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SOME SOLVED AND UNSOLVED PROBLEMS IN COMBINATORIAL NUMBER THEORY, II

BY

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In an earlier paper [9], the authors discussed some solved and unsolved problems in combinatorial number theory. First we will give an update of some of these problems. In the remaining part of this paper we will discuss some further problems of the two authors.

1. Remarks on an earlier paper. In this section we will give a survey of the recent results related to the problems discussed in [9]. Throughout this paper, the counting function of a set $\mathcal{A}$ of positive integers will be denoted by $A(n)$:

$$A(n) = \sum_{a\le n, a\in\mathcal{A}} 1 .$$

Problem 1 in [9] was the following: Show the existence of an infinite sequence $\mathcal{A}$ of positive integers $a_1 < a_2 < \ldots$ such that it is a Sidon set, i.e., all the sums $a_i + a_j$ (with $i \le j$) are distinct, and

$$\liminf_{n\to\infty} \frac{A(n)}{n^{1/3}} = \infty .$$

The existence of such a sequence $\mathcal{A}$ was shown by Ajtai, Komlós and Szemerédi [1]. In fact, they proved that there is an infinite Sidon set $\mathcal{A}$ such that

$$A(n) > 10^{-3}(n\log n)^{1/3} \quad \text{for } n > n_0 .$$

However, this seems to be far from the truth, and almost certainly, for all $\varepsilon > 0$ there is an infinite Sidon set $\mathcal{A}$ such that

$$A(n) > n^{1/2-\varepsilon} \quad \text{for } n > n_0(\varepsilon) .$$

See Erdős and Freud [4] for further related problems.

In Problem 2, we discussed the following question: If $\mathcal{A} = \{a_1, \ldots, a_N\}$ (with $a_1 < \ldots < a_N$) is a sequence of positive integers and t is a positive

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integer, then let $f(N,\mathcal{A},t)$ denote the number of solutions of

$$\sum_{i=1}^{N} \varepsilon_i a_i = t \quad \text{where} \quad \varepsilon_i = 0 \text{ or } 1\,.$$

Improving on a result of Erdős and Moser, Sárközy and Szemerédi proved that

$$f(N,\mathcal{A},t) < c\frac{2^N}{N^{3/2}}\,.$$

In Problem 2, we asked for the best possible value of this constant c. This question has been settled by Stanley [33] and, in fact, he has shown that for fixed N, $f(N,\mathcal{A},t)$ is maximal if and only if the elements of $\mathcal{A}$ form an arithmetic progression and $t = \frac{1}{2}\sum_{i=1}^{N} a_i$.

In Problem 5, we asked for the proof of the existence of sequences $\mathcal{A} \subset \{1,\dots,N\}$ such that

$$a - a' = p - 1, \qquad a \in \mathcal{A},\ a' \in \mathcal{A},\ p \text{ prime}, \tag{1}$$

cannot be solved and we have

$$\frac{A(N)}{\log N} \to \infty \qquad \text{as } N \to \infty\,.$$

This question was settled by the authors in [8] where we showed the existence of sequences $\mathcal{A} \subset \{1,\dots,N\}$ such that (1) cannot be solved and

$$\frac{A(N)}{\log N} \gg \frac{\log_2 N \log_4 N}{(\log_3 N)^2} \tag{2}$$

(where $\log_k N$ denotes the k-fold logarithm), and Ruzsa [32] improved (2) to

$$A(N) \gg \exp\left((1-\varepsilon)\log 2 \frac{\log N}{\log_2 N}\right).$$

In Problem 9, we asked the following question: Let $a_1 < \ldots < a_n$ be a sequence of positive integers which contains the first k positive integers:

$$a_1 = 1,\ a_2 = 2,\ \ldots,a_k = k\,. \tag{3}$$

What can be said on the number of distinct products of the form $\prod_{i=1}^{n} a_i^{\varepsilon_i}$ (where $\varepsilon_i = 0$ or 1 for $i = 1,\dots,n$)? For fixed n and k, let $F(n,k)$ denote the minimal number of the distinct products (the minimum taken over all sequences $a_1,\dots,a_n$ satisfying (3)). We asked whether for all $\omega > 0$, there is a $k_0(\omega)$ such that

$$F(n,k) > n^2 k^{\omega} \qquad \text{for } k > k_0(\omega)\,.$$

Moreover, we conjectured that for $k > k_1$,

$$n^2 \exp(c_1 k/\log k) < F(n,k) < n^2 \exp(c_2 k/\log k)\,.$$

In [10], the authors proved the slightly weaker inequality

$$n^2 \exp(c_1 k/(\log k)^2) < F(n,k) < n^2 \exp(c_2 k/\log k)\,.$$

In Problem 10, we studied the following problem: Let N be a positive integer. Let E_N denote the set of the 2^N sequences $\varepsilon = \{\varepsilon_1, \dots, \varepsilon_N\}$, where $\varepsilon_i = -1$ or $+1$ for $i = 1, \dots, N$. Let

$$F(N) = \min_{\varepsilon \in E_N} \max_{\substack{k,n,q \\ 1 \le n \le n+(k-1)q \le N}} \Big| \sum_{i=0}^{k-1} \varepsilon_{n+iq} \Big| .$$

By a result of Roth [31] we have

$$F(N) \gg N^{1/4}\,,$$

and in [9] we proved that

$$F(N) \ll N^{1/3} (\log N)^{2/3}\,.$$

The problem was to tighten the gap between these bounds. Moreover, we conjectured that for $\varepsilon > 0$, $N > N_0(\varepsilon)$ we have

$$F(N) \ll N^{1/4+\varepsilon}\,.$$

This conjecture has been proved by Beck [2] who showed that

$$F(N) \ll N^{1/4} (\log N)^{5/2}\,.$$

In Problems 15–17, we studied sets of points in the n-dimensional Euclidean space such that for every pair of distinct points P, Q selected from the given set, the distance between P and Q is "far" from the closest integer. No progress has been made in connection with these problems. Here we would like to add a further related problem.

Problem 1. Denote the area of the triangle PQR by $A(P,Q,R)$, and denote the distance from the real number x to the closest integer by $\|x\|$ so that $\|x\| = \min(x - [x], [x] + 1 - x)$. Does there exist a positive number ε and an infinite sequence of points $P_1, P_2, \dots$ in the plane so that $\|A(P_i, P_j, P_k)\| > \varepsilon$ for all $(1 \le)\ i < j < k$? Moreover, for $\varepsilon > 0$ and $N \to \infty$, estimate the maximal number $t = t(\varepsilon, N)$ of points $P_1, \dots, P_t$ in a circle of radius N with the property that $\|A(P_i, P_j, P_k)\| > \varepsilon$ for all $1 \le i < j < k \le t$. Note that this problem is closely related to a problem of Heilbronn; see [29] and [30].

2. Additive problems.

If $\mathcal{A}$ is a set of positive integers, then let $R_1(n)$, $R_2(n)$, $R_3(n)$ denote the number of solutions of

$$a + a' = n, \quad a, a' \in \mathcal{A},$$
$$a + a' = n, \quad a, a' \in \mathcal{A},\ a < a'\,,$$

and

$$a + a' = n, \quad a, a' \in \mathcal{A},\ a \le a',$$

respectively. In [16], [17], and [21]–[23], Erdős, Sárközy and T. Sós studied the functions $R_1(n)$, $R_2(n)$ and $R_3(n)$. We have not been able to settle the following problems:

Problem 2. Put

$$B(\mathcal{A},n) = \sum_{a\le n, a-1\notin\mathcal{A}, a\in\mathcal{A}} 1\,.$$

In [21] we proved that

(i) if $\mathcal{A}$ is an infinite set such that

$$\lim_{n\to\infty} \frac{B(\mathcal{A},n)}{n^{1/2}} = \infty\,,$$

then $|R_1(n+1) - R_1(n)|$ cannot be bounded;

(ii) for all $\varepsilon > 0$, there exists an infinite sequence $\mathcal{A}$ such that

$$B(\mathcal{A},n) \gg N^{1/2-\varepsilon}$$

and $R_1(n)$ is bounded so that also $|R_1(n+1) - R_1(n)|$ is bounded;

(iii) there exists an infinite sequence $\mathcal{A}$ such that

$$\limsup_{n\to\infty} \frac{B(\mathcal{A},n)}{n^{1/2}} > 0$$

and $|R_1(n+1) - R_1(n)|$ is bounded.

The problem is to show that (i) can be sharpened in the following way: If $\mathcal{A}$ is an infinite set such that

$$\limsup_{n\to\infty} \frac{B(\mathcal{A},n)}{n^{1/2}} = \infty$$

or

$$\liminf_{n\to\infty} \frac{B(\mathcal{A},n)}{n^{1/2}} > 0$$

(perhaps, it suffices to assume that $\liminf_{n\to\infty} B(\mathcal{A},n)(\log n)/n^{1/2} = \infty$), then $|R_1(n+1) - R_1(n)|$ cannot be bounded.

Problem 3. In [22], Erdős, Sárközy and T. Sós proved that if $\mathcal{A}$ is an infinite set such that

$$A(n) = o(n/\log n)\,, \tag{4}$$

then the function $R_2(n)$ cannot be increasing (from a certain point on). On the other hand, we showed that there is an infinite set $\mathcal{A}$ such that

$$A(n) < n - cn^{1/2} \tag{5}$$

and $R_2(n)$ is increasing. The problem is to tighten the gap between (4) and (5). The latter seems to be closer to truth, and, in fact, we conjecture that if $R_2(n)$ is increasing, then

$$A(n) \sim n$$

must hold.

PROBLEM 4. Erdős, Sárközy and T. Sós [23] proved that if $\mathcal{A}$ is an infinite set such that $R_3(n)$ is increasing from a certain point on, then

$$\lim_{n\to\infty} \frac{n - A(n)}{\log n} = \infty$$

cannot hold. We conjecture that $R_3(n)$ is increasing from a certain point on if and only if $\mathcal{A}$ contains all the integers from a certain point on:

$$\mathcal{A} \cap \{n_0, n_0+1, n_0+2, \dots\} = \{n_0, n_0+1, n_0+2, \dots\}$$

for some positive integer n_0. The problem is to prove this conjecture.

PROBLEM 5. Let $\mathcal{A}$ be a finite or infinite Sidon set (see Section 1), and let $b_1 < b_2 < \dots$ denote the integers that can be represented in the form $a + a' = b_i$ with $a, a' \in \mathcal{A}$. The problem is to estimate the function $F(\mathcal{A}, n) = \max_{b_i \le n}(b_{i+1} - b_i)$. Note that it follows from a result of Erdős (see [34]) that for infinite Sidon sets $\mathcal{A}$ we have

$$\limsup_{n\to\infty} \frac{F(\mathcal{A}, n)}{\log n} > 0\,.$$

PROBLEM 6. Let $\mathcal{A}$ be an infinite Sidon set, define $b_1, b_2, \dots$ as in Problem 5, write $\mathcal{B}(\mathcal{A}) = \{b_1, b_2, \dots\}$, and let $G(\mathcal{A}, n)$ denote the number of positive integers k such that $k \le n$ and $2k \in \mathcal{B}(\mathcal{A})$ but $2k+1 \notin \mathcal{B}(\mathcal{A})$. Erdős, Sárközy and T. Sós proved that $G(\mathcal{A}, n) \to \infty$ and, in fact, $G(\mathcal{A}, n) \gg A(n)$. The problem is to estimate $G(\mathcal{A}, n)$ (in terms of $A(n)$), in particular, to decide whether $G(\mathcal{A}, n)/A(n) \to \infty$ must hold.

In the next three problems, we shall use the following notations: If $\mathcal{S}$ is a given set and $\mathcal{A}_1, \dots, \mathcal{A}_k$ are subsets of $\mathcal{S}$ with

$$\mathcal{S} = \bigcup_{i=1}^{k} \mathcal{A}_i, \qquad \mathcal{A}_i \cap \mathcal{A}_j = \emptyset \quad \text{for } i \ne j\,,$$

then $\{\mathcal{A}_1, \dots, \mathcal{A}_k\}$ is called a *k-partition* (or *k-colouring*) of $\mathcal{S}$, and the subsets $\mathcal{A}_1, \dots, \mathcal{A}_k$ are referred to as *classes*. If $f(x_1, \dots, x_t)$ is a given function of t variables and

$$f(a_1, \dots, a_t) = n \tag{6}$$

with $a_1, \dots, a_t$ belonging to the same class, then this is called a *monochromatic representation* of n in the form (6). If N is a positive integer and $\mathcal{P}_k = \{\mathcal{A}_1, \dots, \mathcal{A}_k\}$ is a k-partition of $\{1, \dots, N\}$, then let $F(\mathcal{P}_k, N)$ denote the number of even integers not exceeding N that have a monochromatic representation in the form

$$a_1 + a_2 = n \quad \text{with } a_1 \neq a_2 .$$

PROBLEM 7. Erdős, Sárközy and T. Sós [24] proved that for every positive integer N and every 2-partition $\mathcal{P}_2$ of $\{1, \dots, N\}$ we have

$$F(\mathcal{P}_2, N) > \frac{N}{2} - c \log N ;$$

moreover, for every positive integer N, every integer $k > 2$ and every k-partition $\mathcal{P}_k$ of $\{1, \dots, N\}$ we have

$$F(\mathcal{P}_k, N) > \frac{N}{2} - cN^{1-2^{k-1}} ; \tag{7}$$

finally, for every positive integer N and every integer $k \geq 2$ there is a k-partition $\mathcal{P}_k$ of $\{1, \dots, N\}$ such that

$$F(\mathcal{P}_k, N) < \frac{N}{2} - ck \log N .$$

The problem is to improve on (7); in fact, we guess that for every integer $k \geq 2$ there is an $\alpha(k)$ such that for every positive integer $N > N_0(k)$ and every k-partition $\mathcal{P}_k$ of $\{1, \dots, N\}$ we have

$$F(\mathcal{P}_k, N) > \frac{N}{2} - (\log N)^{\alpha(k)} .$$

PROBLEM 8. Let $f(x)$ be a polynomial with integer coefficients such that 2 is a prime divisor of it. Is it true that for any k-partition of the set of positive integers there is a positive integer x (or there are infinitely many positive integers x) such that

$$a_1 + a_2 = f(x)$$

has a monochromatic solution with $a_1 \neq a_2$? (Some further problems can be found in [24].)

3. Multiplicative problems

PROBLEM 9. In [18], the authors studied multiplicative analogues of the problems studied in [24]. In particular, we proved that for every $\varepsilon > 0$ there is a 3-partition $\{\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3\}$ of the set of positive integers such that the lower asymptotic density of the sequence of distinct monochromatic products aa' is less than ε. On the other hand, we have not been able to decide whether it is necessary to take three classes here. In other words, the problem is to decide whether for every $\varepsilon > 0$, there is a 2-partition $\{\mathcal{A}_1, \mathcal{A}_2\}$ of the set of positive integers such that the lower asymptotic density of the sequence of distinct monochromatic products aa' is less than ε.

PROBLEM 10. Let $\mathcal{A}$ be a finite or infinite sequence of positive integers, and write

$$f(\mathcal{A},x) = \sum_{a\in\mathcal{A}, a\le x} \frac{1}{a},$$

$$d(\mathcal{A},n) = \sum_{a\in\mathcal{A}, a|n} 1, \qquad D(\mathcal{A},x) = \max_{n\le x} d(\mathcal{A},n).$$

In [12]–[15] the authors estimated $D(\mathcal{A},x)$ in terms of $f(\mathcal{A},x)$. Next we shall discuss three related problems that we have not been able to settle. The first problem:

In [14] we proved that for all $\omega > 0$ and for $x > x_0(\omega)$,

$$f(\mathcal{A},x) > (\log\log x)^{20} \tag{8}$$

implies that

$$D(\mathcal{A},x) > \omega f(\mathcal{A},x).$$

Is it true that for $x > x_0(\varepsilon,\omega)$, (8) can be replaced by

$$f(\mathcal{A},x) > (\log\log x)^{1+\varepsilon}?$$

PROBLEM 11. It follows from Theorem 2 in [14] that if $\mathcal{A}$ is an infinite sequence such that

$$\liminf_{x\to\infty} \frac{f(\mathcal{A},x)}{\log\log\log x} > 22,$$

then we have

$$\lim_{x\to\infty} \frac{D(\mathcal{A},x)}{f(\mathcal{A},x)} = \infty. \tag{9}$$

On the other hand, we showed that there is an infinite sequence $\mathcal{A}$ such that

$$\liminf_{x\to\infty} \frac{f(\mathcal{A},x)}{\log\log\log x} \ge c \qquad (>0)$$

(we did not compute c explicitly) and

$$\lim_{x\to\infty} \frac{D(\mathcal{A},x)}{f(\mathcal{A},x)} < \infty.$$

The problem is to determine the smallest constant c such that

$$\liminf_{x\to\infty} \frac{f(\mathcal{A},x)}{\log\log\log x} > c$$

implies (9).

PROBLEM 12. Is it true that for all $\omega > 0$, there exist constants $c = c(\omega)$ and $x_0 = x_0(\omega)$ such that $x > x_0$ and

$$f(\mathcal{A},x) > c$$

imply that

$$D(\mathcal{A}, x^2) > (f(\mathcal{A}, x))^{\omega}?$$

PROBLEM 13. Let k and x be positive integers such that $k \le x$, and consider all the sets $\mathcal{A}$ such that $\mathcal{A} \subset \{1, \dots, x\}$ and if $a_1 < \dots < a_k$, $a_1 \in \mathcal{A}, \dots, a_k \in \mathcal{A}$, then the product $a_1 \dots a_k$ is never a square. Let $F(k, x)$ denote the maximum of the cardinalities of these sets $\mathcal{A}$. The function $F(k, x)$ was studied by Erdős, Sárközy and T. Sós in [25]. In particular, we proved that for $x \to \infty$ we have

$$F(2, x) = \left(\frac{6}{\pi^2} + o(1)\right)x\,,$$

$$x - \frac{x}{(\log x)^{c_1}} < F(3, x) < x - \frac{x}{(\log x)^{c_2}}\,,$$

$$F(2k, x) = o(x) \qquad \text{for } k \ge 2$$

and

$$F(2k+1, x) \ge (\log 2 + o(1))x \qquad \text{for } k \ge 2\,.$$

The most interesting problem that we have not been able to settle is the following: Is it true that

$$\lim_{x \to \infty} \frac{F(2k+1, x)}{x} = 1$$

for $k \ge 2$ (as for $k = 1$)?

4. Arithmetic functions. $\omega(n)$ denotes the number of distinct prime factors of n, and $\varphi(n)$ denotes Euler's function. Erdős, Pomerance and Sárközy [5]–[7] studied the solvability of the equation

$$x + \omega(x) = y + \omega(y)$$

and other related questions. The most interesting related unsolved problems are the following:

PROBLEM 14. Show that for infinitely many positive integers n the equation

$$x + \omega(x) = n$$

has at least 3 solutions.

PROBLEM 15. Show that for every positive integer k there are infinitely many positive integers n such that the equation

$$x + \omega(x) = n + i$$

has at least 2 solutions for each $i = 1, \dots, k$.

PROBLEM 16. Show that for all $K > 0$ there is a positive integer n such that the equation

$$x + \varphi(x) = n$$

has more than K solutions.

5. Miscellaneous problems

PROBLEM 17. Let m_k denote the smallest positive integer m such that

$$\omega\left(\binom{m}{k}\right) > k$$

(where $\omega(n)$ denotes the number of distinct prime factors of n). In [11] the authors proved that for $k > k_0(\varepsilon)$ we have

$$m_k > k^2(\log k)^{4/3-\varepsilon},$$

and it follows from results of Erdős and Selfridge [28] that

$$m_k < k^{e+\varepsilon}.$$

The problem is to show that there is a positive constant c such that

$$m_k \gg k^{2+c}.$$

PROBLEM 18. Denote the greatest prime factor of n by $P(n)$, and let n_k denote the smallest integer n for which

$$P(n+i) > k \qquad \text{for all } 1 \le i \le k.$$

In [11] we proved that

$$n_k \gg k^{5/2}$$

and Erdős [3] showed that

$$n_k < k^{\log k/\log\log k}. \tag{10}$$

The problem is to show that for all t we have

$$n_k > k^t \qquad \text{if } k > k_0(t).$$

(In fact, (10) seems to be close to the "truth".)

PROBLEM 19. If k, n are positive integers, then let $F_k(n)$ denote the cardinality of the largest subset of $\{1, \dots, n\}$ which does not contain k pairwise coprime integers. The function $F_k(n)$ was studied by Erdős, Sárközy and Szemerédi in [26], [27] and [19], and these papers contain many related unsolved problems. The most interesting unsolved problem is the following conjecture: if $G_k(n)$ denotes the number of those integers not exceeding n which are multiples of at least one of the first k primes, then we have $F_k(n) = G_k(n)$.

PROBLEM 20. If $\mathcal{A}$ is a finite set of positive integers, then let $\mathcal{P}(\mathcal{A})$ denote the set of subset sums, i.e., the set of distinct positive integers n that can be represented in the form $n = \sum_{a\in\mathcal{A}} \varepsilon_a a$ where $\varepsilon_a = 0$ or 1 for all a. In [20] the authors studied the occurrence of arithmetic progressions in $\mathcal{P}(\mathcal{A})$. The most interesting related problem that we have not been able to settle is the following: If N is an integer with $N \geq 3$, then let $t(N)$ denote the least integer t such that for every $\mathcal{A} \subset \{1, \dots, N\}$ with $|\mathcal{A}| \geq t$ the set $\mathcal{P}(\mathcal{A})$ contains three consecutive multiples of a positive integer. We have proved that

$$t(N) \geq \left[\frac{\log N}{\log 3}\right] + 2$$

but we do not have any reasonable upper bound for $t(N)$. The problem is to give a good upper bound and possibly an asymptotics for $t(N)$.

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