Obviously, $V_n = V_n^{-1}$ and $\lambda(V_n) > 0$ $(n = 1, 2, \ldots)$. Further, we define the sequence of symmetric probability distributions $\mu_1, \mu_2, \ldots$ by the following formula:

$$\mu_n(E) = \frac{1 - a_n + a_n \lambda(V_n)}{\lambda(V_n)} \lambda(E \frown V_n) + a_n \lambda(E \setminus V_n) \quad (n = 1, 2, \ldots).$$

It is easy to verify that the inequality

$$\mu_n(E) \geqslant a_n \lambda(E) \quad (n = 1, 2, \ldots)$$

is true for every Borel subset $E \subset G$. Moreover, in virtue of (16),

$$\|\varphi_{\mu_n}(\mathcal{U}_0) - \mathcal{E}\| \leqslant \int_G \|\mathcal{U}_0(g) - \mathcal{E}\| \mu_n(dg)$$

$$= \frac{1 - a_n + a_n \lambda(V_n)}{\lambda(V_n)} \int_{V_n} \|\mathcal{U}_0(g) - \mathcal{E}\| \lambda(dg) + a_n \int_{G \setminus V_n} \|\mathcal{U}_0(g) - \mathcal{E}\| \lambda(dg) \leqslant n^{-2} + 2 a_n$$

$$(n = 1, 2, \ldots).$$

Hence, according to (15), we get the inequality

$$\sum_{n=1}^{\infty} \|\varphi_{\mu_n}(\mathcal{U}_0) - \mathcal{E}\| < \infty,$$

which implies the convergence

$$\lim_{n \to \infty} \varphi_{\mu_1 * \mu_2 * \ldots * \mu_n}(\mathcal{U}_0) = \lim_{n \to \infty} \varphi_{\mu_1}(\mathcal{U}_0) \varphi_{\mu_2}(\mathcal{U}_0) \ldots \varphi_{\mu_n}(\mathcal{U}_0) \neq 0.$$

Consequently, the Haar measure is not the weak limit of the probability distributions $\mu_1 * \mu_2 * \ldots * \mu_n$ $(n = 1, 2, \ldots)$. The Theorem is thus proved.

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ON THE POWER OF COMPACT SPACES

BY

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Let X be a compact ($=$ bicompact) infinite space. The character $\Theta(p)$ of a point $p \, \epsilon \, X$ is the least cardinal of a family of open sets containing the point p and having that point as its intersection.

Recently S. Mrówka [4] has shown the following

THEOREM. *If* $\mathfrak{m} \leqslant \Theta(p)$ *for all* $p \, \epsilon \, X$, *then* $\overline{\overline{X}} \geqslant 2^{\mathfrak{m}}$.

In this note we present a somewhat simpler proof of this theorem. The main idea of the proof is similar to that of my papers [1] and [2] and has some connections with the idea used by F. B. Jones in [3].

Proof. Denote by I^a the set of all 0-1 sequences of the ordinal type a. Let $\xi \, \epsilon \, I^a$ and $\beta < a$. Denote by ξ_β the segment of the type β of the sequence ξ. For each a we are going to define a family $\{V(\xi) \colon \xi \, \epsilon \, I^a\}$ of open sets of the space X. We use the transfinite induction: for $a = 1$ we have two one-element sequences 0 and 1. Let $V(0)$ and $V(1)$ be two arbitrary disjoint open subsets of X. Suppose we have defined $V(\xi)$ for all $\xi \, \epsilon \, I^\beta$ and $\beta < a$. In order to define $V(\xi)$ for $\xi \, \epsilon \, I^a$ consider two possibilities: (a) a has a precedent and (b) a is a limit-number.

(a) Put $a = a' + 1$. Then either for some $\xi \, \epsilon \, I^{a'}$ the set $\bigcap_{\beta \leqslant a'} V(\xi_\beta)$ contains at most one point or for every $\xi \, \epsilon \, I^{a'}$ it consists of two points at least. In the first case, the sets $V(\xi)$ for $\xi \, \epsilon \, I^a$ will not be defined. In the second case, given a $\xi \, \epsilon \, I^{a'}$, let x_0 and x_1 be two points of $\bigcap_{\beta \leqslant a'} V(\xi_\beta)$.

There are exactly two different sequences η^0 and η^1 belonging to I^a such that $\eta^0_{a'} = \eta^1_{a'} = \xi$. Let $V(\eta^0)$ and $V(\eta^1)$ be two disjoint open sets such that $x_0 \, \epsilon \, V(\eta^0)$, $x_1 \, \epsilon \, V(\eta^1)$ and $\overline{V(\eta^0)} \subset V(\xi)$, $\overline{V(\eta^1)} \subset V(\xi)$.

(b) If a is a limit-number, then for $\xi \, \epsilon \, I^a$ put $V(\xi) = X$.

Note that:

(i) For every a the set $\bigcap_{\beta \leqslant a} V(\xi_\beta)$ is non-void.

It is obvious if α has a precedent, for α being a limit-number, it follows from the equality

$$\bigcap_{\beta \leqslant \alpha} \overline{V(\xi_p)} = \bigcap_{\beta \leqslant \alpha} V(\xi_p)$$

and from the compactness of X.

(ii) If for some γ the sets $V(\xi)$ with $\xi \in I^\gamma$ are defined, then the family $\{\bigcap_{\beta \leqslant \gamma} V(\xi_\beta) : \xi \in I^\gamma\}$ contains a subfamily of cardinal $2^{\overline{\gamma}}$ consisting of disjoint sets. Indeed, if two sequences $\xi^1, \xi^2 \in I^\gamma$ differ in the β-th position, and β is not a limit-number, then the sets $V(\xi_\beta^1)$ and $V(\xi_\beta^2)$ are disjoint; hence $\bigcap_{\beta \leqslant \gamma} V(\xi_\beta^1)$ and $\bigcap_{\beta \leqslant \gamma} V(\xi_\beta^2)$ are disjoint.

Let γ be the first ordinal number such that the set $V(\xi)$ is not defined for $\xi \in I^{\gamma+1}$. Then for some $\xi \in I^\gamma$ the set $\bigcap_{\beta \leqslant \gamma} V(\xi_\beta)$ contains less than two elements. Hence, by (i), we get

$$\bigcap_{\beta \leqslant \gamma} V(\xi_p) = (p).$$

The last equality implies $\Theta(p) \leqslant \overline{\gamma}$ and, if $\Theta(p) \geqslant \mathfrak{m}$, we get, by (ii), $\overline{\overline{X}} \geqslant 2^{\overline{\gamma}} \geqslant 2^{\Theta(p)} \geqslant 2^{\mathfrak{m}}$.

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SUPPLÉMENT À L'ARTICLE „SUR DEUX PROBLÈMES D'ANALYSE NON RÉSOLUS"

PAR

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I. Une modification d'une démonstration. Dans la réciproque de la p. 39 de mon travail [1], on suppose „que la somme de deux courbes ne soit définie que dans le cas où ces deux courbes sont des lignes polygonales orientées". Cette supposition n'ayant pas été respectée dans la démonstration, il y a lieu de modifier celle-ci de la manière suivante.

On aura

$$(Q_\varepsilon, Q_\omega) = (P_\varepsilon + P'_\varepsilon, P_\omega + P'_\omega) = \|(P_\varepsilon + P'_\varepsilon) + (-1)(P_\omega + P'_\omega)\|$$

$$= \|(P_\varepsilon + (-1)P_\omega) + (P'_\varepsilon + (-1)P'_\omega)\|$$

$$\leqslant \|P_\varepsilon + (-1)P_\omega\| + \|P'_\varepsilon + (-1)P'_\omega\| = (P_\varepsilon, P_\omega) + (P'_\varepsilon, P'_\omega).$$

Or

$$(P_\varepsilon; P_\omega) \leqslant (P_\varepsilon, c) + (c, P_\omega) \leqslant \varepsilon + \omega.$$

De même pour les P'. On a donc

$$(8) \qquad (Q_\varepsilon, Q_\omega) \leqslant 2\varepsilon + 2\omega$$

comme dans le texte déjà paru. D'après ce même texte: „L'espace Γ, considéré comme espace distancié, (...) est complet". Dès lors, il résulte de la formule (8) que Q_ε tend vers une courbe déterminée σ quand $\varepsilon \to 0$. Cette courbe limite ne dépend pas du choix des lignes polygonales $P_\varepsilon, P'_\varepsilon$. Si on leur substitue deux lignes polygonales orientées $p_\varepsilon, p'_\varepsilon$ et si l'on pose $q_\varepsilon = p_\varepsilon + p'_\varepsilon$, on aura (mais ici nous modifions encore une fois les formules):

$$(Q_\varepsilon, q_\varepsilon) \equiv (P_\varepsilon + P'_\varepsilon, p_\varepsilon + p'_\varepsilon) = \|P_\varepsilon + P'_\varepsilon + (-1)(p_\varepsilon + p'_\varepsilon)\|$$

$$= \|(P_\varepsilon + (-1)p_\varepsilon) + (P'_\varepsilon + (-1)p'_\varepsilon)\|$$

$$\leqslant \|P_\varepsilon + (-1)p_\varepsilon\| + \|P'_\varepsilon + (-1)p'_\varepsilon\| = (P_\varepsilon, p_\varepsilon) + (P'_\varepsilon, p'_\varepsilon).$$