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# CURVATURE PROPERTIES OF CARTAN HYPERSURFACES

BY

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**1. Introduction.** Every Einsteinian as well as every conformally flat hypersurface $M$ in $M^{n+1}(c)$, $n \geq 4$, is a pseudosymmetric manifold ([23], Proposition 3.2). Thus every quasi-umbilical hypersurface $M$ in $M^{n+1}(c)$, $n \geq 4$, is pseudosymmetric. An $n$-dimensional hypersurface $M$, $\dim M \geq 3$, is called quasi-umbilical if $M$ has at every point a principal curvature of multiplicity $\geq n-1$. In [25] (Theorem 1) it was shown that every hypersurface $M$ in $M^{n+1}(c)$, $n \geq 3$, having at every point at most two distinct principal curvatures is also pseudosymmetric. A necessary and sufficient condition for hypersurfaces in $M^{n+1}(c)$, $n \geq 4$, to be pseudosymmetric will be presented in the subsequent paper [17]. Pseudosymmetric hypersurfaces in $M^4(c)$ were considered in [25]. A Cartan hypersurface in a sphere $S^{n+1}(c)$ is a compact hypersurface with principal curvatures $-(3c)^{1/2}$, 0, $(3c)^{1/2}$ with the same multiplicity ([27]). Thus every Cartan hypersurface is minimal. Cartan hypersurfaces only exist when $n = 3, 6, 12, 24$. These hypersurfaces were discovered by E. Cartan in his work about isoparametric hypersurfaces, i.e. hypersurfaces with constant principal curvatures, in spaces of constant curvature ([2], [3]). We refer to [4] (Section 3) for a review on isoparametric hypersurfaces. In this note we prove that 3-dimensional Cartan hypersurfaces are not semisymmetric pseudosymmetric manifolds. Further, we show that Cartan hypersurfaces of dimensions 6, 12 or 24 are not pseudosymmetric. However, these hypersurfaces realize a weaker condition of pseudosymmetry type. Namely, we prove that such hypersurfaces are Ricci-pseudosymmetric. We note that every pseudosymmetric manifold is Ricci-pseudosymmetric. The converse statement is not true ([22], [13]). In addition, we verify that Cartan hypersurfaces of dimensions 6, 12 or 24 have non-pseudosymmetric Weyl tensor.

**2. Conditions of pseudosymmetry type.** Let $(M, g)$ be a connected $n$-dimensional, $n \geq 3$, semi-Riemannian manifold of class $C^\infty$. We define on $M$ the endomorphisms $\widetilde{R}(X, Y)$ and $X \wedge Y$ by

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$$\widetilde{R}(X,Y)Z = [\nabla_X, \nabla_Y]Z - \nabla_{[X,Y]}Z, \qquad (X \wedge Y)Z = g(Y,Z)X - g(X,Z)Y,$$

respectively, where $\nabla$ is the Levi-Civita connection of $(M,g)$ and $X, Y, Z \in \Xi(M)$, $\Xi(M)$ being the Lie algebra of vector fields on $M$. We define the *Riemann–Christoffel curvature tensor* $R$ and the *concircular tensor* $Z(R)$ of $(M,g)$ by $R(X_1,\ldots,X_4) = g(\widetilde{R}(X_1,X_2)X_3,X_4)$, $Z(R) = R - \frac{\kappa}{n(n-1)}G$ respectively, where $\kappa$ is the scalar curvature of $(M,g)$ and $G$ is given by $G(X_1,\ldots,X_4) = g((X_1 \wedge X_2)X_3,X_4)$. For a $(0,k)$-tensor field $T$ on $M$, $k \geq 1$, we define the $(0,k+2)$-tensors $R \cdot T$ and $Q(g,T)$ by

$$\begin{aligned}
&(R \cdot T)(X_1,\ldots,X_k;X,Y)\\
&\quad = -T(\widetilde{R}(X,Y)X_1,X_2,\ldots,X_k) - \ldots - T(X_1,\ldots,X_{k-1},\widetilde{R}(X,Y)X_k),\\
&Q(g,T)(X_1,\ldots,X_k;X,Y)\\
&\quad = T((X \wedge Y)X_1,X_2,\ldots,X_k) + \ldots + T(X_1,\ldots,X_{k-1},(X \wedge Y)X_k).
\end{aligned}$$

A semi-Riemannian manifold $(M,g)$ is said to be *pseudosymmetric* ([19]) if at every point of $M$ the following condition is satisfied:

$(*)$ the tensors $R \cdot R$ and $Q(g,R)$ are linearly dependent.

The manifold $(M,g)$ is pseudosymmetric if and only if

$$R \cdot R = L_R Q(g,R) \tag{1}$$

holds on the set $U_R = \{x \in M \mid Z(R) \neq 0 \text{ at } x\}$, where $L_R$ is some function on $U_R$. It is clear that any semisymmetric manifold ($R \cdot R = 0$, [28]) is pseudosymmetric. The condition $(*)$ arose in the study of totally umbilical submanifolds of semisymmetric manifolds ([1]) as well as when considering geodesic mappings of semisymmetric manifolds (e.g. [21], [6]). There exist many examples of pseudosymmetric manifolds which are not semisymmetric ([19], [21], [12], [16]).

A semi-Riemannian manifold $(M,g)$ is said to be *Ricci-pseudosymmetric* ([22], [13]) if at every point of $M$ the following condition is satisfied:

$(**)$ the tensors $R \cdot S$ and $Q(g,S)$ are linearly dependent.

The manifold $(M,g)$ is Ricci-pseudosymmetric if and only if

$$R \cdot S = L_S Q(g,S) \tag{2}$$

holds on the set $U_S = \{x \in M \mid S - \frac{\kappa}{n}g \neq 0 \text{ at } x\}$, where $L_S$ is some function on $U_S$. It is clear that if at a point $x \in U_R$ the relation (1) is satisfied then also (2) holds at $x$. The converse statement is not true. E.g. every warped product $M_1 \times_F M_2$, $\dim M_1 = 1$, $\dim M_2 = n-1 \geq 3$, of a manifold $M_1$ and a not pseudosymmetric Einstein manifold $M_2$ is a non-pseudosymmetric Ricci-pseudosymmetric manifold (cf. [22], Corollary 3.2 and [19], Theorem 4.1).

For any $X, Y \in \Xi(M)$ we define the endomorphism $\widetilde{C}(X,Y)$ by

$$\widetilde{C}(X,Y) = \widetilde{R}(X,Y) - \frac{1}{n-2}(X \wedge \widetilde{S}Y + \widetilde{S}X \wedge Y) + \frac{\kappa}{(n-1)(n-2)} X \wedge Y,$$

where the *Ricci operator* $\widetilde{S}$ of $(M,g)$ is defined by $S(X,Y) = g(X, \widetilde{S}Y)$. We denote by $C$ the *Weyl conformal curvature tensor* of $(M,g)$, $C(X_1,\ldots,X_4) = g(\widetilde{C}(X_1,X_2)X_3,X_4)$. Now we define the $(0,6)$-tensor $C \cdot C$ by

$$\begin{aligned}&(C \cdot C)(X_1,\ldots,X_4;X,Y)\\ &\quad = -C(\widetilde{C}(X,Y)X_1,X_2,X_3,X_4) - \ldots - C(X_1,X_2,X_3,\widetilde{C}(X,Y)X_4).\end{aligned}$$

A semi-Riemannian manifold $(M,g)$, $\dim M \geq 4$, is said to be a *manifold with pseudosymmetric Weyl tensor* ([24]) if at every point of $M$ the following condition is satisfied:

$(***)$ the tensors $C \cdot C$ and $Q(g,C)$ are linearly dependent.

The manifold $(M,g)$ is a manifold with pseudosymmetric Weyl tensor if and only if

$$C \cdot C = L_C Q(g,C) \tag{3}$$

holds on the set $U_C = \{x \in M \mid C \neq 0 \text{ at } x\}$, where $L_C$ is some function on $U_C$. The condition $(***)$ arose in the study of 4-dimensional warped products ([15]). Namely, in [15] (Theorem 2) it was proved that at every point of a warped product $M_1 \times_F M_2$, with $\dim M_1 = \dim M_2 = 2$, $(***)$ is satisfied. Many examples of manifolds satisfying $(***)$ are presented in [8]. For instance, the Cartesian product of two manifolds of constant curvature is a manifold realizing $(***)$. Recently, warped products satisfying $(***)$ were considered in [24]. In [8] it was shown that the classes of manifolds realizing $(*)$ and $(***)$ do not coincide. However, there exist pseudosymmetric manifolds satisfying (3), e.g. Einsteinian pseudosymmetric manifolds ([8], Theorem 3.1). We note that $(***)$ is invariant under conformal deformations of the metric tensor $g$.

R e m a r k 1. In [16] an example is presented of a pseudosymmetric warped product manifold $S^p \times_F S^{n-p}$, $p \geq 2$, $n-p \geq 2$, with pseudosymmetric Weyl tensor. This manifold cannot be realized as a hypersurface isometrically immersed in a manifold of constant curvature ([16], Theorem 4.1).

R e m a r k 2. Applying Theorem 4.5 of [28] and Theorem 4.1 of [8] in the Corollary of [26] we conclude that every minimal hypersurface imbedded in an $(n+1)$-dimensional sphere, $n \geq 4$, with two principal curvatures of multiplicities $\geq 2$ is a semisymmetric manifold with pseudosymmetric Weyl tensor.

Further, we define the $(0,6)$-tensor $Q(S,R)$ by

$$Q(S,R)(X_1,\dots,X_4;X,Y)$$
$$= R((X \wedge_S Y)X_1,X_2,X_3,X_4) + \dots + R(X_1,X_2,X_3,(X \wedge_S Y)X_4),$$

where $X \wedge_S Y$ is the endomorphism defined by $(X \wedge_S Y)Z = S(Y,Z)X - S(X,Z)Y$.

A semi-Riemannian manifold $(M,g)$ is said to be *Ricci-generalized pseudosymmetric* [5] if at every point of $M$ the following condition is satisfied:

$(****)$ the tensors $R \cdot R$ and $Q(S,R)$ are linearly dependent.

A very important subclass of Ricci-generalized pseudosymmetric manifolds is formed by the manifolds satisfying ([20], [5], [7])

$$R \cdot R = Q(S,R). \tag{4}$$

Any 3-manifold $(M,g)$ satisfies (4) ([14], Theorem 3.1). Moreover, any hypersurface $M$ isometrically immersed in an $(n+1)$-dimensional Euclidean space $\mathbb{E}^{n+1}$, $n \geq 4$, satisfies (4) ([23], Corollary 3.1).

As was proved in [23], at every point of a hypersurface $M$ isometrically immersed in a manifold of constant curvature $M^{n+1}(c)$, $n \geq 4$, the following condition is satisfied:

$(****)$ the tensors $R \cdot R - Q(S,R)$ and $Q(g,C)$ are linearly dependent.

More precisely, in [23] (Proposition 3.1) it was proved that every hypersurface $M$ isometrically immersed in a space of constant curvature $M^{n+1}(c)$, $n \geq 4$, satisfies the equality

$$R \cdot R - Q(S,R) = -\frac{n-2}{n(n+1)}\widetilde{\kappa}Q(g,C), \tag{5}$$

where $\widetilde{\kappa}$ is the scalar curvature of $M^{n+1}(c)$ and $R$, $S$ and $C$ are the Riemann–Christoffel curvature tensor, the Ricci tensor and the Weyl tensor of $M$, respectively. Using Theorem 3.1 of [14] which was mentioned above and the fact that the Weyl tensor of any 3-dimensional manifold vanishes identically, we conclude immediately that $(****)$ is trivially satisfied on any 3-dimensional semi-Riemannian manifold. Recently, warped products realizing $(****)$ were considered in [9]. For instance, in [9] it was shown that every warped product $M_1 \times_F M_2$ with $\dim M_1 = 1$ and $\dim M_2 = 3$, satisfies $(****)$. The relations $(*)$–$(****)$ are called *conditions of pseudosymmetry type*. We refer to [18] and [10] for reviews on semi-Riemannian manifolds satisfying such conditions. A hypersurface satisfying a curvature conditon of pseudosymmetry type is said to be a *hypersurface of pseudosymmetry type* [10].

**3. Main results.** The Gauss equation of a hypersurface $M$ isometrically immersed in a Riemannian space of constant curvature $M^{n+1}(c)$, $n \geq 3$, can be written in the form

$$\begin{aligned}(6)\qquad R(X_1,\dots,X_4) &= \frac{\widetilde{\kappa}}{n(n+1)} G(X_1,\dots,X_4)\\ &\quad + H(X_1,X_4)H(X_2,X_3) - H(X_1,X_3)H(X_2,X_4),\end{aligned}$$

where $\widetilde{\kappa}$ is the scalar curvature of the ambient space, $H$ is the second fundamental form of $M$ in $M^{n+1}(c)$, $R$ is the Riemann–Christoffel curvature tensor of $M$, $G$ is the $(0,4)$-tensor corresponding to the metric tensor $g$ of $M$ and $X_1, X_2, X_3, X_4$ are vector fields tangent to $M$. Further, we denote by $S$ and $\kappa$ the Ricci tensor and the scalar curvature of $M$, respectively.

PROPOSITION 1. *Let $M$ be a hypersurface in $M^{n+1}(c)$, $n = 3p$, $p \geq 1$. Moreover, suppose $M$ has at a point $x$ eigenvalues $-\lambda, 0$ and $\lambda$, where $\lambda \in \mathbb{R} - \{0\}$, with the same multiplicity.*

(i) *If $p \geq 1$ then*

$$(7)\qquad R \cdot S = \frac{1}{n(n+1)} \widetilde{\kappa} Q(g,S)$$

*holds at $x$.*

(ii) *If $p = 1$ then*

$$(8)\qquad R \cdot R = \frac{1}{n(n+1)} \widetilde{\kappa} Q(g,R)$$

*holds at $x$.*

(iii) *If $p > 1$ then the tensors $R \cdot R$ and $Q(g,R)$ as well as the tensors $C \cdot C$ and $Q(g,C)$ are linearly independent at $x$ and moreover the equality*

$$(9)\qquad R \cdot R - Q(S,R) = -\frac{n-2}{n-1}\left(\frac{1}{n}\kappa + \frac{2}{3}\lambda^2\right) Q(g,C)$$

*holds at $x$.*

Proof. (i) We can choose an orthonormal basis $E_1, \dots, E_n$ at $x$ such that

$$(10)\qquad \begin{aligned} H(E_a,E_b) &= H_{ab} = -\lambda g(E_a,E_b) = -\lambda\delta_{ab},\\ H(E_\alpha,E_\beta) &= H_{\alpha\beta} = 0 g(E_\alpha,E_\beta) = 0,\\ H(E_r,E_s) &= H_{rs} = \lambda g(E_r,E_s) = \lambda\delta_{rs},\end{aligned}$$

where $a,b,c,d \in \{1,\dots,p\}$, $\alpha,\beta,\gamma,\delta \in \{p+1,\dots,2p\}$, $r,s,t,u \in \{2p+1,\dots,n\}$. For a $(0,k)$-tensor $T$ at $x$ we put $T_{hijklm\dots} = T(E_h,E_i,E_j,E_k,E_l,E_m,\dots)$, where $h,i,j,k,l,m \in \{1,2,\dots,n\}$. Thus (6) takes the form

$$(11)\qquad R_{hijk} = H_{hk}H_{ij} - H_{hj}H_{ik} + \frac{1}{n(n+1)}\widetilde{\kappa} G_{hijk}.$$

Now we have

$$S_{hk} = -H^2_{hk} + \frac{n-1}{n(n+1)}\widetilde{\kappa} g_{hk}, \tag{12}$$

$$\kappa = -\frac{2n}{3}\lambda^2 + \frac{n-1}{n+1}\widetilde{\kappa}, \tag{13}$$

$$g^{ij} H_{hi} S_{kj} = -H^3_{hk} + \frac{n-1}{n(n+1)}\widetilde{\kappa} H_{hk}, \tag{14}$$

where $H^2_{hk} = g^{ij} H_{hi} H_{kj}$, $H^3_{hk} = g^{ij} H^2_{hi} H_{kj}$ and $g^{ij} = \delta^{ij}$ are the components of the tensor $g^{-1}$ with respect to the given basis at $x$. Since $H^3_{hk} = \lambda^2 H_{hk}$, (14) reduces to

$$g^{ij} H_{hi} S_{kj} = \Big( -\lambda^2 + \frac{n-1}{n(n+1)}\widetilde{\kappa}\Big) H_{hk}.$$

Using this and (11) we find

$$\begin{aligned} g^{lm} S_{hl} R_{mijk} = {} & \frac{1}{n(n+1)}\widetilde{\kappa}(S_{hk} g_{ij} - S_{hj} g_{ik}) \\ & + \Big( -\lambda^2 + \frac{n-1}{n(n+1)}\widetilde{\kappa}\Big)(H_{hk} H_{ij} - H_{hj} H_{ik}), \end{aligned}$$

which yields (7).

(ii) This assertion is a consequence of Lemma 1.2 of [12] (see also Lemma 2 of [20]) and the fact that (7) holds at $x$.

(iii) Using (10), (11) and the definitions of $R \cdot R$ and $Q(g, R)$ we get

$$R \cdot R_{\alpha abcd\beta} = \lambda^2 \frac{1}{n(n+1)}\widetilde{\kappa} g_{\alpha\beta} G_{dabc},$$

$$R \cdot R_{cabrsd} = 2\lambda^2 \Big( -\lambda^2 + \frac{1}{n(n+1)}\widetilde{\kappa}\Big) g_{rs} G_{cabd},$$

$$Q(g, R)_{\alpha abcd\beta} = \lambda^2 g_{\alpha\beta} G_{dabc}, \qquad Q(g, R)_{cabrsd} = -2\lambda^2 g_{rs} G_{cabd}.$$

From these equalities it follows that the tensors $R \cdot R$ and $Q(g, R)$ are linearly independent at $x$. Further, using again (10) and (11) we can easily verify that the following components of the Weyl tensor $C$ of $M$ are non-zero at $x$:

$$C_{abcd} = \tau_1 G_{abcd}, \quad C_{a\alpha\beta b} = \varrho G_{a\alpha\beta b}, \quad C_{rstu} = \tau_1 G_{rstu},$$
$$C_{arsb} = (\tau_1 - 2\lambda^2) G_{arsb}, \quad C_{\alpha\beta\gamma\delta} = \tau_2 G_{\alpha\beta\gamma\delta}, \quad C_{r\alpha\beta s} = \varrho G_{r\alpha\beta s},$$

where

$$\varrho = \frac{1}{n-2}(\lambda^2 + \tau), \qquad \tau = \frac{1}{n-1}\kappa - \frac{1}{n+1}\widetilde{\kappa},$$
$$\tau_1 = \frac{1}{n-2}(n\lambda^2 + \tau) \quad \text{and} \quad \tau_2 = \frac{1}{n-2}\tau.$$

Now, using the definitions of the tensors $C \cdot C$ and $Q(g, C)$ we get

$$C \cdot C_{\alpha abcd\beta} = \frac{n-1}{n-2}\lambda^2 \varrho g_{\alpha\beta} G_{dabc}, \qquad Q(g,C)_{\alpha abcd\beta} = \frac{n-1}{n-2}\lambda^2 g_{\alpha\beta} G_{dabc},$$

$$C \cdot C_{rabcds} = 2\lambda^2(\tau_1 - 2\lambda^2) g_{rs} G_{dabc}, \quad Q(g,C)_{rabcds} = 2\lambda^2 g_{rs} G_{dabc}.$$

From these equalities it follows that the tensors $C\cdot C$ and $Q(g,C)$ are linearly independent at $x$. Finally, the relation (5), by (13), turns into (9). Our proposition is thus proved.

As a consequence of the proposition above we obtain the following.

THEOREM 1. *Every Cartan hypersurface $M$ in $S^{n+1}(c)$, $n = 6, 12, 24$, is a non-pseudosymmetric Ricci-pseudosymmetric manifold with non-pseudosymmetric Weyl tensor. Moreover, the condition*

$$R \cdot R - Q(S,R) = -\frac{n-2}{n-1}\left(\frac{\kappa}{n} + \frac{2}{3}\lambda^2\right) Q(g,C)$$

*holds on $M$. Furthermore, every Cartan hypersurface $M$ in $S^4(c)$ is a non-semisymmetric pseudosymmetric manifold.*

Remark 3. From the results presented in [11] (see Theorems 1 and 2) we conclude that, at every point of a hypersurface $M$ in $\mathbb{E}^{n+1}$, $n \geq 4$, at which the tensor $R\cdot S$ is non-zero, the conditions $(*)$ and $(**)$ are equivalent. Proposition 1 states that this is not the case when the ambient space is a space of non-zero constant curvature.

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