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ROUGH SINGULAR INTEGRAL OPERATORS WITH HARDY SPACE FUNCTION KERNELS ON A PRODUCT DOMAIN

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In this paper we introduce atomic Hardy spaces on the product domain $S^{n-1}\times S^{m-1}$ and prove that rough singular integral operators with Hardy space function kernels are L^p bounded on $\mathbb{R}^n\times\mathbb{R}^m$. This is an extension of some well known results.

1. Introduction. Let S^{n-1}, S^{m-1} be unit spheres in $\mathbb{R}^n$, $\mathbb{R}^m$ $(n\ge 2,\ m\ge 2)$ respectively and $\Omega(x,y)$ be a function on the product domain $\mathbb{R}^n\times\mathbb{R}^m$ satisfying

$$\Omega(\lambda_1 x',\lambda_2 y')=\Omega(x',y')\quad\text{for any } \lambda_1,\lambda_2>0 \tag{1.1}$$

and

$$\begin{aligned}\int_{S^{n-1}}\Omega(x',y')\,dx'&=0\quad\text{for any } y'\in S^{m-1},\\ \int_{S^{m-1}}\Omega(x',y')\,dy'&=0\quad\text{for any } x'\in S^{n-1}.\end{aligned}\tag{1.2}$$

A singular integral operator T on $\mathbb{R}^n\times\mathbb{R}^m$ is defined by

$$Tf(x,y)=\text{p.v.}\iint_{\mathbb{R}^n\times\mathbb{R}^m}\frac{\Omega(u,v)}{|u|^n|v|^m}f(x-u,y-v)\,du\,dv.$$

It is well known that T is an $L^p(\mathbb{R}^n\times\mathbb{R}^m)$ bounded operator $(1<p<\infty)$ when Ω satisfies some regularity conditions [3]. Using the idea developed in [2], J. Duoandikoetxea [1] proved the $L^p(\mathbb{R}^n\times\mathbb{R}^m)$ boundedness $(1<p<\infty)$ of T with the rough condition $\Omega\in L^q(S^{n-1}\times S^{m-1})$ instead of regularity. Recently, Y. S. Jiang and S. Z. Lu improved the above results in [4]. They set up a class of block-spaces $B_q^{\phi}(S^{n-1}\times S^{m-1})$ $(q>1)$ on $S^{n-1}\times S^{m-1}$ and proved that T is $L^2(\mathbb{R}^n\times\mathbb{R}^m)$ bounded if $\Omega\in B_q^{\phi}(S^{n-1}\times S^{m-1})$.

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Under inspiration from [5], in this paper we shall introduce the atomic Hardy spaces $H^1_{\rm a}(S^{n-1}\times S^{m-1})$ and prove that T is $L^p(\mathbb{R}^n\times\mathbb{R}^m)$ bounded $(1<p<\infty)$ if $\Omega\in H^1_{\rm a}(S^{n-1}\times S^{m-1})$. This is an extension of the above mentioned results.

Let us begin with the definition of $(1,\infty)$-atoms on $S^{n-1}\times S^{m-1}$.

Definition 1. A function $a(x',y')$ on $S^{n-1}\times S^{m-1}$ is called a $(1,\infty)$-*atom* if it satisfies the following conditions:

(i)
$$\int_{S^{n-1}} a(x',y')\,dx'=0 \quad \text{for any } y'\in S^{m-1},$$
$$\int_{S^{m-1}} a(x',y')\,dy'=0 \quad \text{for any } x'\in S^{n-1},$$

(ii)
$$\operatorname{supp} a\subset B,\qquad B=B_n\times B_m,$$

where
$$B_n=\{x'\in S^{n-1}:|x'-x'_0|<\alpha,\ x'_0\in S^{n-1}\},$$
$$B_m=\{y'\in S^{m-1}:|y'-y'_0|<\beta,\ y'_0\in S^{m-1}\}.$$

(iii)
$$\|a\|_\infty\le\alpha^{-(n-1)}\beta^{-(m-1)}.$$

Now, we may define the atomic Hardy space $H^1_{\rm a}(S^{n-1}\times S^{m-1})$.

Definition 2 . The *atomic Hardy space* $H^1_{\rm a}(S^{n-1}\times S^{m-1})$ is defined by

$$H^1_{\rm a}(S^{n-1}\times S^{m-1})=\Big\{f\in L^1(S^{n-1}\times S^{m-1}):f(x',y')=\sum_{l=0}^\infty\lambda_l a_l(x',y'),$$
$$a_l(x',y') \text{ is a } (1,\infty)\text{-atom and } \sum_{l=0}^\infty|\lambda_l|<\infty\Big\}.$$

Moreover, we set $\|f\|_{H^1_{\rm a}(S^{n-1}\times S^{m-1})}=\inf\sum_{l=0}^\infty|\lambda_l|$, where the infimum is taken over all decompositions $f=\sum_{l=0}^\infty\lambda_l a_l$ of f.

The main result of this paper is

Theorem 1. *Suppose that* Ω *satisfies* (1.1), (1.2) *and* $\Omega(x',y')\in H^1_{\rm a}(S^{n-1}\times S^{m-1})$. *Then* T *is* $L^p(\mathbb{R}^n\times\mathbb{R}^m)$ *bounded* $(1<p<\infty)$.

In proving Theorem 1 we shall use a result of [1]:

Theorem A. *Let* $\{\sigma_{j,k}\}_{j,k\in\mathbb{Z}}$ *be a double sequence of uniformly bounded Borel measures in* $\mathbb{R}^n\times\mathbb{R}^m$ *and*
$$|\widehat{\sigma}_{j,k}(\xi,\eta)|\le C|a^j\xi|^{\pm\delta}|b^k\eta|^{\pm\varrho}$$
for some $a,b>1$, $\delta,\varrho>0$ *and for all* $j,k\in\mathbb{Z}$. *If* $\sigma^*(f)=\sup_{j,k}||\sigma_{j,k}|*f|$

is bounded in $L^q(\mathbb{R}^n \times \mathbb{R}^m)$ *for some* $q > 1$, *then*

$$Tf(x,y) = \sum_j \sum_k \sigma_{j,k} * f(x,y)$$

is bounded in $L^p(\mathbb{R}^n \times \mathbb{R}^m)$ *for* $|1/p - 1/2| < 1/(2q)$.

2. Proof of Theorem 1. By Definition 2, we may write $\Omega(x',y') = \sum_{l=0}^{\infty} \lambda_l \Omega_l(x',y')$, where $\Omega_l(x',y')$ is a $(1,\infty)$-atom and $\sum_{l=0}^{\infty} |\lambda_l| < \infty$. Then Ω_l satisfies the following conditions:

$$\int_{S^{n-1}} \Omega_l(x',y')\,dx' = 0 \quad \text{for any } y' \in S^{m-1},$$

$$\int_{S^{m-1}} \Omega_l(x',y')\,dy' = 0 \quad \text{for any } x' \in S^{n-1}, \tag{2.1}$$

$$\operatorname{supp} \Omega_l \subset B^l, \quad B^l = B_n^l \times B_m^l, \tag{2.2}$$

where

$$B_n^l = \{x' \in S^{n-1} : |x' - x_0'| < \alpha_l,\ x_0' \in S^{n-1}\},$$

$$B_m^l = \{y' \in S^{m-1} : |y' - y_0'| < \beta_l,\ y_0' \in S^{m-1}\},$$

$$\|\Omega_l\|_{L^\infty(S^{n-1}\times S^{m-1})} \le \alpha_l^{-(n-1)} \beta_l^{-(m-1)}. \tag{2.3}$$

First let us introduce some notation. For $j,k \in \mathbb{Z}$,

$$E_{j,k}(x,y) = \{(x,y) \in \mathbb{R}^n \times \mathbb{R}^m : 2^{j-1} < |x| \le 2^j, 2^{k-1} < |y| \le 2^k\},$$
$$E_{j,k}^{\mathrm{c}}(x,y) = (\mathbb{R}^n \times \mathbb{R}^m) \setminus E_{j,k}(x,y),$$
$$K_{j,k}(x,y) = \begin{cases} \Omega(x,y)|x|^{-n}|y|^{-m} & \text{for } (x,y) \in E_{j,k}(x,y), \\ 0, & \text{for } (x,y) \in E_{j,k}^{\mathrm{c}}(x,y), \end{cases}$$
$$K_{j,k}^l(x,y) = \begin{cases} \Omega_l(x,y)|x|^{-n}|y|^{-m} & \text{for } (x,y) \in E_{j,k}(x,y), \\ 0, & \text{for } (x,y) \in E_{j,k}^{\mathrm{c}}(x,y). \end{cases}$$

Then we have

$$K_{j,k}(x,y) = \sum_{l=0}^{\infty} \lambda_l K_{j,k}^l(x,y)$$

and

$$Tf(x,y) = \sum_j \sum_k K_{j,k} * f(x,y) = \sum_j \sum_k \sum_{l\ge 0} \lambda_l K_{j,k}^l * f(x,y).$$

Let j_l, k_l be integers such that

$$1 < 2^{j_l}\alpha_l \le 2 \quad \text{and} \quad 1 < 2^{k_l}\beta_l \le 2, \tag{2.4}$$

where α_l, β_l are determined by (2.2). Obviously, when l is fixed, j_l, k_l are unique. Then we may write

$$Tf(x,y)=\sum_j\sum_k \sigma_{j,k}*f(x,y), \tag{2.5}$$

where

$$\sigma_{j,k}(x,y)=\sum_{l=0}^{\infty}\lambda_l K^l_{j+j_l,k+k_l}(x,y). \tag{2.6}$$

We now give the Fourier transform estimates of $K^l_{j,k}(x,y)$.

Lemma 1. *For any δ with $0<\delta<1/2$, there are $0<\varepsilon,\theta<1$ and a constant $\overline{C}=C(\delta,\varepsilon,\theta)$ such that*

$$|\widehat{K^l_{j,k}}(\xi,\eta)|\le \overline{C}\min\{|2^j\alpha_l\xi|^{1/2}|2^k\beta_l\eta|^{1/2},|2^j\alpha_l\xi|^{-\delta}|2^k\beta_l\eta|^{-\delta},\\ |2^j\alpha_l\xi|^{\varepsilon}|2^k\beta_l\eta|^{-\theta},|2^j\alpha_l\xi|^{-\theta}|2^k\beta_l\eta|^{\varepsilon}\}.$$

Proof. By the cancellation condition (2.1), we have

$$\int_{S^{n-1}}\Omega_l(x',y')\int_{2^{k-1}}^{2^k}\int_{2^{j-1}}^{2^j}e^{-2\pi i(r\xi\cdot x'_0+s\eta\cdot y')}\frac{dr\,ds}{rs}\,dx'=0.$$

Hence

$$\iint_{S^{n-1}\times S^{m-1}}\Omega_l(x',y')\int_{2^{k-1}}^{2^k}\int_{2^{j-1}}^{2^j}e^{-2\pi i(r\xi\cdot x'_0+s\eta\cdot y')}\frac{dr\,ds}{rs}\,dx'dy'=0. \tag{2.7}$$

Again using (2.1) we get

$$\iint_{S^{n-1}\times S^{m-1}}\Omega_l(x',y')\int_{2^{k-1}}^{2^k}\int_{2^{j-1}}^{2^j}e^{-2\pi i(r\xi\cdot x'+s\eta\cdot y'_0)}\frac{dr\,ds}{rs}\,dx'dy'=0. \tag{2.8}$$

By (2.7),

$$\begin{aligned}|\widehat{K^l_{j,k}}(\xi,\eta)|&=\Big|\iint_{\mathbb{R}^n\times\mathbb{R}^m}e^{-2\pi i(\xi\cdot x+\eta\cdot y)}K^l_{j,k}(x,y)\,dx\,dy\Big|\\ &=\Big|\iint_{E_{j,k}(x,y)}e^{-2\pi i(\xi\cdot x+\eta\cdot y)}\frac{\Omega_l(x',y')}{|x|^n|y|^m}\,dx\,dy\Big|\\ &=\Big|\int_{2^{k-1}}^{2^k}\int_{2^{j-1}}^{2^j}\iint_{S^{n-1}\times S^{m-1}}\Omega_l(x',y')e^{-2\pi i(r\xi\cdot x'+s\eta\cdot y')}\,dx'dy'\frac{dr\,ds}{rs}\Big|\end{aligned}$$

$$= \Big| \iint_{S^{n-1}\times S^{m-1}} \Omega_l(x', y') \int_{2^{k-1}}^{2^k} \int_{2^{j-1}}^{2^j} [e^{-2\pi i(r\xi\cdot x' + s\eta\cdot y')} - e^{-2\pi i(r\xi\cdot x_0' + s\eta\cdot y')}] \frac{dr\,ds}{rs}\, dx'dy' \Big|$$

$$\le \iint_{S^{n-1}\times S^{m-1}} |\Omega_l(x', y')| \int_{2^{k-1}}^{2^k} \int_{2^{j-1}}^{2^j} 2\pi |r\xi\cdot(x' - x_0')| \frac{dr\,ds}{rs}\, dx'dy'$$

$$= C2^j|\xi| \iint_{S^{n-1}\times S^{m-1}} |\Omega_l(x', y')| \cdot |x' - x_0'|\, dx'\, dy' \le C|2^j\alpha_l\xi|,$$

where the last inequality follows from $\iint_{S^{n-1}\times S^{m-1}} |\Omega_l(x', y')|\, dx'\, dy' \le 1$ (by (2.2) and (2.3)). From (2.8) and using the same method we can prove

$$|\widehat{K^l_{j,k}}(\xi, \eta)| \le C|2^k\beta_l\eta|.$$

Thus we obtain

$$|\widehat{K^l_{j,k}}(\xi, \eta)| \le C \min\{|2^j\alpha_l\xi|, |2^k\beta_l\eta|\}. \tag{2.9}$$

On the other hand,

$$|\widehat{K^l_{j,k}}(\xi, \eta)|^2 = \Big| \int_{2^{k-1}}^{2^k} \int_{2^{j-1}}^{2^j} \iint_{S^{n-1}\times S^{m-1}} \Omega_l(x', y') e^{-2\pi i(r\xi\cdot x' + s\eta\cdot y')}\, dx'dy'\, \frac{dr\,ds}{rs} \Big|^2$$

$$\le C \int_{2^{k-1}}^{2^k} \int_{2^{j-1}}^{2^j} \Big| \iint_{S^{n-1}\times S^{m-1}} \Omega_l(x', y') e^{-2\pi i(r\xi\cdot x' + s\eta\cdot y')}\, dx'dy' \Big|^2 \frac{dr\,ds}{rs}$$

and

$$\Big| \iint_{S^{n-1}\times S^{m-1}} \Omega_l(x', y') e^{-2\pi i(r\xi\cdot x' + s\eta\cdot y')}\, dx'dy' \Big|^2 = \iint_{(S^{n-1}\times S^{m-1})^2} \Omega_l(x', y')\overline{\Omega_l(u', v')} \times e^{-2\pi i(r\xi\cdot x' + s\eta\cdot y')} e^{2\pi i(r\xi\cdot u' + s\eta\cdot v')}\, dx'dy'du'dv'.$$

Set

$$I = \int_{2^{k-1}}^{2^k} \int_{2^{j-1}}^{2^j} e^{-2\pi i[r\xi\cdot(x'-u') + s\eta\cdot(y'-v')]} \frac{dr\,ds}{rs}.$$

Then we have $|I| \le (\log 2)^2$. Moreover, from [2] there is a constant C such that

$$|I| \le C \frac{1}{|2^j\xi\cdot(x'-u')|\cdot|2^k\eta\cdot(y'-v')|}.$$

Thus, for any $0<\sigma<1$ we have

$$|I| \le C_\sigma \frac{1}{|2^j\xi\cdot(x'-u')|^\sigma|2^k\eta\cdot(y'-v')|^\sigma}.$$

Hence

$$\begin{aligned}
&|\widehat{K^l_{j,k}}(\xi,\eta)|^2\\
&\quad\le C_\sigma \iint\limits_{(S^{n-1}\times S^{m-1})^2} |\Omega_l(x',y')\overline{\Omega_l(u',v')}| \frac{dx'\,dy'\,du'\,dv'}{|2^j\xi\cdot(x'-u')|^\sigma|2^k\eta\cdot(y'-v')|^\sigma}\\
&\quad\le \frac{C_\sigma}{|B^l_n|^2|B^l_m|^2}\Bigg(\iint\limits_{\substack{|x'-x'_0|<\alpha_l\\|u'-x'_0|<\alpha_l}} \frac{dx'\,du'}{|2^j\xi\cdot(x'-u')|^\sigma}\Bigg)\\
&\qquad\times\Bigg(\iint\limits_{\substack{|y'-y'_0|<\beta_l\\|v'-y'_0|<\beta_l}} \frac{dy'\,dv'}{|2^k\eta\cdot(y'-v')|^\sigma}\Bigg).
\end{aligned}$$

From [5] we know that

$$\frac{1}{|B^l_n|^2}\Bigg(\iint\limits_{\substack{|x'-x'_0|<\alpha_l\\|u'-x'_0|<\alpha_l}} \frac{dx'\,du'}{|2^j\xi\cdot(x'-u')|^\sigma}\Bigg) \le \frac{C}{|2^j\alpha_l\xi|^\sigma}$$

and

$$\frac{1}{|B^l_m|^2}\Bigg(\iint\limits_{\substack{|y'-y'_0|<\beta_l\\|v'-y'_0|<\beta_l}} \frac{dy'\,dv'}{|2^k\eta\cdot(y'-v')|^\sigma}\Bigg) \le \frac{C}{|2^k\beta_l\eta|^\sigma}.$$

Thus, we obtain

$$|\widehat{K^l_{j,k}}(\xi,\eta)| \le \frac{C_\sigma}{|2^j\alpha_l\xi|^{\sigma/2}|2^k\beta_l\eta|^{\sigma/2}}. \tag{2.10}$$

Combining (2.9) and (2.10), we see that

$$|\widehat{K^l_{j,k}}(\xi,\eta)| \le C_\sigma \min\left\{|2^j\alpha_l\xi|, |2^k\beta_l\eta|, \frac{1}{|2^j\alpha_l\xi|^{\sigma/2}|2^k\beta_l\eta|^{\sigma/2}}\right\}.$$

By interpolation we get

$$\begin{aligned}
|\widehat{K^l_{j,k}}(\xi,\eta)| \le \overline{C}\min\bigg\{&|2^j\alpha_l\xi|^{1/2}|2^k\beta_l\eta|^{1/2}, \frac{1}{|2^j\alpha_l\xi|^{\delta}|2^k\beta_l\eta|^{\delta}},\\
&|2^j\alpha_l\xi|^{\varepsilon}|2^k\beta_l\eta|^{-\theta}, |2^j\alpha_l\xi|^{-\theta}|2^k\beta_l\eta|^{\varepsilon}\bigg\},
\end{aligned} \tag{2.11}$$

where $0<\delta=\sigma/2<1/2$, $0<\varepsilon,\theta<1$.

In fact, taking $\delta/(1+\delta) < \tau < 1$, we obtain

$$\begin{aligned}|\widehat{K^l_{j,k}}(\xi,\eta)| &= |\widehat{K^l_{j,k}}(\xi,\eta)|^{\tau}|\widehat{K^l_{j,k}}(\xi,\eta)|^{1-\tau}\\ &\le |2^j\alpha_l\xi|^{\tau}\{|2^j\alpha_l\xi|^{-\delta}|2^k\beta_l\eta|^{-\delta}\}^{1-\tau}\\ &= |2^j\alpha_l\xi|^{\tau-\delta(1-\tau)}|2^k\beta_l\eta|^{-\delta(1-\tau)} = |2^j\alpha_l\xi|^{\varepsilon}|2^k\beta_l\eta|^{-\theta},\end{aligned}$$

where $\varepsilon = \tau - \delta(1-\tau)$ and $\theta = \delta(1-\tau)$. Using the same method, we may get

$$|\widehat{K^l_{j,k}}(\xi,\eta)| \le |2^j\alpha_l\xi|^{-\theta}|2^k\beta_l\eta|^{\varepsilon}.$$

This is the conclusion of Lemma 1.

LEMMA 2. *For $\sigma_{j,k}$ as defined above in* (2.6), *the maximal operator σ^* defined in Theorem* A *is bounded on $L^p(\mathbb{R}^n\times\mathbb{R}^m)$ for $1<p<\infty$.*

P r o o f. For any $j,k\in\mathbb{Z}$, the measures $\{\sigma_{j,k}\}_{j,k\in\mathbb{Z}}$ are uniformly bounded Borel measures in $\mathbb{R}^n\times\mathbb{R}^m$. Indeed,

$$\begin{aligned}\|\sigma_{j,k}\| &= \iint\limits_{\mathbb{R}^n\times\mathbb{R}^m} |\sigma_{j,k}(x,y)|\,dx\,dy\\ &\le \sum_{j=0}^{\infty}|\lambda_l| \iint\limits_{E_{j+j_l,k+k_l}} |K^l_{E_{j+j_l,k+k_l}}(x,y)|\,dx\,dy\\ &= \sum_{j=0}^{\infty}|\lambda_l| \iint\limits_{S^{n-1}\times S^{m-1}} |\Omega_l(x',y')| \int\limits_{2^{j+j_l-1}}^{2^{j+j_l}}\int\limits_{2^{k+k_l-1}}^{2^{k+k_l}} \frac{dr\,ds}{rs}\,dx'dy'\\ &\le \sum_{j=0}^{\infty}|\lambda_l|,\end{aligned}$$

where we use again the fact that $\iint_{S^{n-1}\times S^{m-1}}|\Omega_l(x',y')|\,dx'\,dy' \le 1$. Moreover, from (2.11), (2.4) and (2.6) we deduce immediately that $\sigma_{j,k}(x,y)$ satisfies the following Fourier transform estimates:

$$|\widehat{\sigma}_{j,k}(\xi,\eta)| \le \overline{C}\sum_{j=0}^{\infty}|\lambda_l|\cdot\min\Big\{|2^j\xi|^{1/2}|2^k\eta|^{1/2}, \frac{1}{|2^j\xi|^{\delta}|2^k\eta|^{\delta}}, |2^j\xi|^{\varepsilon}|2^k\eta|^{-\theta}, |2^j\xi|^{-\theta}|2^k\eta|^{\varepsilon}\Big\} \tag{2.12}$$

To complete the proof of Lemma 2 we need to introduce the following variances of maximal operators.

The maximal operator in direction θ is defined by

$$M_\theta f(x) = \sup_{r>0}\frac{1}{r}\int\limits_0^r |f(x-t\theta)|\,dt \quad \text{for } \theta\in S^{n-1},$$

and the maximal operator in directions $(\theta_1,\theta_2)\in S^{n-1}\times S^{m-1}$ is defined by

$$M_{\theta_1,\theta_2}f(x,y)=\sup_{r_1,r_2>0}\frac{1}{r_1r_2}\int_0^{r_2}\int_0^{r_1}|f(x-t_1\theta_1,y-t_2\theta_2)|\,dt_1\,dt_2.$$

Moreover, if $\Omega(x,y)$ is homogeneous of degree zero, i.e. (1.1) holds, then the maximal operator M_Ω on $\mathbb{R}^n\times\mathbb{R}^m$ is defined by

$$M_\Omega f(x,y)=\sup_{r>0,s>0}\frac{1}{r^ns^m}\iint_{\substack{|u|<r\\|v|<s}}|\Omega(u,v)|\cdot|f(x-u,y-v)|\,du\,dv.$$

From the above definitions of maximal operators we see that

$$M_{\theta_1,\theta_2}f(x,y)\le M_{\theta_1}(M_{\theta_2}f)(x,y) \tag{2.13}$$

and

$$M_{\Omega_l}f(x,y)\le\iint_{S^{n-1}\times S^{m-1}}|\Omega_l(\theta_1,\theta_2)|M_{\theta_1,\theta_2}f(x,y)\,d\theta_1\,d\theta_2. \tag{2.14}$$

By the strong maximal theorem on $\mathbb{R}^1\times\mathbb{R}^1$ and Fubini's theorem we find that M_{θ_1} (M_{θ_2}) is uniformly bounded on $L^p(\mathbb{R}^n\times\mathbb{R}^m)$ $(1<p<\infty)$ for any $(\theta_1,\theta_2)\in S^{n-1}\times S^{m-1}$ and so is M_{θ_1,θ_2} by (2.13).

Now, let us turn to the proof of the boundedness for σ^* on $L^p(\mathbb{R}^n\times\mathbb{R}^m)$ $(1<p<\infty)$. Using the method of rotations and (2.14) we see that for any $j,k\in\mathbb{Z}$,

$$||K^l_{j+j_l,k+k_l}|*f(x,y)|\le\iint_{S^{n-1}\times S^{m-1}}|\Omega_l(\theta_1,\theta_2)|M_{\theta_1,\theta_2}f(x,y)\,d\theta_1\,d\theta_2.$$

Thus, from (2.6) we have

$$||\sigma_{j,k}|*f(x,y)|\le\iint_{S^{n-1}\times S^{m-1}}\Big(\sum_{l\ge0}|\lambda_l|\cdot|\Omega_l(\theta_1,\theta_2)|\Big)M_{\theta_1,\theta_2}f(x,y)\,d\theta_1\,d\theta_2,$$

uniformly in j, k, so the inequality still holds upon replacing the left side with $\sigma^*f(x,y)$. Since

$$\iint_{S^{n-1}\times S^{m-1}}\sum_{l\ge0}|\lambda_l|\cdot|\Omega_l(\theta_1,\theta_2)|\,d\theta_1\,d\theta_2\le\sum_{l\ge0}|\lambda_l|<\infty.$$

The $L^p(\mathbb{R}^n\times\mathbb{R}^m)$ boundedness $(1<p<\infty)$ of σ^* now follows from the uniform L^p bounds for M_{θ_1,θ_2} by the Minkowski integral formula, and the proof of Lemma 2 is finished.

Now, the conclusion of Theorem 1 is a straightforward consequence of Lemma 2 and Theorem A.

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