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THE NIKODYM PROPERTY AND LOCAL PROPERTIES OF BOOLEAN ALGEBRAS

BY

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We study local interpolation properties and local supremum properties for a Boolean algebra. In particular, we present a new condition that is sufficient for the Nikodym property.

1. Introduction. Schachermayer [9] defined the concept of a Boolean algebra with the properties of Vitali–Hahn–Saks (VHS), Grothendieck (G) and Nikodym (N). He proved that $\mathcal{F}$ is (VHS) if and only if $\mathcal{F}$ is (G) and (N) and that the Boolean algebra $\mathcal{J}$ of Jordan-measurable sets in $[0,1]$ is (N) but lacks (G). Talagrand [11] proved, assuming the continuum hypothesis, that there exists a Boolean algebra that is (G) but lacks (N). Several algebraic conditions on a Boolean algebra which imply any of these properties have been studied. These algebraic conditions can be classified in the following way:

- Supremum properties: σ-completeness (σ), E-property (E) [9], subsequential completeness (SC) [7] and weak subsequential completeness (WSC) [1].
- Interpolation properties: Seever property (S) [10], f-property (f) [8], subsequential interpolation property (SI) [4] and weak subsequential interpolation property (WSI) [5].

In this paper we introduce, by means of the Stone space $S = \mathrm{St}(\mathcal{F})$ of a Boolean algebra $\mathcal{F}$, new properties of $\mathcal{F}$ that are similar to those algebraic properties but have a local character. We will only study the local versions of (SC) and (SI). Dashiell [3] studied the Boolean algebra $\mathcal{G}$ of subsets of $[0,1]$ which are simultaneously F_σ and G_δ, and proved that $\mathcal{G}$ is (VHS) but lacks any of the above mentioned interpolation and supremum properties. In contrast, $\mathcal{G}$ does have the local properties that we introduce.

As usual, we identify elements of $\mathcal{F}$ with clopen subsets of S. By a measure we understand a complex bounded finitely additive function on $\mathcal{F}$.

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Definition 1.1. Let $\mathcal{F}$ be a Boolean algebra and $(A_i)_{i\in\omega}$ a disjoint sequence in $\mathcal{F}$. We say that $(A_i)_{i\in\omega}$ is (SC) if there exists an infinite set $M\subset\omega$ such that the sequence $(A_i)_{i\in M}$ has a supremum; $\mathcal{F}$ is (SC) if every disjoint sequence in $\mathcal{F}$ is (SC); and $\mathcal{F}$ is *locally subsequential complete* (LSC) if for every $x\in S=\mathrm{St}(\mathcal{F})$ there exists a decreasing sequence $(T_n)_{n\in\omega}$ of clopen neighborhoods of x such that if $(A_n)_{n\in\omega}$ is a disjoint sequence in $\mathcal{F}$ with $A_n\subset T_n$ for $n\in\omega$, then $(A_n)_{n\in\omega}$ is (SC).

In a similar way, one can define local versions of other supremum properties: σ-completeness, (E) and (WSC). The corresponding local properties will be denoted by (Lσ), (LE) and (LWSC).

Definition 1.2. Let $\mathcal{F}$ be a Boolean algebra and $[(A_n)_{n\in\omega},(B_n)_{n\in\omega}]$ a disjoint couple of sequences of disjoint elements of $\mathcal{F}$. We say that this couple is (SI) if there exist an infinite set $N\subset\omega$ and an element $A\in\mathcal{F}$ such that $\bigcup_{i\in N}A_i\subset A$ and $(\bigcup_{i\in\omega\setminus N}A_i)\cup(\bigcup_{i\in\omega}B_i)\subset A^{\mathrm{c}}$. We say that $\mathcal{F}$ is (SI) if every pair of disjoint sequences of disjoint element of $\mathcal{F}$ has the (SI) property.

We say that $\mathcal{F}$ is *locally subsequential interpolation* (LSI) if for every $x\in S=\mathrm{St}(\mathcal{F})$ there exists a decreasing sequence $(T_n)_{n\in\omega}$ of clopen neighborhoods of x such that if $(A_n)_{n\in\omega}$ and $(B_n)_{n\in\omega}$ are disjoint sequences of disjoint clopen sets in $\mathcal{F}$ with $A_n,B_n\subset T_n$ for $n\in\omega$, then the pair $[(A_n)_{n\in\omega},(B_n)_{n\in\omega}]$ is (SI).

For the remaining interpolation properties (S), (f) and (WSI) one can also define their corresponding local properties; they will be denoted by (LS), (Lf) and (LWSI).

It is clear that if $x\in S=\mathrm{St}(\mathcal{F})$ and $\{x\}$ is a clopen set then it has all the above mentioned local properties: it is sufficient to choose $T_i=\{x\}$ for every $i\in\omega$.

It is obvious that if $\mathcal{F}$ has any of the properties (σ), (S), (E), (f), (SC), (SI), (WSC) or (WSI) then $\mathcal{F}$ has the corresponding local property. However, we shall prove that the converse is false (see Remark 2.4).

The relations between the interpolation and supremum properties of a Boolean algebra are (see [1]):

$$\begin{array}{ccccccc}\sigma & \to & \mathrm{E} & \to & \mathrm{SC} & \to & \mathrm{WSC}\\ \downarrow & & \downarrow & & \downarrow & & \downarrow\\ \mathrm{S} & \to & \mathrm{f} & \to & \mathrm{SI} & \to & \mathrm{WSI}\end{array}$$

It is easy to prove that the above mentioned local properties of a Boolean algebra are related by a completely similar diagram.

The interpolation properties (S), (f), (SI) and (WSI) are hereditary with respect to quotient algebras and it is easy to prove that so are the corresponding local properties.

It is proved in [1] that if a Boolean algebra $\mathcal{F}$ is atomic and has any of the properties (σ), (E), (SC) or (WSC) then, if $\mathcal{I}$ is the ideal of $\mathcal{F}$ of finite elements, $\mathcal{F}/\mathcal{I}$ is (nσ); i.e., there is no nontrivial disjoint sequence in $\mathcal{F}$ with supremum. Hence $\mathcal{F}/\mathcal{I}$ is not (WSC). This shows that the properties (σ), (E), (SC), (WSC), as well as the corresponding local properties, are not hereditary with respect to quotient algebras.

If $\mathcal{F}$ is a Boolean algebra with the countable chain condition (ccc) then the properties (σ) and (S) [resp. (E) and (f), (SC) and (SI), (WSC) and (WSI)] are equivalent (cf. [1]). We shall prove here that a similar result holds for the local properties (LSC) and (LSI). The equivalence between the remaining pairs of local properties could be proved in a similar way.

In [6] the property (Lf) is introduced (denoted there by (LI)): $\mathcal{F}$ is (Lf) if for every $x \in S = \mathrm{St}(\mathcal{F})$ there exists a decreasing sequence $(T_n)_{n\in\omega}$ of clopen neighborhoods of x such that if $(A_n)_{n\in\omega}$ and $(B_n)_{n\in\omega}$ are two disjoint sequences of disjoint elements of $\mathcal{F}$ with $A_n, B_n \subset T_n$ for $n \in \omega$, then there exists an $A^M \in \mathcal{F}$ such that $\bigcup_{i\in M} A_i \subset A^M$ and $(\bigcup_{i\in N\setminus M} A_i)\cup(\bigcup_{i\in\omega} B_i) \subset (A^M)^{\mathrm{c}}$. In [6] it is proved that the property (Lf) is sufficient for (N) but not for (G) and the question is raised whether there exists a Boolean algebra that is (N) and is not (Lf). In this paper this problem is solved (Prop. 2.5) and it is proved that a Boolean algebra with the property (LSI) is (N).

Remark 1.3. Let $\mathcal{F}$ be a Boolean algebra and $S = \mathrm{St}(\mathcal{F})$. If μ is a measure in $\mathcal{F}$ we denote by $[\mu = 0]$ the ideal of μ-null elements of $\mathcal{F}$. The Stone space of the quotient algebra $\mathcal{F}/[\mu = 0]$ is the support of μ in S. It is well known ([10], [1]) that a Boolean algebra $\mathcal{F}$ is (VHS) [resp. (G), (N)] if and only if for every measure μ on $\mathcal{F}$ the quotient algebra $\mathcal{F}/[\mu = 0]$ is (VHS) [resp. (G), (N)]. Let us observe that this quotient algebra is (ccc).

Remark 1.4. Let $\mathcal{F}$ be a Boolean algebra and $S = \mathrm{St}(\mathcal{F})$. If $\mathcal{F}$ is (ccc) then, using Zorn's lemma, it is easy to prove ([1]) that if $(A_i)_{i\in\omega}$ is a disjoint sequence in $\mathcal{F}$ then there exists another disjoint sequence $(B_i)_{i\in\omega}$, disjoint from $(A_i)_{i\in\omega}$, such that S is the supremum of the sequence formed with these two sequences.

2. Main results

Proposition 2.1. *Let $\mathcal{F}$ be a Boolean algebra with the property* (ccc). *Then $\mathcal{F}$ has the property* (LSC) *if and only if $\mathcal{F}$ is* (LSI).

Proof. Suppose that $\mathcal{F}$ is (LSI). Let $x \in S = \mathrm{St}(\mathcal{F})$ and let $(T_n)_{n\in\omega}$ be the decreasing sequence of clopen neighborhoods of x, the existence of which is assured by the property (LSI). We can suppose that $T_1 = S$. Let $(A_i)_{i\in\omega}$ be a disjoint sequence of clopen sets such that $A_i \subset T_i$ for $i \in \omega$. Consider the sequence $(D_i)_{i\in\omega}$ of disjoint clopen sets defined, for $i \in \omega$, by

$D_i = (T_i \setminus T_{i+1}) \setminus \bigcup_{j=1}^{i} A_j$. Since $\mathcal{F}$ is (ccc), by Remark 1.4 there exists a disjoint sequence $(B_i)_{i\in\omega}$ of clopen sets that is disjoint from both $(A_i)_{i\in\omega}$ and $(D_i)_{i\in\omega}$ and such that S is the supremum of the sequence formed with these three sequences. Clearly $\bigcup_{i\in\omega} B_i \subset \bigcap_{j\in\omega} T_j$. Since $\mathcal{F}$ is (LSI), we apply this property to the pair $[(A_i)_{i\in\omega}, (H_i)_{i\in\omega}]$, where $H_i = D_i \cup B_i$ for $i \in \omega$.

There exists an infinite set $N \subset \omega$ and an $A \in \mathcal{F}$ such that $(\bigcup_{i\in N} A_i) \subset A$ and $(\bigcup_{i\in\omega\setminus N} A_i) \cup (\bigcup_{i\in\omega} H_i) \subset A^{c}$. It is easy to prove that $\overline{\bigcup_{i\in N} A_i} = A$. ∎

THEOREM 2.2. *If $\mathcal{F}$ is a Boolean algebra with the properties* (ccc) *and* (LSC) *then $\mathcal{F}$ is* (N).

Proof. If $\mathcal{F}$ is not (N) then there exists a sequence $(\mu_n)_{n\in\omega}$ of measures which is pointwise bounded in $\mathcal{F}$ but is not uniformly bounded. There exists an $x_0 \in S$ such that for every clopen neighborhood T of x_0, $(\mu_n)_{n\in\omega}$ is not uniformly bounded in the algebra $\mathcal{F}_T$ of clopen subsets of T [2, Corollary 3.3(a)]. Let $(T_n)_{n\in\omega}$ be the sequence of clopen neighborhoods of x_0, the existence of which is assured by the property (LSC). We can suppose that $T_1 = S$.

Let $\alpha_1 = \sup_{n\in\omega} |\mu_n(T_1)|$. Since $(\mu_n)_{n\in\omega}$ is not uniformly bounded there exist a clopen set $E_1 \subset T_1$ and $n_1 \in \omega$ such that $|\mu_{n_1}(E_1)| > 1 + \alpha_1$. The clopen set $F_1 = T_1 \setminus E_1$ satisfies $|\mu_{n_1}(F_1)| > 1$. The point x_0 is in either E_1 or F_1. Suppose that $x_0 \in E_1$ and set $A_1 = F_1$. Since $x_0 \in E_1 \cap T_2$, $(\mu_n)_{n\in\omega}$ is not uniformly bounded in $\mathcal{F}_{E_1\cap T_2}$. The clopen set $E_1 \cap T_2$ can be partitioned in two clopen sets E_2 and F_2 such that $|\mu_{n_2}(E_2)| > 2$ and $|\mu_{n_2}(F_2)| > 2$ for some $n_2 > n_1$. The point x_0 is in either E_2 or F_2. Suppose that $x_0 \in E_2$ and set $A_2 = F_2$. Proceeding in a similar way we can find a disjoint sequence $(A_i)_{i\in\omega}$ such that $A_i \subset T_i$ for $i \in \omega$ and a subsequence $(\mu_{n_i})_{i\in\omega}$ of $(\mu_n)_{n\in\omega}$ (that will be denoted by $(\mu_j)_{j\in\omega}$) such that $|\mu_i(A_i)| > i$ for every $i \in \omega$. Consider the sequence $(D_i)_{i\in\omega}$ of disjoint clopen sets defined, for $i \in \omega$, by $D_i = (T_i \setminus T_{i+1}) \setminus \bigcup_{j=1}^{i} A_j$. Since $\mathcal{F}$ is (ccc), by Remark 1.4 there exists a disjoint sequence $(B_i)_{i\in\omega}$ of clopen sets that is disjoint from both $(A_i)_{i\in\omega}$ and $(D_i)_{i\in\omega}$ and such that S is the supremum of the sequence formed with these three sequences. It is clear that $\bigcup_{i\in\omega} B_i \subset \bigcap_{j\in\omega} T_j$.

Let $(H_i)_{i\in\omega}$ be the sequence defined by $H_{3i} = B_i$, $H_{3i-1} = D_i$ and $H_{3i-2} = A_i$ for $i \in \omega$. Let $\mathcal{G}$ be the family of those clopen sets A of S such that there exists an $M \subset \omega$ with $\overline{\bigcup_{i\in M} H_i} = A$. It is easy to prove that $\mathcal{G}$ is a subalgebra of $\mathcal{F}$. Let us prove that $\mathcal{G}$ has the property (SC).

Let $(C_i)_{i\in\omega}$ be a disjoint sequence in $\mathcal{G}$. For every $i \in \omega$, there exists an $M_i \subset \omega$ such that $C_i = \overline{\bigcup_{j\in M_i} H_j}$. Set $i_1 = 1$; we have $C_{i_1} \subset T_1$. There exists an $i_2 > i_1$ such that neither A_1 nor D_1 is in the sequence $(H_j)_{j\in M_{i_2}}$; hence this sequence is contained in T_2 and $C_{i_2} \subset T_2$. Similarly, there exists an $i_3 > i_2$ such that none of A_1, A_2, D_1 and D_2 is in the sequence $(H_j)_{j\in M_{i_3}}$. Hence $C_{i_3} \subset T_3$.

We can obtain inductively an increasing sequence $i_1 < i_2 < \ldots$ of elements of ω such that $C_{i_j} \subset T_j$ for every $j \in \omega$. There exists an infinite set $M \subset \omega$ such that the sequence $(C_i)_{i\in M}$ has a supremum $C \in \mathcal{F}$. Let us prove that $C \in \mathcal{G}$.

Define $K = \bigcup_{i\in M} M_i$. It is clear that $\bigcup_{i\in K} H_i \subset C$. For every $j \in \omega \setminus K$ we have $H_j \cap C = \emptyset$, since otherwise there would exist an $i \in M$ such that $H_j \cap C_i \neq \emptyset$ and we could find $l \in M_i \subset K$ such that $H_i \cap H_l \neq \emptyset$, which is not possible. Hence $C \cap (\bigcup_{i\in\omega\setminus K} H_i) = \emptyset$, $\overline{\bigcup_{i\in K} H_i} = C$ and $C \in \mathcal{G}$.

The Boolean algebra $\mathcal{G}$ has, by [7] and [4], the property (N). This contradicts the fact that $(\mu_i)_{i\in\omega}$ is pointwise bounded in $\mathcal{G}$, $(A_i)_{i\in\omega}$ is a disjoint sequence in $\mathcal{G}$ and $|\mu_i(A_i)| > i$ for every $i \in \omega$. ■

COROLLARY 2.3. *If $\mathcal{F}$ has the property* (LSI) *then $\mathcal{F}$ is* (N).

Proof. If $\mathcal{F}$ is (LSC) or if $\mathcal{F}$ is (LSI) then for every measure μ in $\mathcal{F}$, $\mathcal{F}/[\mu = 0]$ is (LSI) and (ccc). By Proposition 2.1 and Theorem 2.2 this quotient algebra is (N) and, hence, $\mathcal{F}$ is (N) (Remark 1.3). ■

Let us observe that the property (LSC) is not hereditary with respect to quotient algebras, although (LSI) is. It $\mathcal{F}$ is (LSC) then $\mathcal{F}$ is (LSI) but the converse is not, in general, true.

Remark 2.4. A Boolean algebra $\mathcal{F}$ is said to be *up-down-semi-complete* (udsc) ([9]) if for every sequence of disjoint elements of $\mathcal{F}$ with a supremum, every subsequence also has a supremum. Let $\mathcal{F}$ be a (udsc) Boolean algebra, $S = \mathrm{St}(\mathcal{F})$, $(T_i)_{i\in\omega}$ a decreasing sequence of clopen subsets of S such that $\mathrm{int}(\bigcap_{i\in\omega} T_i) = \emptyset$ and $(A_i)_{i\in\omega}$ a sequence of disjoint clopen subsets of S such that, for every $i \in \omega$, $A_i \subset T_i$. We are going to prove that every subsequence of $(A_i)_{i\in\omega}$ has a supremum [3, Th. 1.8]. Consider a disjoint sequence $(C_i)_{i\in\omega}$ of subsets of S such that $C_{2i-1} = A_i$ and $C_{2i} = (T_i \setminus T_{i+1}) \setminus \bigcup_{j=1}^{i} A_j$ for every $i \in \omega$. It is easy to prove that $\overline{\bigcup C_i} = T_1$ and that, as a consequence, the sequence $(A_i)_{i\in\omega}$ has a supremum.

Consider the Boolean algebra $\mathcal{J}$ of Jordan measurable sets in $[0,1]$ and let $(A_i)_{i\in\omega}$ be a sequence of disjoint elements in $\mathcal{J}$ with a supremum. We have $\lambda(\mathrm{Fr}(\bigcup_{i\in\omega} A_i)) = 0$, where λ denotes the Lebesgue measure. It is easy to prove that for every $M \subset \omega$ we also have $\lambda(\mathrm{Fr}(\bigcup_{i\in M} A_i)) = 0$. Hence $\mathcal{J}$ is (udsc). It is also easy to prove that the Boolean algebra $\mathcal{G}$ of subsets of $[0,1]$ which are simultaneously F_σ and G_δ is (udsc).

Let $\mathcal{F}$ be either the Boolean algebra $\mathcal{J}$ or $\mathcal{G}$. We shall prove that $\mathcal{F}$ is (Lσ), i.e. for every $x \in S = \mathrm{St}(\mathcal{F})$ there exists a decreasing sequence $(T_n)_{n\in\omega}$ of clopen neighborhoods of x such that if $(A_n)_{n\in\omega}$ is a disjoint sequence in $\mathcal{F}$ with $A_n \subset T_n$ for $n \in \omega$, then $(A_n)_{n\in\omega}$ has a supremum in $\mathcal{F}$. If $x \in S$ is such that $\{x\}$ is clopen then $\mathcal{F}$ has the property (Lσ) at x. If $\{x\}$ is not clopen, then x is a nonprincipal maximal filter of $\mathcal{F}$. Let I_1 be either $[0, 1/2]$

or $[1/2, 1]$—we take the one that is in x. We divide I_1 in two subintervals whose intersection is the middle point of I_1. We denote by I_2 the subinterval that is in x. Inductively, we can find a decreasing sequence $(I_n)_{n\in\omega}$ of intervals in $[0, 1]$ such that $\lim_{n\to\infty} \operatorname{diam}(I_n) = 0$ and $I_n \in x$ for every $n \in \omega$.

We define $\{t_0\} = \bigcap I_n$ and $T_n = I_n \setminus \{t_0\}$ for every $n \in \omega$. Since x is not a principal filter, the sequence $(T_n)_{n\in\omega}$ is contained in x. Hence, by considering $x \in S$, $(T_n)_{n\in\omega}$ is a decreasing sequence of clopen neighborhoods of x. Since $\mathcal{F}$ is (udsc) and $\operatorname{int}(\bigcap T_n) = \emptyset$, for every disjoint sequence $(A_i)_{i\in\omega}$ of clopen sets such that $A_i \subset T_i$ for $i \in \omega$, we see that $(A_i)_{i\in\omega}$ (and every subsequence of it) has a supremum.

Observe that for the sequence $(B_i)_{i\in\omega}$ in $\mathcal{G}$ defined, for $i \in \omega$, by $B_i = \{1/2^i, 3/2^i, \ldots, (2^i - 1)/2^i\}$, there is no element $A \in \mathcal{G}$ such that A contains an infinite number of elements of $(B_i)_{i\in\omega}$ and such that A is disjoint from another infinite number of elements of $(B_i)_{i\in\omega}$ ([5]). Hence, $\mathcal{G}$ lacks any supremum and interpolation property. Nevertheless, $\mathcal{G}$ has all the local properties that appear in this paper.

PROPOSITION 2.5. *There exists a Boolean algebra with the* (N) *property which lacks the* (Lf) *property.*

Proof. Haydon [7, Proposition 1E] obtains, by transfinite induction, a Boolean algebra $\mathcal{H}$ with the following characteristics:

1. $\mathcal{H}$ is a subalgebra of $\mathcal{P}(\omega)$.
2. $\mathcal{H}$ is atomic and the set of atoms of $\mathcal{H}$ is $\{\{i\} : i \in \omega\}$. Hence, $\mathcal{H}$ is (ccc).
3. $\mathcal{H}$ is (SC) and, as a consequence ([4]), it is (G) and (N).
4. For no infinite $K \subset \omega$ do we have $\mathcal{P}(K) = \{K \cap A : A \in \mathcal{H}\}$.

We prove that $\mathcal{H}$ is not (Lf). Otherwise, let $x \in S = \operatorname{St}(\mathcal{H})$ be such that $\{x\}$ is not clopen and let $(T_n)_{n\in\omega}$ be the decreasing sequence of clopen neighborhoods of x whose existence given by the property (Lf). We can suppose that $T_1 = S$. It is easy to obtain a sequence $(n_j)_{j\in\omega}$ of different elements of ω such that $n_j \in T_j$ for every $j \in \omega$. We define, for $i \in \omega$, $D_i = (T_i \setminus T_{i+1}) \setminus \bigcup_{j=1}^{i} \{n_j\}$. Since $\mathcal{H}$ is (ccc) there exists a disjoint sequence $(B_i)_{i\in\omega}$ of clopen sets that is disjoint from both $(\{n_j\})_{j\in\omega}$ and $(D_i)_{i\in\omega}$ and such that S is the supremum of the sequence formed with these three sequences. We apply the (Lf) property to the pair $[(\{n_j\})_{j\in\omega}, (C_j)_{j\in\omega}]$, where $C_j = D_j \cup B_j$ for $j \in \omega$. There exists an infinite set $N \subset \omega$ such that for every $M \subset N$ there exists an A^M with $\bigcup_{j\in M} \{n_j\} \subset A^M$ and $(\bigcup_{j\in\omega\setminus M} \{n_j\}) \cup (\bigcup_{j\in\omega} C_j) \subset (A^M)^{\mathrm{c}}$. It is easy to prove that A^M is the supremum of $(\{n_j\})_{j\in M}$. Hence the set $K = \{n_j : j \in N\}$ contradicts property 4 of $\mathcal{H}$. ∎

Remark 2.6. We do not know if there exists a Boolean algebra with Nikodym's property that is not (LSI).

REFERENCES

[1] A. Aizpuru, *Relaciones entre propiedades de supremo y propiedades de interpolación en álgebras de Boole*, Collect. Math. 39 (1988), 115–125.
[2] —, *On the Grothendieck and Nikodym properties of Boolean algebras*, Rocky Mountain J. Math. 22 (1992), 1–10.
[3] F. K. Dashiell, *Nonweakly compact operators from order Cauchy complete $C(S)$ lattices, with application to Baire classes*, Trans. Amer. Math. Soc. 266 (1981), 397–413.
[4] F. J. Freniche, *The Vitali–Hahn–Saks theorem for Boolean algebras with the subsequential interpolation property*, Proc. Amer. Math. Soc. 92 (1984), 362–366.
[5] —, *Some classes of Boolean algebras related to the Vitali–Hahn–Saks and Nikodym theorems*, communication at the Meeting on Boolean Algebras, Oberwolfach, 1985.
[6] M. T. Gassó, *Algebras with the local interpolation property*, Rocky Mountain J. Math. 22 (1992), 181–196.
[7] R. Haydon, *A non-reflexive Grothendieck space that does not contain l_∞*, Israel J. Math. 40 (1981), 65–73.
[8] A. Moltó, *On the Vitali–Hahn–Saks theorem*, Proc. Roy. Soc. Edinburgh Sect. A 90 (1981), 163–173.
[9] W. Schachermayer, *On some classical measure theoretic theorems for non-σ-complete Boolean algebras*, technical report, Linz University, 1978.
[10] G. Seever, *Measures on F-spaces*, Trans. Amer. Math. Soc. 133 (1968), 267–280.
[11] M. Talagrand, *Propriété de Nikodym et propriété de Grothendieck*, Studia Math. 68 (1984), 165–171.

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