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DISJOINTNESS PRESERVING MAPPINGS BETWEEN FOURIER ALGEBRAS

BY

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1. Introduction. In recent years considerable attention has been given to disjointness preserving operators on Banach lattices (see e.g. [1], [5], [15] or [17]), on spaces of continuous functions (see e.g. [2], [10], [12], [14] or [16]) and on group algebras of locally compact Abelian groups ([11]). Three major directions of investigation have been followed in the above papers, namely the automatic continuity of disjointness preserving operators, their multiplicative representation and their spectral theory.

In [9], the automatic continuity results for disjointness preserving mappings between group algebras in [11] were extended to the class of regular Banach function algebras (see Section 2). Let us recall that a linear map T defined from a regular Banach function algebra A into such an algebra B is said to be *disjointness preserving* or *separating* if $f \cdot g \equiv 0$ implies $T(f) \cdot T(g) \equiv 0$ for all $f, g \in A$. In this paper we focus on the behaviour of disjointness preserving mappings defined between Fourier algebras on locally compact amenable (not necessarily Abelian) groups. These algebras and t Fourier–Stieltjes algebras were introduced by P. Eymard in his fundamental article [7]. For a locally compact group G, $B(G)$ denotes its *Fourier–Stieltjes algebra*, that is, the linear span of all continuous positive definite functions on G. Regarded as the dual of the universal C^*-algebra, $C^*(G)$, the algebra $B(G)$ is a Banach space with norm $\|\cdot\|$ (the dual norm of $C^*(G)$) and even a commutative semisimple Banach algebra under pointwise multiplication. $A(G)$ will denote the *Fourier algebra* of G, which is the closed subalgebra of $B(G)$ generated by functions with compact support. It is well known (see [7]) that $A(G)$ is a regular semisimple commutative Banach algebra. Indeed, $A(G)$ is a closed ideal in $B(G)$ and its structure space is G. Furthermore, $A(G)$ satisfies Ditkin's condition ([7, 4.11]), i.e., it is a Ditkin algebra.

If G is allowed to be Abelian and $\widehat{G}$ denotes its dual group, then $B(G)$ consists of the Fourier–Stieltjes transforms of $M(\widehat{G})$, the regular complex

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Borel measures on $\widehat{G}$. Similarly, $A(G)$ is nothing but the Fourier transforms of the elements of the group algebra $L^1(G)$.

In Section 3, we shall prove that the Fourier algebras of two locally compact amenable groups, G_1 and G_2, are (algebra) isomorphic if and only if there exists a disjointness preserving bijection between them. Such disjointness preserving bijection of $A(G_1)$ onto $A(G_2)$ can be extended, in a unique way, to a weighted composition map between the Fourier–Stieltjes algebras of G_1 and G_2. These results extend the ones in [11] given for $L^1(G)$ and $M(G)$, where G is a locally compact Abelian group. As corollaries, we show that if there exists either a supremum norm isometry or a bipositive bijection which preserves cozero sets between two Fourier algebras, then they are (algebra) isomorphic.

2. Preliminaries and background. Throughout, $\mathbb{N}$ (resp. $\mathbb{R}$, $\mathbb{C}$) stands for the set of all natural numbers (resp. real, complex numbers). If X is locally compact, then $X \cup \{\infty\}$ denotes its Aleksandrov compactification. As usual, $C_0(X)$ denotes the Banach algebra of all complex-valued continuous functions on X which are zero at infinity provided with the supremum norm $\|\cdot\|_\infty$. If $f \in C(X)$ (the linear space of all complex-valued continuous functions on X), the *cozero set* of f is the set $\mathrm{coz}(f) := \{x \in X : f(x) \neq 0\}$ and $\mathrm{supp}(f)$ denotes the support of f, i.e., the closure of $\mathrm{coz}(f)$. When U is any subset of X, we denote by $\mathrm{int}(U)$ the interior of U and by $\mathrm{cl}(U)$ the closure of U in X. For any $f \in C(X)$, $f_{|U}$ stands for the restriction of f to U.

Let A be a commutative Banach algebra which may or may not have an identity element. Let Φ_A be the (locally compact) *structure space* of A. The *Gelfand transform* of $f \in A$ is denoted by $\widehat{f}$. We write $\widehat{A}$ for the point-separating subalgebra of $C_0(\Phi_A)$ consisting of all $\widehat{f}$, $f \in A$.

Next we gather the main results concerning disjointness preserving maps between regular Banach function algebras, which can be found in [9]:

In the sequel, let A and B be regular semisimple commutative Banach algebras, which is to say, regular Banach function algebras. Associated with a disjointness preserving map $T : A \to B$, we can define a linear mapping $\widehat{T}$: $\widehat{A} \to \widehat{B}$ as $\widehat{T}(\widehat{f}\,) := \widehat{T(f)}$ for every $f \in A$. Since A and B are semisimple, it is easy to check that T is disjointness preserving if and only if $\widehat{T}$ is disjointness preserving. In like manner, T is injective (resp. surjective) if and only if $\widehat{T}$ is injective (resp. surjective).

If $\gamma \in \Phi_B$, let $\delta_\gamma \circ \widehat{T} : \widehat{A} \to \mathbb{C}$ be the functional defined as $(\delta_\gamma \circ \widehat{T})(\widehat{f}\,) := \widehat{T}(\widehat{f}\,)(\gamma)$ for all $f \in A$.

In general, a disjointness preserving map $T : A \to B$ induces a continuous mapping h of Φ_B into $\Phi_A \cup \{\infty\}$, which may make no sense if A and B are not regular. We call h the *support map* of T. If T is continuous, then it is a *weighted composition map*; i.e., $(\delta_\gamma \circ \widehat{T})(\widehat{f}) = \widehat{T}(\widehat{f})(\gamma) = \kappa(\gamma)\widehat{f}(h(\gamma))$ for all $\gamma \in \Phi_B$ and all $f \in A$, where the weight function $\kappa : \Phi_B \to \mathbb{C}$ is continuous, and the range of h is contained in Φ_A. If, in addition, T is surjective, then the point-separating property of $\widehat{B}$ easily implies that κ is nonvanishing on Φ_B.

Proposition 1 [9, Proposition 3]. *Let U be an open subset of $\Phi_A \cup \{\infty\}$ and suppose that $f \in A$. Then*:

(1) $\widehat{f}_{|U\cap\Phi_A} \equiv 0$ *implies that* $\widehat{T}(\widehat{f})_{|h^{-1}(U)} \equiv 0$.
(2) $h(\mathrm{coz}(\widehat{T}(\widehat{f}))) \subset \mathrm{cl}_{\Phi_A\cup\{\infty\}}(\mathrm{coz}(\widehat{f}))$.
(3) *If T is injective, then $h(\Phi_B)$ is a dense subset of $\Phi_A \cup \{\infty\}$.*

The main result in [9] is the following:

Theorem 1. *Let $T : A \to B$ be a disjointness preserving bijection. If A satisfies Ditkin's condition* (*i.e., if A is a Ditkin algebra*), *then*:

(1) *T is continuous.*
(2) *T^{-1} is disjointness preserving.*
(3) *If also B satisfies Ditkin's condition, then h, the support map of T, is a homeomorphism of Φ_A onto Φ_B.*

As a consequence of this theorem and the above paragraphs, if there exists a disjointness preserving bijection T of A onto B, then $\widehat{T}(\widehat{f})(\gamma) = \kappa(\gamma)\widehat{f}(h(\gamma))$ for all $f \in A$ and all $\gamma \in \Phi_B$. Since T^{-1} is also disjointness preserving and, consequently, continuous, we can write $\widehat{T}^{-1}(\widehat{g})(\zeta) = \Psi(\zeta)\widehat{g}(h^{-1}(\zeta))$ for all $g \in B$ and all $\zeta \in \Phi_A$, where h^{-1} can be proved to be the inverse of the homeomorphism h. We will call $\kappa \in C(\Phi_B)$ and $\Psi \in C(\Phi_A)$ the *weight functions associated with T*.

3. Characterizations of (algebra) isomorphisms between Fourier algebras by means of disjointness preserving mappings. Let $\mathcal{A}$ be a semisimple commutative Banach algebra. A *multiplier* T on $\mathcal{A}$ is a bounded linear operator on $\mathcal{A}$ into itself which satisfies $T(f\cdot g) = f\cdot T(g) = T(f)\cdot g$ for all $f, g \in \mathcal{A}$. We use $M(\mathcal{A})$ to denote the commutative Banach algebra consisting of all multipliers on $\mathcal{A}$. By [18, Corollary 1.2.1], we may identify $M(\mathcal{A})$ with the normed algebra of all bounded continuous functions ϕ on $\Phi_{\mathcal{A}}$ such that $\phi\widehat{\mathcal{A}} \subset \widehat{\mathcal{A}}$. It is then apparent that multipliers are examples of disjointness preserving mappings.

Theorem 2. *Let A and B regular semisimple commutative Banach algebras. Then A and B are* (*algebra*) *isomorphic if and only if there exists a continuous disjointness preserving bijection between them whose* (*associated*) *weight functions are multipliers.*

Proof. Let us suppose that there exists a continuous disjointness preserving bijection T of A onto B. First we claim that $\widehat{g} \circ h^{-1} \in \widehat{A}$ for all $g \in B$. To prove this, let $\zeta \in \Phi_A$ and $f \in A$ such that $\widehat{f}(\zeta) = 1$. Hence

$$\begin{aligned} 1 = \widehat{f}(\zeta) &= \widehat{T}^{-1}(\widehat{T}(\widehat{f}))(\zeta) = \Psi(\zeta) \cdot \widehat{T}(\widehat{f})(h^{-1}(\zeta)) \\ &= \Psi(\zeta) \cdot \kappa(h^{-1}(\zeta)) \cdot \widehat{f}(h(h^{-1}(\zeta))) = \Psi(\zeta) \cdot \kappa(h^{-1}(\zeta)); \end{aligned}$$

that is, $\Psi(\zeta) \cdot \kappa(h^{-1}(\zeta)) = 1$ for all $\zeta \in \Phi_A$. On the other hand, from the fact that $\widehat{B}$ is an ideal in $M(B)$ (see [18]) and since, by hypothesis, $\kappa : \Phi_B \to \mathbb{C}$ belongs to $M(B)$, we infer that $\kappa \cdot \kappa \cdot (\widehat{f} \circ h)$ belongs to $\widehat{B}$ for every $f \in A$. Consequently,

$$\begin{aligned} \widehat{T}^{-1}(\kappa \cdot \kappa \cdot (\widehat{f} \circ h))(\zeta) &= \Psi(\zeta) \cdot \kappa(h^{-1}(\zeta)) \cdot \kappa(h^{-1}(\zeta)) \cdot \widehat{f}(h(h^{-1}(\zeta))) \\ &= \kappa(h^{-1}(\zeta)) \cdot \widehat{f}(\zeta) \end{aligned}$$

for all $\zeta \in \Phi_A$. This implies that the function $(\kappa \circ h^{-1}) \cdot \widehat{f}$ belongs to $\widehat{A}$ for all $f \in A$, which is to say that $\kappa \circ h^{-1}$ belongs to $M(A)$. Hence, since $\widehat{A}$ is an ideal in $M(A)$ and the function $\Psi \cdot (\widehat{g} \circ h^{-1})$ belongs to $\widehat{A}$, we see that $(\kappa \circ h^{-1}) \cdot \Psi \cdot (\widehat{g} \circ h^{-1}) = (\widehat{g} \circ h^{-1})$ belongs to $\widehat{A}$ for all $g \in B$.

In like manner, we can prove that $\widehat{f} \circ h$ belongs to $\widehat{B}$ for all $f \in A$. Hence, it is now clear, since $h : \Phi_B \to \Phi_A$ is a homeomorphism, that the mapping $\widehat{T}_h : \widehat{A} \to \widehat{B}$, defined as $\widehat{T}_h(\widehat{f}) := \widehat{f} \circ h$, is a surjective algebra isomorphism, which, by semisimplicity, provides the desired algebra isomorphism of A onto B.

The converse is clear. ∎

THEOREM 3. *Let A and B be Ditkin algebras. Then A and B are* (*algebra*) *isomorphic if and only if there exists a disjointness preserving bijection between them whose weight functions are multipliers.*

Proof. Combine Theorems 1 and 2. ∎

We now turn our attention to Fourier algebras. Let us first recall that a locally compact group G is said to be *amenable* if it has a bounded approximate identity or, equivalently, if there exists a left-invariant mean on $L^\infty(G)$ (see, e.g., [6]).

Let Σ be the set of all unitary continuous representations of G. For $\sigma \in \Sigma$ and an arbitrary bounded Radon measure μ, we denote by $\|\sigma(\mu)\|$ the norm of $\int \sigma(x)\, d\mu(x)$, and $\|\mu\|_\Sigma := \sup\{\|\sigma(\mu)\| : \sigma \in \Sigma\}$. For $\gamma \in G$, δ_γ denotes the Dirac measure concentrated at γ.

LEMMA 1. *Let G_1 and G_2 be locally compact groups. Assume further that G_1 is amenable. Let $T : A(G_1) \to A(G_2)$ be a disjointness preserving bijection. Then the weight function κ belongs to $B(G_2)$.*

P r o o f. Let $\{\gamma_1,\ldots,\gamma_n\}$ be a subset of G_2 and $\varepsilon>0$. By [6, Theorem 7], there exists $f\in A(G_1)$ such that $\|f\|<1+\varepsilon$ and $f(h(\gamma_i))=1$ for $i=1,\ldots,n$.

Let $\{c_1,\ldots,c_n\}\subset\mathbb{C}$. Then, since T is continuous, we have

$$\Big|\sum_{i=1}^n c_i\cdot\kappa(\gamma_i)\Big| = \Big|\sum_{i=1}^n c_i\cdot T(f)(\gamma_i)\Big| \le \|T(f)\|\Big\|\sum_{i=1}^n c_i\delta_{\gamma_i}\Big\|_{\Sigma} \le \|T\|(1+\varepsilon)\Big\|\sum_{i=1}^n c_i\delta_{\gamma_i}\Big\|_{\Sigma}.$$

Consequently,

$$\Big|\sum_{i=1}^n c_i\cdot\kappa(\gamma_i)\Big| \le \|T\|\Big\|\sum_{i=1}^n c_i\delta_{\gamma_i}\Big\|_{\Sigma}.$$

From the continuity of κ and from the Bochner–Schoenberg–Eberlein-type characterization of the elements of $B(G_2)$ in [7, Corollary 2.24], we infer that $\kappa\in B(G_2)$. ■

REMARK 1. If the amenability of G_1 is dropped, then Lemma 1 may fail to be true. Namely, in [8], the authors provide a locally compact group G and a function ϕ on G such that $\phi f\in A(G)$ for every $f\in A(G)$, while $\phi\notin B(G)$.

THEOREM 4. *Let G_1 and G_2 be locally compact amenable groups. Then $A(G_1)$ and $A(G_2)$ are algebra isomorphic if and only if there exists a disjointness preserving bijection between them.*

P r o o f. Since $A(G_2)$ is an ideal in $B(G_2)$ ([7, 3.4]), it is clear that $B(G_2)\subseteq M(A(G_2))$. Hence the result follows from Lemma 1 and Theorem 3. ■

COROLLARY 1. *Let G_1 and G_2 be locally compact amenable groups. Then $A(G_1)$ and $A(G_2)$ are algebra isomorphic if they are $\|\cdot\|_\infty$-isometric*; *i.e.*, *there exists a linear bijection T of $A(G_1)$ onto $A(G_2)$ such that $\|f\|_\infty=\|T(f)\|_\infty$ for all $f\in A(G_1)$.*

P r o o f. Let $\tau(A(G_i))$ $(i=1,2)$ stand for the set of all strong boundary points for $A(G_i)$. Recall that $\gamma\in G_i$ is a *strong boundary point* for $A(G_i)$ if for every open neighbourhood U of γ, there exists $f\in A(G_i)$ with $\|f\|_\infty=|f(x)|$ and $|f_{|G_i\setminus U}|<\|f\|_\infty$. Define the following subset of G_2:

$$G_2^0 := \bigcup_{\zeta\in\tau(A(G_1))}\{\gamma\in G_2 : |f(\zeta)|=|T(f)(\gamma)| \text{ for all } f\in A(G_1)\}.$$

Since $A(G_i)$ is a regular subalgebra of $C_0(G_i)$ (see Section 2), [3, Corollary 4.3] entails that the set G_2^0 coincides with $\tau(A(G_2))$.

On the other hand, by the Bishop–de Leeuw Theorem (see, e.g., [13, Corollary 12.10]) the set $\tau(A(G_i))$ is dense in the Shilov boundary of $A(G_i)$. Furthermore, it is well known that the Shilov boundary of $A(G_i)$ coincides with G_i since $A(G_i)$ is regular. As a consequence, we deduce that G_2^0 is dense in G_2.

The remainder of the proof consists in checking that T is disjointness preserving and applying Theorem 4. Assume, contrary to what we claim, that there are $f, g \in A(G_1)$ with disjoint cozero sets such that $T(f)\cdot T(g) \not\equiv 0$. In virtue of the density of G_2^0, we can choose $\gamma_0 \in G_2^0$ such that $|T(f)(\gamma_0)| > 0$ and $|T(g)(\gamma_0)| > 0$. By the definition of G_2^0, there exists $\zeta_0 \in \tau(A(G_1))$ such that $|f(\zeta_0)| = |T(f)(\gamma_0)|$ for all $f \in A(G_1)$. Since the cozero sets of f and g are disjoint, we see that either $f(\zeta_0)$ or $g(\zeta_0)$ is zero, which yields that either $T(f)(\gamma_0) = 0$ or $T(g)(\gamma_0) = 0$. This contradiction proves that T is disjointness preserving. ∎

REMARK 2. As a straightforward consequence of Corollary 1 and the main results (Theorem 3 and its corollary) of [19], we infer that two locally compact amenable groups, G_1 and G_2, are topologically isomorphic if and only if $A(G_1)$ and $A(G_2)$ are $\|\cdot\|$-isometric and $\|\cdot\|_\infty$-isometric.

DEFINITION 1. Let $T : \mathcal{X} \to \mathcal{Y}$ be a map defined between spaces of functions. It is said that T *preserves cozero sets* if $\operatorname{coz}(f) \subseteq \operatorname{coz}(g)$ yields $\operatorname{coz}(T(f)) \subseteq \operatorname{coz}(T(g))$ for any $f, g \in \mathcal{X}$.

In [4], the authors show that Fourier algebras are ordered vector spaces for two order relations, namely the pointwise and the positive definite ordering. Among other results, they prove ([4, Proposition 4.2]) that *continuous* linear bipositive (pointwise) bijections between Fourier algebras are weighted composition maps. The following result shows that preserving cozero sets is a sufficient condition for such bipositive bijections to be automatically continuous. Indeed, they are disjointness preserving, which yields that $A(G_1)$ and $A(G_2)$ are (algebra) isomorphic.

COROLLARY 2. *Let G_1 and G_2 be locally compact amenable groups. If there exists a linear bipositive* (*pointwise*) *bijection T of $A(G_1)$ onto $A(G_2)$ which preserves cozero sets*, *then T is automatically continuous. Furthermore*, *in that case*, *$A(G_1)$ and $A(G_2)$ are* (*algebra*) *isomorphic.*

P r o o f. We shall check that T is disjointness preserving and thus the results will follow from Theorems 1 and 4. Let $A(G_i)^+$ $(i = 1, 2)$ denote the positive cone of $A(G_i)$. Let $f, g \in A(G_1)^+$ such that $\operatorname{coz}(T(f)) \cap \operatorname{coz}(T(g)) \neq \emptyset$. By the regularity of $A(G_2)$, one can find a function $0 \not\equiv k \in A(G_2)^+$ such that $k \le T(f)$ and $k \le T(g)$. Hence, the positivity of $T(f)$, $T(g)$ and k yields $\operatorname{coz}(k) \subseteq \operatorname{coz}(T(f)) \cap \operatorname{coz}(T(g))$. Since T^{-1} also preserves positive functions, we have $0 \not\equiv T^{-1}(k) \subseteq \operatorname{coz}(f) \cap \operatorname{coz}(g)$. As a consequence, the restriction of T to $A(G_1)^+$ is disjointness preserving.

Given a function $f \in A(G_i)$, let $\overline{f}$ denote its complex conjugate. By [7, 3.8], we have $\overline{f} \in A(G_i)$. Let $f, g \in A(G_1)$ be such that $\operatorname{coz}(f) \cap \operatorname{coz}(g) = \emptyset$. Hence $\operatorname{coz}(f \cdot \overline{f}) \cap \operatorname{coz}(g \cdot \overline{g}) = \emptyset$. From the above paragraph, we know that $\operatorname{coz}(T(f \cdot \overline{f})) \cap \operatorname{coz}(T(g \cdot \overline{g})) = \emptyset$. Since T preserves cozero sets, we conclude that $\operatorname{coz}(T(f)) \cap \operatorname{coz}(T(g)) = \emptyset$. ■

In closing, we study the possibility of extending disjointness preserving mappings from Fourier algebras to Fourier–Stieltjes algebras.

THEOREM 5. *Let G_1 and G_2 be locally compact amenable groups and let T be a disjointness preserving bijection of $A(G_1)$ onto $A(G_2)$. Then T has a unique extension to a weighted composition bijection of $B(G_1)$ onto $B(G_2)$.*

Proof. From Section 2, we know that $T(f)(\gamma) = \kappa(\gamma) f(h(\gamma))$ for all $\gamma \in G_2$ and all $f \in A(G_1)$. We also know that h is a homeomorphism of G_1 onto G_2, κ does not vanish on G_2 and, from Lemma 1, it belongs to $B(G_2)$.

We now claim that $\phi \circ h$ belongs to $B(G_2)$ for all $\phi \in B(G_1)$. Let $\{\gamma_1, \ldots, \gamma_n\}$ be a subset of G_2 and let $\varepsilon > 0$. By [6, Theorem 7], there exists $g \in A(G_1)$ such that $\|g\| < 1 + \varepsilon$ and $g(h(\gamma_i)) = 1$ for $i = 1, \ldots, n$. Fix $\phi \in B(G_1)$ and put $\varphi := \phi \cdot g$, which belongs to $A(G_1)$. Let $\{c_1, \ldots, c_n\} \subset \mathbb{C}$. By Theorem 2, we may consider the continuous algebra homomorphism $T_h : A(G_1) \to A(G_2)$ defined to be $T_h(f) := f \circ h$. Then

$$\Big|\sum_{i=1}^{n} c_i \cdot \phi(h(\gamma_i))\Big| = \Big|\sum_{i=1}^{n} c_i \cdot T_h(\varphi)(\gamma_i)\Big| \\ \le \|T_h(\varphi)\| \Big\|\sum_{i=1}^{n} c_i \delta_{\gamma_i}\Big\|_{\Sigma} \le \|T_h\|(1+\varepsilon) \Big\|\sum_{i=1}^{n} c_i \delta_{\gamma_i}\Big\|_{\Sigma}.$$

Since ε is arbitrary, we infer that

$$\Big|\sum_{i=1}^{n} c_i \cdot \kappa(\gamma_i)\Big| \le \|T_h\| \Big\|\sum_{i=1}^{n} c_i \delta_{\gamma_i}\Big\|_{\Sigma}.$$

From the continuity of $\phi \circ h$ and from [7, Corollary 2.24], we deduce that $\phi \circ h \in B(G_2)$. In like manner, we can prove that $\psi \circ h^{-1} \in B(G_1)$ for all $\psi \in B(G_2)$.

As a consequence of the above paragraph, the mapping $\widetilde{T} : B(G_1) \to B(G_2)$ defined as $\widetilde{T}(\varphi)(\gamma) := \kappa(\gamma)\varphi(h(\gamma))$, for all $\varphi \in B(G_1)$ and all $\gamma \in G_2$, is an extension of T. From the first paragraph of the proof, it is apparent that $\widetilde{T}$ is injective.

In order to prove the surjectivity of $\widetilde{T}$, let $\psi \in B(G_2)$. If Ψ is as in the proof of Theorem 2, then we know that Ψ belongs to $B(G_1)$ and $\kappa \cdot (\Psi \circ h) = 1$ on G_2. Hence, it is a routine matter to verify that $\widetilde{T}(\Psi \cdot (\psi \circ h^{-1})) = \psi$.

Finally, suppose that there is another extension of T to a weighted composition bijection $\breve{T}$ of $B(G_1)$ onto $B(G_2)$. Assume that $\breve{T}(\varphi)(\gamma) = \omega(\gamma)\varphi(h'(\gamma))$, for all $\varphi \in B(G_1)$ and all $\gamma \in G_2$. Fix $\gamma_0 \in G_2$ and choose $f \in A(G_1)$ such that $f(h(\gamma_0)) = f(h'(\gamma_0)) = 1$. Since $\widetilde{T} = \breve{T}$ on $A(G_1)$, we infer that $\kappa(\gamma_0) = \omega(\gamma_0)$. Hence, $\kappa \equiv \omega$ on the whole G_2 and, consequently, $f(h(\gamma)) = f(h'(\gamma))$ for all $f \in A(G_1)$ and each $\gamma \in G_2$. The regularity of $A(G_1)$ shows that $h \equiv h'$ on G_2. ■

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