is an additive function for $\mathbb{Z}\Delta$ then*

$$f(n+1)^{\mathrm{T}} = Af(n)^{\mathrm{T}}$$

for any $n \in \mathbb{Z}$.

Proof. By induction on the number of predecessors of a given vertex of Δ we get

$$\begin{aligned} f(n+1,x) &= \sum_{z\in x^-} d_{z,x} f(n+1,z) + \sum_{y\in x^+} d_{x,y} f(n,y) - f(n,x) \\ &= \sum_{z\in x^-} \Big(d_{z,x} \sum_{y\in\Delta_0} a_{z,y} f(n,y)\Big) + \sum_{y\in\Delta_0} d_{x,y} f(n,y) - f(n,x) \\ &= \sum_{y\in\Delta_0} \Big(\sum_{z\in x^-} d_{z,x} a_{z,y} + d_{x,y} - \delta_{x,y}\Big) f(n,y) = \sum_{y\in\Delta_0} a_{x,y} f(n,y) \end{aligned}$$

for any $x \in \Delta_0$. ■

Together with the above lemma the next property of the matrix A is not suprising.

LEMMA 3.2. *The matrix A is invertible over $\mathbb{Z}$.*

Proof. Define a matrix $B = (b_{x,y})_{x,y\in\Delta_0}$ by the following inductive formula for the number of successors of the vertex x:

$$b_{x,y} := \sum_{z\in x^+} d_{x,z} b_{z,y} + d_{y,x} - \delta_{x,y}$$

for any $x, y \in \Delta_0$. It is an easy exercise to check inductively on the number of predecessors of a given vertex of Δ that B is the inverse matrix of A. ■

Our final lemma completes the above observations.

LEMMA 3.3. *If $\mathbf{d} \in \mathbb{Z}^{\Delta_0}$ then the function $f : \mathbb{Z} \times \Delta_0 \to \mathbb{Z}$ given by*

$$f(n)^{\mathrm{T}} := A^n \mathbf{d}^{\mathrm{T}}$$

for any $n \in \mathbb{Z}$ is an additive function for $\mathbb{Z}\Delta$.

Proof. It follows again by induction on the number of predecessors, for $n > 0$ (successors, for $n \leq 0$), that

$$f(n,x) + f(n-1,x) = \sum_{y\in x^-} d_{y,x} f(n,x) + \sum_{y\in x^+} d_{x,y} f(n-1,x)$$

for any $x \in \Delta_0$. ■

Now we are ready to identify the matrix A.

PROPOSITION 3.4. *If Δ is a finite directed quiver then*

$$A_\Delta = \Phi_\Delta^{-1}.$$

Proof. Fix a vertex $x \in \Delta_0$. Define a function $f : \mathbb{Z} \times \Delta_0 \to \mathbb{Z}$ by

$$f(n)^{\mathrm{T}} := A^n \mathbf{p}(x)^{\mathrm{T}}$$

for any $n \in \mathbb{Z}$. By Lemma 3.3 it is an additive function for $\mathbb{Z}\Delta$. On the other hand, using induction on the number of successors of a given vertex of Δ and the formulas listed in Section 2 we conclude that $f(-1) = -\mathbf{q}(x)$. Indeed, for $y \in \Delta_0$ we have

$$\begin{aligned} f(-1,y) &= \sum_{z \in y^+} d_{y,z} f(-1,z) + \sum_{z \in y^-} d_{z,y} f(0,z) - f(0,y) \\ &= -\sum_{z \in y^+} d_{y,z} \mathbf{q}(x)_z + \sum_{z \in y^-} d_{z,y} \mathbf{p}(x)_z - \mathbf{p}(x)_y \\ &= -\sum_{z \in y^+} d_{y,z} \#\{\text{paths from } z \text{ to } x\} \\ &\quad + \sum_{z \in y^-} d_{z,y} \#\{\text{paths from } x \text{ to } z\} - \#\{\text{paths from } x \text{ to } y\} \\ &= -\#\{\text{paths from } y \text{ to } x\} = -\mathbf{q}(x)_y. \end{aligned}$$

As a consequence of the above considerations we obtain

$$A^{-1}\mathbf{p}(x)^{\mathrm{T}} = -\mathbf{q}(x)_y = \Phi \mathbf{p}(x)^{\mathrm{T}}$$

for any $x \in \Delta_0$. Since the vectors $\mathbf{p}(x)$, $x \in \Delta_0$, form a basis of the vector space $\mathbb{Q}^{\Delta_0}$ (because the Cartan matrix C is invertible), we conclude that $A^{-1} = \Phi$. ■

4. Proofs of the main results. Recall that $\Delta = (\Delta_0, \Delta_1)$ is a finite connected directed quiver. Assume $\mathbf{d} = \mathbf{dim}\, V$ for a regular representation V of Δ (so Δ is not of Dynkin type). Then it follows from Lemma 3.3, Proposition 3.4 and the results mentioned in Section 2 that $f(n)^{\mathrm{T}} := \Phi^{-n}\mathbf{d}^{\mathrm{T}} = (\mathbf{dim}\, \tau^{-n} V)^{\mathrm{T}}$ defines a positive additive function for $\mathbb{Z}\Delta$. Conversely, assume that $f : \mathbb{Z} \times \Delta_0 \to \mathbb{Z}$ is an arbitrary positive additive function for $\mathbb{Z}\Delta$. We shall prove that f is of the required form.

Since f is positive, it follows from Lemma 3.1 that $f(0)$ is a nonzero vector of $\mathbb{N}^{\Delta_0}$. Choose a representation V in rep_Δ with $\mathbf{dim}\, V = f(0)$ and the smallest possible dimension of the endomorphism ring. We know (see for example [1, VII.3.2]) that then $\mathrm{Ext}(V_1, V_2) = 0$ for any decomposition $V_1 \oplus V_2$ of V.

Consider first the case when V has an indecomposable preprojective direct summand. Choose such a summand V_0 with the smallest possible number m such that $\tau^m V_0$ is projective. If we decompose V into the direct sum $V_0 \oplus \ldots \oplus V_l$ of indecomposable representations then we know by our assumption that $\tau^m V_i \neq 0$ for any $i = 0, \ldots, l$. In addition, using the

Auslander–Reiten formulas we obtain $\operatorname{Ext}(\tau^m V_i, \tau^m V_0) = 0$ for any $i = 1, \dots, l$. As a consequence, using again the same formulas we conclude that $\operatorname{Hom}(\tau^m V_0, \tau^{m+1} V_i) = 0$ for any $i = 1, \dots, l$. However, $\tau^m V_0 = P(x)$ for some vertex $x \in \Delta_0$. Thus the above equalities mean that $(\mathbf{dim}\,\tau^{m+1} V_i)_x = 0$ for all $i = 1, \dots, l$, so that $\tau^m V_i$ is not projective. Without loss of generality we may assume that the representations $\tau^m V_1, \dots, \tau^m V_k$ are projective and $\tau^m V_{k+1}, \dots, \tau^m V_l$ are not. For each $i = 1, \dots, k$, we may write $\tau^m V_i = P(x_i)$ for some vertex $x_i \in \Delta_0$. Using Proposition 3.4, Lemma 3.1 and the facts listed in Section 2, we get

$$\begin{aligned} f(-m-1)^{\mathrm{T}} &= \Phi^{m+1}(\mathbf{dim}\, V)^{\mathrm{T}} = \Phi(\mathbf{dim}\,\tau^m V)^{\mathrm{T}} \\ &= \Phi(\mathbf{dim}\,\tau^m V_0)^{\mathrm{T}} + \sum_{i=1}^{k} \Phi(\mathbf{dim}\,\tau^m V_i)^{\mathrm{T}} + \sum_{i=k+1}^{l} \Phi(\mathbf{dim}\,\tau^m V_i)^{\mathrm{T}} \\ &= \Phi\mathbf{p}(x)^{\mathrm{T}} + \sum_{i=1}^{k} \Phi\mathbf{p}(x_i)^{\mathrm{T}} + \sum_{i=k+1}^{l} \mathbf{dim}(\tau^{m+1} V_i)^{\mathrm{T}} \\ &= -\mathbf{q}(x)^{\mathrm{T}} - \sum_{i=1}^{k} \mathbf{q}(x_i)^{\mathrm{T}} + \sum_{i=k+1}^{l} \mathbf{dim}(\tau^{m+1} V_i)^{\mathrm{T}}. \end{aligned}$$

In particular, we have

$$\begin{aligned} f(-m-1)_x &= -\mathbf{q}(x)_x - \sum_{i=1}^{k} \mathbf{q}(x_i)_x + \sum_{i=k+1}^{l} \mathbf{dim}(\tau^{m+1} V_i)_x \\ &= -1 - \sum_{i=1}^{k} \mathbf{q}(x_i)_x < 0 \end{aligned}$$

and this contradicts the positivity of f. We proceed dually when V has an indecomposable preinjective direct summand. Therefore, V is regular and $\mathbf{d} := f(0) = \mathbf{dim}\, V \in \mathcal{D}_r(\Delta)$. A direct application of Lemmas 3.1, 3.2 and Proposition 3.4 shows that f is of the required from. This finishes the proof of the theorem.

Assume now that Δ is of Euclidean type. Then it is well known (see [7]) that for any regular representation X of Δ there exists $m \geq 1$ such that $\tau^m X \simeq X$. Hence, it follows from the theorem that every positive additive function for $\mathbb{Z}\Delta$ is periodic, and hence bounded. Assume now that Δ is neither of Dynkin nor of Euclidean type. Then it follows from [6], [8] that for any indecomposable regular representation M of Δ the dimension-vectors $\mathbf{dim}\,\tau^n M$, $n \in \mathbb{Z}$, are pairwise different. In particular, for any regular representation V of Δ, the associated positive additive function f for $\mathbb{Z}\Delta$, with $f(n) := \mathbf{dim}\,\tau^{-n} V$, is not bounded, and hence is not periodic. Therefore, by the theorem, every positive additive function for $\mathbb{Z}\Delta$ is neither periodic nor bounded. This completes the proof of Corollary 1.

Finally, Corollary 2 follows from the theorem and the fact that the quivers of Dynkin type do not admit regular indecomposable representations.

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