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ON SOME PROBLEM OF A. ROSŁANOWSKI

BY

SZYMON PLEWIK (KATOWICE)

We present a negative answer to problem 3.7(b) posed on page 193 of [2], where, in fact, A. Rosłanowski asked: *Does every set of Lebesgue measure zero belong to some Mycielski ideal*?

We identify a set $X \in [\omega]^\omega$ with its characteristic function, i.e. with the sequence $(X(0), X(1), \ldots) \in 2^\omega$ such that $X(n) = 1$ iff $n \in X$. A set $X \in [\omega]^\omega$ has *asymptotic density* d whenever

$$\lim_{n\to\infty} \frac{|X \cap n|}{n} = d,$$

where $|X \cap n|$ denotes the number of natural numbers from X less than n.

We consider the family of all sets of asymptotic density not equal to $1/2$, i.e. the set

$$A = 2^\omega \setminus \{X \in [\omega]^\omega : X \text{ is of asymptotic density } 1/2\}.$$

An old result of E. Borel [1] says: *The set A has Lebesgue measure zero.* A direct consequence of this result is

THEOREM. *The set A does not belong to any Mycielski ideal.*

Proof. Our notation follows [2]. If K is a normal system, i.e. for each $X \in K$ there exist two disjoint subsets of X which belong to K, then K contains three disjoint sets X, Y and Z. Since

$$|X \cap n| + |Y \cap n| + |Z \cap n| \le n,$$

one of the sets: X, Y or Z does not contain any subset of asymptotic density $1/2$. Suppose X is a such set. If Player I always chooses zero, then he wins the game $\Gamma(X, A)$, because any set (sequence) which can be the result of that game is not of asymptotic density $1/2$ and thus belongs to A. This means that the set A does not belong to the Mycielski ideal generated by K. ∎

If one considers Mycielski ideals on k^ω, where $k > 2$ is a natural number, then our theorem can be slightly modified. The Lebesgue measure and

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Mycielski ideals can also be considered on k^ω because of the definition of the Lebesgue measure given in [2], p. 188. Similarly to the asymptotic density, one can define the *asymptotic frequency* of functions from k^ω. Again, it is a result of E. Borel [1] that: *The set of all sequences from k^ω in which every natural number n occurs asymptotically with frequency $1/k$ has full measure.* Its complement A^* has Lebesgue measure zero and does not belong to any Mycielski ideal, since Player I wins the game $\Gamma(\omega \setminus X, A^*)$ whenever he always chooses the same number and X does not contain any subset with asymptotic frequency $(k-1)/k$.

REFERENCES

[1] E. Borel, *Sur les probabilités dénombrables et leurs applications arithmétiques*, Rend. Circ. Mat. Palermo 29 (1909), 247–271.
[2] A. Rosłanowski, *Mycielski ideals generated by uncountable systems*, Colloq. Math. 66 (1994), 187–200.

INSTITUTE OF MATHEMATICS
SILESIAN UNIVERSITY
BANKOWA 14
40-007 KATOWICE, POLAND
E-mail: PLEWIK@GATE.MATH.US.EDU.PL

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