C O L L O Q U I U M   M A T H E M A T I C U M

VOL. 76 | 1998 | NO. 2

SIDONICITY IN COMPACT, ABELIAN HYPERGROUPS

BY

KATHRYN E. HARE (WATERLOO, ONTARIO)

1. Introduction. Lacunarity has been extensively studied in a variety of settings. In this paper we investigate Sidon-type sets in duals of compact abelian hypergroups, and from our study derive new conclusions about central Sidonicity in compact, non-abelian groups.

It is well known that every infinite subset of the dual of a compact, abelian group contains an infinite Sidon set ([13]) but that there are compact, abelian hypergroups (and compact, non-abelian groups) whose duals contain no infinite (central) Sidon sets (cf. [14]). The main result of this note is to show, in contrast, that p-Sidon sets, for $p > 1$, are plentiful. In particular, we prove that every infinite subset of the dual of a compact group contains an infinite central p-Sidon set for all $p > 1$. This extends results previously established for all infinite, compact, connected groups [8], which in turn extended work of [3].

Formally, we prove more. We introduce a generalization of Sidon sets in duals of hypergroups, called (a,p)-Sidon sets, which arise by considering classical Sidonicity with the Fourier transform weighted by $(-2a)$th powers of the 2-norm of the characters; $(1,p)$-Sidon sets are p-Sidon sets, $(1,1)$-Sidon sets are the usual Sidon sets. By an essentially constructive method, we prove that any infinite subset E of the dual of a compact, abelian hypergroup K which satisfies

$$\inf\{\|\chi\|_2 : \chi \in E\} = 0,$$

contains an infinite set which is (a,p)-Sidon for all $a < p$. The condition on the 2-norms of characters is a natural one, satisfied by many hypergroups. If it is not satisfied and if, in addition, the dual, $\widehat{K}$, is also a hypergroup, then it is known that E contains an infinite Sidon set [18]; the case when the 2-norms are bounded away from zero and the dual is not a hypergroup remains open.

1991 *Mathematics Subject Classification*: Primary 43A46; Secondary 43A62.

Key words and phrases: Sidon set, lacunary set, hypergroups, non-abelian compact groups.

This research is partially supported by NSERC.

We also investigate (a,p)-Sidonicity properties of the entire dual object, and again obtain new results for compact groups.

For definitions and basic facts about hypergroups we refer the reader to [1] and [11]. References [10] and [13] are good resources for lacunarity on groups. Lacunarity on hypergroups has been studied by a number of authors, e.g. in [7], [12], [16] and [18].

2. Definition and basic properties. Throughout this paper K will denote a compact, abelian hypergroup with dual $\widehat{K}$.

NOTATION. For $E \subseteq \widehat{K}$, $1 \le p < \infty$ and $a \in \mathbb{R}$ we define

$$l_{a,p}(E) \equiv \Big\{(a_\chi)_{\chi\in E} : \|(a_\chi)\|_{a,p} \equiv \Big(\sum |a_\chi|^p \|\chi\|_2^{-2a}\Big)^{1/p} < \infty\Big\}$$

and

$$l_{a,\infty}(E) \equiv \{(a_\chi)_{\chi\in E} : \|(a_\chi)\|_{a,\infty} \equiv \sup_{\chi\in E}\{|a_\chi|\|\chi\|_2^{-2a}\} < \infty\}.$$

If $a = 1$ and $p < \infty$ or $a = 0$ and $p = \infty$ we will just write $l_p(E)$ for the weighted l_p space. Of course, if $\inf\{\|\chi\|_2 : \chi \in E\} > 0$ then the $l_{a,p}(E)$ spaces are identical for all a (and fixed p).

We let $\mathrm{Trig}_E(K)$ denote the space of trigonometric polynomials on K whose Fourier transform is supported on E. More generally, a subscript on a function space X will denote the subspace of functions whose Fourier transform is supported on E.

Motivated by [9] we make the following definition:

DEFINITION 1. Let $E \subseteq \widehat{K}$, $1 \le p \le 2$ and $a \in \mathbb{R}$. We call E an (a,p)-*Sidon set* if

$$\sup\Big\{\|(\widehat{f(\chi)})\|_{a,p} : f = \sum \widehat{f(\chi)}\chi\|\chi\|_2^{-2} \in \mathrm{Trig}_E(K),\ \|f\|_\infty \le 1\Big\} < \infty.$$

A $(1,p)$-Sidon set is customarily called a p-*Sidon set* and a $(1,1)$-Sidon set is the classical *Sidon set*.

Obviously, it is easier to be an (a,p)-Sidon set as p increases and a decreases. More generally, $l_{a,p} \subseteq l_{aq/p,q}$ if $1 \le p \le q \le 2$, so that if E is (a,p)-Sidon then E is (b,q)-Sidon for all $q \ge p$ and $b \le aq/p$.

As with Sidon sets there are a number of equivalent properties; we list some below.

PROPOSITION 2. *Let $E \subseteq \widehat{K}$, $1 \le p \le 2$ and $a \in \mathbb{R}$. Let $1/p + 1/p' = 1$ and let $b = p'(1 - a/p)$ if $p \ne 1$ or $b = 1 - a$ if $p = 1$. The following are equivalent*:

(1) *E is an (a,p)-Sidon set*;

(2) *there is a constant c such that $\|\widehat{f}\|_{a,p} \le c\|f\|_\infty$ for all $f \in L^\infty_E(K)$*;

(3) *there is a constant c such that whenever $\phi \in l_{b,p'}(E)$ then there is a measure μ on K such that $\widehat{\mu(\chi)} = \phi(\chi)$ for all $\chi \in E$ and $\|\mu\|_{M(K)} \leq c\|\phi\|_{b,p'}$.*

The proofs are similar to those found in [10] §37.2 or [5] for compact groups.

EXAMPLE 3. An important example of a compact, abelian hypergroup, whose dual is also a hypergroup under pointwise operations, is the space G_I of conjugacy classes of a compact group G [11]. A function f on G which is constant on the conjugacy classes may be viewed as defined on the hypergroup G_I and we will denote this function by $f^{\#}$. It is known that

$$\widehat{G}_I = \{(\operatorname{Tr}\chi)^{\#}/\deg\chi : \chi \in \widehat{G}\}.$$

Given $P \subseteq \widehat{G}$, we will denote by $P^{\#}$ the corresponding subset of $\widehat{G}_I$. It is easy to see that P is central Sidon if and only if $P^{\#}$ is a Sidon set in $\widehat{G}_I$. A subset E of $\widehat{G}$ is called a *central* (b,p)*-Sidon set* in [9] if there is a constant c such that whenever $f = \sum_{\chi\in E} d_\chi a_\chi \operatorname{Tr}\chi \in \operatorname{Trig}_E(G)$ then

$$\|f\|_\infty \geq c\Big(\sum d_\chi^{b+1}|a_\chi|^p\Big)^{1/p}.$$

Since $\|\operatorname{Tr}\chi/\deg\chi\|_2^{-2} = d_\chi^2$, it follows that if f is as above, then

$$\|\widehat{f^{\#}}\|_{a,p} = \Big(\sum d_\chi^{2a}|a_\chi|^p\Big)^{1/p},$$

and consequently $P^{\#}$ is (a,p)-Sidon if and only if P is central $(2a-1,p)$-Sidon in the sense of [9]. It is due to this relationship that the main results of the present paper apply to give new results for central (a,p)-Sidon sets in duals of compact groups.

The work of Pisier shows the important connection in compact groups between Sidonicity and the Λ property. These (a,p)-Sidon sets satisfy similar Λ-like properties. It is an open problem if such conditions are sufficient in any sense.

PROPOSITION 4. *If E is an (a,p)-Sidon set then there is a constant c such that for all $s \geq 1$ and for all $f \in L_E^{2s}(K)$,*

$$\|f\|_{2s} \leq c\sqrt{s}\|\widehat{f}\|_{b,q}$$

where $1/q = 1/p' + 1/2$ and $b = (4p-2a)/(3p-2)$.

The proof is similar to the analogous results for Sidon sets in groups and hypergroups (see [2] and [16]) and is omitted.

REMARK 1. Notice that a difference between this proposition and the corresponding result for the group case is that when $p = a = 1$ we obtain the inequality

$$\|f\|_{2s} \le c\sqrt{s}\|\widehat{f}\|_{b,2}$$

with $b = 2$, rather than $b = 1$. The necessity of this choice of b can be seen in [15]. There, in Example 1, Rider constructs a compact group G and a central Sidon set $\{\chi_n\}$ consisting of representations of degree 2^n which have the property that if $f = \operatorname{Tr}\chi_n$, then $\|f\|_{2s} = 2^{n(1-1/s)}$ and $\|\widehat{f}\|_{b,2} = 2^{n(b-1)}$. Obviously, we cannot have the inequality $\|f\|_{2s} \le c\sqrt{s}\|\widehat{f}\|_{b,2}$ satisfied for all $s \ge 1$ and $n \in \mathbb{N}$ with any choice of $b < 2$.

3. Existence of (a,p)-Sidon sets. The main purpose of this paper is to demonstrate the plentifulness of (a,p)-Sidon sets when $a < p$. The key tool for showing this is the following theorem.

THEOREM 5. *Suppose K is a compact, abelian hypergroup and $\{\chi_n\} \subseteq \widehat{K}$ satisfies* $\inf\|\chi_n\|_2 = 0$. *Let $0 < \varepsilon_n < 1/3$, $n = 1, 2, \ldots$ There is a subsequence $\{\chi_{n_k}\}$ such that if*

$$f = \sum a_k \chi_{n_k} \|\chi_{n_k}\|_2^{-2} \in \operatorname{Trig}(K)$$

with $\|f\|_\infty \le 1$, then $|a_k| \le \|\chi_{n_k}\|_2^{2(1-\varepsilon_k)}$.

Proof. As in [17], let $\{D_\alpha\}$ be a bounded, approximate identity in $\operatorname{Trig}(K)$ with $\|D_\alpha\|_1 = 1$. Set $\phi_1 = \chi_{n_1}\|\chi_{n_1}\|_2^{-1}$, where $\|\chi_{n_1}\|_2^{-\varepsilon_1} \ge 16$. We now proceed to construct the subsequence $\{\chi_{n_k}\}$ inductively. Assume $\phi_i = \chi_{n_i}\|\chi_{n_i}\|_2^{-1}$ for $i = 1, \ldots, j-1$ have been chosen, and for each such i select $D_i \in \{D_\alpha\}$ satisfying $\|\phi_i * D_i\|_\infty \ge \frac{3}{4}\|\phi_i\|_\infty$. Since $\inf\|\chi_n\|_2 = 0$ we may choose $\phi_j = \chi_{n_j}\|\chi_{n_j}\|_2^{-1}$ with the following properties:

(1) $\|\phi_j\|_\infty^{\varepsilon_j} \ge 2(j+1)$;
(2) $\|\phi_j\|_\infty^{\varepsilon_j} \ge \min\{\|\phi_i\|_\infty, \|D_i\|_\infty : i = 1, \ldots, j-1\}$;
(3) $\|\phi_j\|_\infty^{1-2\varepsilon_j} \ge 4^{(j+1)}j$.

For each i choose $x_i \in K$ so that $|\phi_i * D_i(x_i)| \ge \frac{2}{3}\|\phi_i * D_i\|_\infty$.

Now assume that

$$f = \sum_{k=1}^{N} a_k \chi_{n_k} \|\chi_{n_k}\|_2^{-2}$$

with $\|f\|_\infty \le 1$. For convenience we write $f = \sum_{k=1}^N b_k\phi_k$ with $b_k = a_k\|\chi_{n_k}\|_2^{-1}$. Observe that as $\{\phi_k\}$ is orthonormal, $|b_k| \le \|f\|_2 \le 1$.

We begin by evaluating the convolution of D_N with f at x_N:

$$f * D_N(x_N) = b_N(\phi_N * D_N(x_N)) + \sum_{j=1}^{N-1} b_j(\phi_j * D_N(x_N)),$$

to obtain the bound

$$|b_N|\,|\phi_N * D_N(x_N)| \le \|f * D_N\|_\infty + \sum_{j=1}^{N-1} |b_j|\,\|\phi_j * D_N\|_\infty.$$

Since $|\phi_N * D_N(x_N)| \ge \frac{1}{2}\|\phi_N\|_\infty$ and $\|\phi_j * D_N\|_\infty \le \|\phi_j\|_\infty \|D_N\|_1 \le \|\phi_j\|_\infty$, this gives the estimate

$$|b_N| \le \frac{2}{\|\phi_N\|_\infty}\Big(1 + \sum_{j=1}^{N-1} \|\phi_j\|_\infty\Big),$$

which by properties (1) and (2) yields

$$|b_N| \le \frac{2N}{\|\phi_N\|_\infty^{1-\varepsilon_N}}.$$

For $j = 1, \dots, N-1$ we similarly have

$$\begin{aligned} f * D_{N-j}(x_{N-j}) = b_{N-j}&(\phi_{N-j} * D_{N-j}(x_{N-j})) \\ &+ \sum_{k=1}^{N-j-1} b_k(\phi_k * D_{N-j}(x_{N-j})) \\ &+ \sum_{k=0}^{j-1} b_{N-k}(\phi_{N-k} * D_{N-j}(x_{N-j})) \end{aligned}$$

and hence

$$|b_{N-j}| \le \frac{2}{\|\phi_{N-j}\|_\infty}\Big(1 + \sum_{k=1}^{N-j-1} \|\phi_k\|_\infty + \sum_{k=0}^{j-1} |b_{N-k}|\,\|\phi_{N-k}\|_1 \|D_{N-j}\|_\infty\Big).$$

If $k \le j-1$ then $N-k > N-j$, so by (2) we have $\|D_{N-j}\|_\infty \le \|\phi_{N-k}\|_\infty^{\varepsilon_{N-k}}$. Thus if we let

$$S_{j-1} = \sum_{k=0}^{j-1} |b_{N-k}|\,\|\phi_{N-k}\|_\infty^{\varepsilon_{N-k}},$$

we can write

$$|b_{N-j}| \le \frac{2}{\|\phi_{N-j}\|_\infty}(1 + (N-j-1)\|\phi_{N-j}\|_\infty^{\varepsilon_{N-j}} + S_{j-1}).$$

Upon noting that $\|\phi_{N-j}\|_\infty^{\varepsilon_{N-j}} \ge 1$ this can be simplified to

$$|b_{N-j}| \le \frac{2}{\|\phi_{N-j}\|_\infty}(S_{j-1} + (N-j)\|\phi_{N-j}\|_\infty^{\varepsilon_{N-j}}).$$

Of course, $S_j = S_{j-1} + |b_{N-j}|\,\|\phi_{N-j}\|_\infty^{\varepsilon_{N-j}}$ so our previous estimates, together with the fact that $\|\phi_{N-j}\|_\infty^{1-\varepsilon_{N-j}} \ge 2$, show that

$$S_j \le 2S_{j-1} + \frac{2(N-j)}{\|\phi_{N-j}\|_\infty^{1-2\varepsilon_{N-j}}}.$$

Iterating, this gives

$$S_j \le 2^j S_0 + \sum_{k=1}^{j} \frac{2^{j-k+1}(N-k)}{\|\phi_{N-k}\|_\infty^{1-2\varepsilon_{N-k}}}.$$

Since $S_0 = |b_N|\,\|\phi_N\|_\infty^{\varepsilon_N}$, our bound for $|b_N|$ established earlier in the proof gives

$$S_j \le \sum_{k=0}^{j} \frac{2^{j-k+1}(N-k)}{\|\phi_{N-k}\|_\infty^{1-2\varepsilon_{N-k}}}.$$

By property (3) it follows that $S_j \le \sum_{k=0}^{j} 2^{k-N-1}$ and this is bounded by 1 since $j \le N-1$. Thus

$$|b_{N-j}| \le \frac{2}{\|\phi_{N-j}\|_\infty} + \frac{2(N-j)}{\|\phi_{N-j}\|_\infty^{1-\varepsilon_{N-j}}} \le \frac{1}{\|\phi_{N-j}\|_\infty^{1-2\varepsilon_{N-j}}}$$

by (1). Hence

$$|a_k| = |b_k|\,\|\chi_{n_k}\|_2 \le \|\chi_{n_k}\|_2^{2(1-\varepsilon_k)}$$

as claimed.

An important corollary of this result is

Corollary 6. *Suppose $1 \le p \le 2$ and E is an infinite subset of $\widehat{K}$ satisfying*

$$(*) \qquad \inf\{\|\chi\|_2 : \chi \in E\} = 0.$$

Then E contains an infinite subset which is (a,p)-Sidon for all $a < p$.

Proof. Choose a sequence $a_k \nearrow p$ and pick $0 < \varepsilon_k < 1/3$ so that $\delta_k \equiv 2(p(1-\varepsilon_k) - a_k) > 0$. Let $\{\chi_k\}$ be a subset of E with decreasing 2-norms satisfying $\|\chi_k\|_2^{\delta_k} < 2^{-k}$. We apply the theorem to $\{\chi_k\}$ to choose a subsequence $\{\chi_{n_k}\} = E''$ with the property described in the statement of the theorem.

Given $a < p$, select N such that $a_k \ge a$ for all $k \ge N$ and assume

$$f = \sum b_k \chi_{n_k} \|\chi_{n_k}\|_2^{-2} \in \mathrm{Trig}_{E''}(K)$$

with $\|f\|_\infty \le 1$. From the theorem we see that

$$\begin{aligned}\|\widehat{f}\|_{a,p}^p &= \sum_{k=1}^{N-1} |b_k|^p \|\chi_{n_k}\|_2^{-2a} + \sum_{k=N}^{\infty} |b_k|^p \|\chi_{n_k}\|_2^{-2a} \\ &\le C + \sum_{k=N}^{\infty} \|\chi_{n_k}\|_2^{2(p(1-\varepsilon_k)-a)}\end{aligned}$$

where C is a constant independent of f. Since $a_k \ge a$ for all $k \ge N$ it follows that

$$\|\widehat{f}\|_{a,p}^p \le C + \sum_{k=N}^{\infty} \|\chi_{n_k}\|_2^{\delta_k} \le C + 1.$$

Thus E'' is an infinite (a,p)-Sidon set for all $a < p$.

REMARK 2. Suppose E satisfies $(*)$. Let $f(x) = \chi(x)\|\chi\|_2^{-2}$ be an E function. Then

$$\|f\|_\infty = \|\widehat{f}\|_{p+\varepsilon,p} \|\chi\|_2^{2\varepsilon/p},$$

and hence if $\varepsilon > 0$ then E is not a $(p+\varepsilon,p)$-Sidon set. It is an open problem if E may contain infinite (p,p)-Sidon sets.

COROLLARY 7. *If K is an infinite, compact, abelian hypergroup, whose dual is a hypergroup, then every infinite subset of the dual of K contains an infinite (a,p)-Sidon for all $a < p$. In particular, it contains an infinite $(a,1)$-Sidon set for all $a < 1$ and infinite p-Sidon sets for all $p > 1$.*

Proof. Let E be an infinite subset of $\widehat{K}$. If $\sup_{\chi \in E} \|\chi\|_2 > 0$ then E contains an infinite Sidon set [18], and hence an infinite (a,p)-Sidon set for all $a \le p$. Otherwise the corollary above applies.

Perhaps the most important consequence of the theorem is the following corollary which was previously obtained for compact connected groups by using the representation theory and structure theory of Lie groups [8].

COROLLARY 8. *If G is any infinite, compact group then every infinite subset of the dual contains an infinite central (a,p)-Sidon set for all $a < 2p-1$.*

Proof. We just need to recall that $E \subseteq \widehat{G}$ is central $(2a-1,p)$-Sidon if and only if $E^\# \subseteq \widehat{G}_I$ is an (a,p)-Sidon set in the hypergroup sense, and that $\widehat{G}_I$ is a hypergroup.

EXAMPLE 9. In [12], Lasser constructs a family of hypergroups $K(\alpha,\beta)$ whose characters are the Jacobi polynomials associated with α and β. When these polynomials are the Legendre polynomials or the Chebyshev polynomials of the second kind, no infinite subset of the dual of $K(\alpha,\beta)$ is Sidon. But since every infinite subset of the dual satisfies $(*)$, each such set contains an infinite (a,p)-Sidon set for all $a < p$.

The same is true for the countable, compact hypergroups introduced by Dunkl and Ramirez (see [4] and [16]).

4. Sidon properties of $\widehat{K}$. Since $\|f\|_\infty \ge \|f\|_2 = \|\widehat{f}\|_{1,2}$, the entire dual $\widehat{K}$ is obviously $(1,2)$-Sidon. The final result of our paper will show that this cannot in general be improved for compact, abelian hypergroups if we assume, in addition, that $\widehat{K}$ is a hypergroup under pointwise operations. First we require a preliminary result on multipliers.

TERMINOLOGY. (i) We will say $(e_\chi) \in M(C(K), l_p(\widehat{K}))$ (the space of multipliers from $C(K)$ to $l_p(\widehat{K})$) if $(e_\chi a_\chi) \in l_p(\widehat{K})$ whenever $f = \sum a_\chi \chi \|\chi\|_2^{-2}$ is a continuous function.

(ii) We will say $(e_\chi) \in M(l_q(\widehat{K}), M(K))$ if $\sum e_\chi a_\chi \chi \|\chi\|_2^{-2} \in M(K)$ whenever $(a_\chi) \in l_q(\widehat{K})$.

LEMMA 10. *If K is a compact, abelian hypergroup whose dual is a hypergroup, then*

$$M(C(K), l_p(\widehat{K})) \simeq \begin{cases} l_{2p/2-p}(\widehat{K}) & \text{if } 1 \le p < 2, \\ l_\infty(\widehat{K}) & \text{if } p = 2. \end{cases}$$

P r o o f. Standard arguments show that if $q \ge 2$ and $1/s = 1/2 - 1/q$ then

$$M(l_q(\widehat{K}), l_2(\widehat{K})) = l_s(\widehat{K}).$$

Obviously, $M(l_q(\widehat{K}), l_2(\widehat{K})) \subseteq M(l_q(\widehat{K}), M(K))$.

Let $E = (e_\chi) \in M(l_q(\widehat{K}), M(K))$ and let $(b_\chi) \in l_q(\widehat{K})$. Only countably many $b_\chi \neq 0$, so without loss of generality we may assume $\widehat{K}$ is countable.

Consider uniformly bounded, independent random variables $\{\xi_\chi\}$ on a probability space Ω, with variance one and mean zero. Clearly, $\{\xi_\chi(\omega) b_\chi\} \in l_q(\widehat{K})$ for a.e. ω and hence

$$\sum e_\chi \xi_\chi(\omega) b_\chi \chi \|\chi\|_2^{-2} \in M(K) \quad \text{for a.e. } \omega.$$

By Thm. 4.8 of [6],

$$\sum e_\chi b_\chi \chi \|\chi\|_2^{-2} \in L^2(K),$$

i.e. $\{e_\chi b_\chi\} \in l_2(\widehat{K})$. Thus E is a multiplier from $l_q(\widehat{K})$ to $l_2(\widehat{K})$ and so belongs to $l_s(\widehat{K})$. Consequently, $M(l_q(\widehat{K}), M(K)) \simeq l_s(\widehat{K})$ and by duality $M(C(K), l_{q'}(\widehat{K})) \simeq l_s(\widehat{K})$. If $p = q'$ then $s = 2p/(2-p)$, which completes the proof.

THEOREM 11. *Suppose K is a compact, abelian hypergroup whose dual is a hypergroup. If $1 \le p < 2$ and $a \ge p/2$ then $\widehat{K}$ is not an (a,p)-Sidon set. If $\inf \|\chi\|_2 = 0$ and $a > 1$, then $\widehat{K}$ is not an $(a,2)$-Sidon set.*

Proof. The proof is similar to [9], (1.11). Assume $\widehat{K}$ is (a,p)-Sidon and f and h are trigonometric polynomials on K. If c is the (a,p)-Sidon constant of $\widehat{K}$ then

$$\|h\|_1\|f\|_\infty \geq \|h*f\|_\infty \geq c\Big(\sum |\widehat{h(\chi)}|^p|\widehat{f(\chi)}|^p\|\chi\|_2^{-2a}\Big)^{1/p}.$$

Define a trigonometric polynomial H by $\widehat{H(\chi)} = \widehat{h(\chi)}\|\chi\|_2^{2(1-a)/p}$. Then

$$\|\widehat{H*f}\|_p = \|\widehat{h*f}\|_{a,p} \leq (1/c)\|h\|_1\|f\|_\infty.$$

Hence H is a multiplier from $C(K)$ to $l_p(\widehat{K})$ with operator norm at most $(1/c)\|h\|_1$. Applying the lemma, we see that $\widehat{H} \in l_s(\widehat{K})$ for $s = 2p/(2-p)$, with operator norm comparable to $\|\widehat{H}\|_s$.

Now take for h a bounded approximate identity $\{h_\alpha\} \subseteq \mathrm{Trig}(K)$, with $\|h_\alpha\|_1 = 1$. By the previous remarks it follows that there is a constant C such that

$$C = C\|h_\alpha\|_1 \leq \|\widehat{H}_\alpha\|_s = \begin{cases} (\sum_\chi |\widehat{h_\alpha(\chi)}|^s\|\chi\|_2^{(2(1-a)s/p)-2})^{1/s} & \text{if } p \neq 2,\\ \sup_\chi |\widehat{h_\alpha(\chi)}|\,\|\chi\|_2^{1-a} & \text{if } p = 2.\end{cases}$$

As $\widehat{h_\alpha(\chi)} \to 1$, the previous inequalities imply that

(i) $$\sum_\chi \|\chi\|_2^{4(1-a)/(2-p)-2} < \infty \quad \text{if } p \neq 2$$

and

(ii) $$\sup_\chi \|\chi\|_2^{1-a} < \infty \quad \text{if } p = 2.$$

As $\|\chi\|_2 \leq 1$, (i) clearly fails if $4(1-a) \leq 2(2-p)$, i.e. $p \leq 2a$, and (ii) fails when $a > 1$ and $\inf\|\chi\|_2 = 0$.

The analogous result is already known for central (a,p)-Sidon sets in duals of compact, connected groups [9]. By our usual arguments this can now be improved to

COROLLARY 12. *If G is an infinite, compact group then $\widehat{G}$ is not central $(p-1,p)$-Sidon for any $1 \leq p < 2$. If the degrees of the irreducible representations of G are unbounded, then $\widehat{G}$ is not central $(a,2)$-Sidon for any $a > 1$.*

REFERENCES

[1] W. Bloom and H. Heyer, *Harmonic Analysis of Probability Measures on Hypergroups*, de Gruyter Stud. Math. 20, de Gruyter, New York, 1995.

[2] A. Dooley, *On lacunary sets for nonabelian groups*, J. Austral. Math. Soc. 25 (1978), 167–176.

[3] A. Dooley, *Central lacunary sets for Lie groups*, ibid. 45 (1988), 30–45.
[4] C. Dunkl and D. Ramirez, *Central Sidon sets of bounded representation type*, Notices Amer. Math. Soc. 18 (1971), A1101.
[5] R. E. Edwards and K. A. Ross, *p-Sidon sets*, J. Funct. Anal. 15 (1974), 404–427.
[6] J. Fournier and K. Ross, *Random Fourier series on hypergroups*, J. Austral. Math. Soc. 37 (1984), 45–81.
[7] O. Gebuhrer and A. Schwartz, *Sidon sets and Riesz sets for some measure algebras on the disk*, Colloq. Math. 72 (1997), 269–279.
[8] K. Hare, *Central Sidonicity for compact Lie groups*, Ann. Inst. Fourier (Grenoble) 45 (1995), 547–564.
[9] K. Hare and D. Wilson, *Weighted p-Sidon sets*, J. Austral. Math. Soc. 61 (1996), 73–95.
[10] E. Hewitt and K. Ross, *Abstract Harmonic Analysis II*, Springer, New York, 1970.
[11] R. Jewett, *Spaces with an abstract convolution of measures*, Adv. Math. 18 (1975), 1–101.
[12] R. Lasser, *Orthogonal polynomials and hypergroups*, Rend. Mat. (7) 3 (1983), 185–209.
[13] J. Lopez and K. Ross, *Sidon Sets*, Lecture Notes in Pure and Appl. Math. 13, Marcel Dekker, New York, 1975.
[14] D. Ragozin, *Central measures on compact simple Lie groups*, J. Funct. Anal. 10 (1972), 212–229.
[15] D. Rider, *Central lacunary sets*, Monatsh. Math. 76 (1972), 328–338.
[16] R. Vrem, *Lacunarity on compact hypergroups*, Math. Z. 164 (1978), 93–104.
[17] —, *Harmonic analysis on compact hypergroups*, Pacific J. Math. 85 (1979), 239–251.
[18] —, *Independent sets and lacunarity for hypergroups*, J. Austral. Math. Soc. 50 (1991), 171–188.

Deptartment of Pure Mathematics
University of Waterloo
Waterloo, Ontario
Canada
E-mail: kehare@math.uwaterloo.ca

Received 22 April 1997