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SMOOTH POINTS OF MUSIELAK–ORLICZ SEQUENCE SPACES EQUIPPED WITH THE LUXEMBURG NORM

BY

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Introduction. In the sequel $\mathbb{N}$ denotes the set of natural numbers, $\mathbb{R}$ the reals and $\mathbb{R}_+$ the nonnegative reals. By a *Musielak–Orlicz function* Φ we understand a sequence $(\Phi_i)_{i=1}^\infty$ of Orlicz functions Φ_i, i.e. $\Phi_i : \mathbb{R} \to [0,\infty]$ and Φ_i vanishes and is continuous at zero, left-continuous on the whole $\mathbb{R}_+$, convex and even on $\mathbb{R}$, and not identically zero. For any Musielak–Orlicz function $\Phi = (\Phi_i)_{i=1}^\infty$ we denote by Φ^* its *complementary function* in the sense of Young, i.e. $\Phi^* = (\Phi_i^*)_{i=1}^\infty$, where

$$\Phi_i^*(u) = \sup_{v>0}\{|u|v - \Phi_i(v)\} \qquad (\forall u \in \mathbb{R}).$$

If Ψ is an Orlicz function and $u \in \mathbb{R}$, we denote by $\Psi^-(u)$ and $\Psi^+(u)$ the left and the right derivatives of Ψ at u, respectively. Moreover, for any Orlicz function Ψ, we define

$$b(\Psi) = \sup\{u \in \mathbb{R}_+ : \Psi(u) < \infty\},$$

$$\partial\Psi(u) = \begin{cases} [\Psi^-(u), \Psi^+(u)] & \text{if } -b(\Psi) < u < b(\Psi), \\ [\Psi^-(u), \infty) & \text{if } u = b(\Psi) \text{ and } \Psi^-(b(\Psi)) < \infty, \\ (-\infty, \Psi^+(u)] & \text{if } u = -b(\Psi) \text{ and } \Psi^+(-b(\Psi)) > -\infty, \\ \{\infty\} & \text{if } u > b(\Psi), \text{ or } u = b(\Psi) \text{ and } \Psi^-(b(\Psi)) = \infty, \\ \{-\infty\} & \text{if } u < -b(\Psi), \text{ or } u = -b(\Psi) \\ & \quad \text{and } \Psi^+(-b(\Psi)) = -\infty. \end{cases}$$

It is easy to show that for any $u \in \mathbb{R}$, we have

$$\partial\Phi(u) = \{v \in \mathbb{R} : \Psi(u) + \Psi^*(v) = uv\}.$$

Let us denote by ℓ^0 the space of all sequences of reals, and for any $x = (x_i)_{i=1}^\infty \in \ell^0$ and $A \subset \mathbb{N}$, define $x^A = \sum_{i\in A} x_i e_i$, where e_i is the ith basic sequence, i.e. $e_i = (0, \ldots, 0, 1, 0, \ldots)$, where 1 stands in the ith place. For any $x \in \ell^0$ define

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$$x^{(n)} = (x_1, \ldots, x_n, 0, \ldots) .$$

If $\Phi = (\Phi_i)_{i=1}^{\infty}$ is a Musielak–Orlicz function and $x = (x_i)_{i=1}^{\infty} \in \ell^0$, we define $\partial\Phi(x) = (\partial\Phi_i(x_i))_{i=1}^{\infty}$. Moreover, we define

$$b_i = b(\Phi_i) , \qquad a_i = a_i(\Phi_i) = \begin{cases} b_i & \text{if } \Phi_i(b_i) \le 1, \\ \Phi_i^{-1}(1) & \text{if } \Phi_i(b_i) > 1. \end{cases}$$

Given a Musielak–Orlicz function $\Phi = (\Phi_i)_{i=1}^{\infty}$, we define on ℓ^0 a convex functional I_Φ by the formula

$$I_\Phi(x) = \sum_{i=1}^{\infty} \Phi_i(x_i) \qquad (\forall x = (x_i) \in \ell^0) .$$

The *Musielak–Orlicz space* ℓ^Φ generated by a Musielak–Orlicz function Φ is defined in the following way:

$$\ell^\Phi = \{x \in \ell^0 : I_\Phi(\lambda x) < \infty \text{ for some } \lambda > 0\} .$$

This space endowed with the *Luxemburg norm*

$$\|x\|_\Phi = \inf\{\lambda > 0 : I_\Phi(x/\lambda) \le 1\}$$

is a Banach space (see [5]–[7] and [10]).

For any Musielak–Orlicz function Φ we define $\mathtt{h}^\Phi$ to be the norm closure in ℓ^Φ of the set $\mathtt{h}$ of all sequences in ℓ^0 with a finite number of coordinates different from zero. This space will be considered with the norm $\| \ \|_\Phi$ induced from ℓ^Φ.

In the case when all the Φ_i, $i = 1, 2, \ldots$, are finite-valued, we have

$$\mathtt{h}^\Phi = \{x \in \ell^0 : I_\Phi(\lambda x) < \infty \text{ for any } \lambda > 0\} .$$

If $d(x, \mathtt{h}^\Phi) = \inf\{\|x - y\| : y \in \mathtt{h}^\Phi\}$ for $x \in \ell^\Phi$, then we have

$$d(x, \mathtt{h}^\Phi) = \inf\Big\{\lambda > 0 : \sum_{i=j}^{\infty} \Phi_i(\lambda^{-1} x_i) < \infty \text{ for some } j \in \mathbb{N}\Big\} \qquad \text{(see [4])} .$$

For any Banach space X denote by $S(X)$ its unit sphere and by X^* its dual space. If $x \in X\backslash\{0\}$ then $x^* \in X^*$ is said to be a *support functional* at x if $\|x^*\| = 1$ and $x^*(x) = \|x\|$.

For any $x \in X\backslash\{0\}$, we denote by $\mathrm{Grad}(x)$ the set of all support functionals at x. We say that $x \in X\backslash\{0\}$ is a *smooth point* if $\mathrm{Card}(\mathrm{Grad}(x)) = 1$. A Banach space X is said to be *smooth* if any point $x \in S(X)$ is smooth (see [2] and [8]).

For any Musielak–Orlicz space ℓ^Φ equipped with the Luxemburg norm, any $x^* \in X^*$ is uniquely represented in the form

$$x^* = x_1^* + x_2^* , \tag{$*$}$$

where x_1^* is a *regular* (= order continuous) *functional*, i.e. a functional represented by an element $y \in \ell^{\Phi^*}$, i.e.

$$x_1^*(x) = \sum_{i=1}^{\infty} x_i y_i \qquad (\forall x = (x_i) \in \ell^{\Phi}),$$

and x_2^* is a *singular functional*, i.e. $x_2^*(x) = 0$ for any $x \in \mathrm{h}^{\Phi}$. Moreover,

$$\|x^*\| = \|x_1^*\| + \|x_2^*\|$$

(see [4] and in the case of Orlicz spaces also [1]).

Auxiliary results. In this section we recall some results from [3] and [4] which will be applied to obtain our main results.

LEMMA 1. *If $x \in S(\ell^{\Phi})$ and $d(x, \mathrm{h}^{\Phi}) < 1$ then every support functional x^* at x is regular.*

LEMMA 2. *Suppose that $x \in S(\ell^{\Phi})$, $x^* \in \mathrm{Grad}(x)$ is regular and x^* corresponds to $\lambda = (\lambda_i)_{i=1}^{\infty} \in \ell^{\Phi^*}$. Then*

(i) *$\lambda_i x_i \geq 0$ for any $i \in \mathbb{N}$,*

(ii) *if $\lambda_{i_0} x_{i_0} > 0$ and $|x_{i_0}| < a_{i_0}$ for some $i_0 \in \operatorname{supp} x^* = \{i \in \mathbb{N} : \lambda_i \neq 0\}$, then $I_{\Phi}(x) = \sup\{I_{\Phi}(y) : \operatorname{supp} y \subset \operatorname{supp} x^*,\ \|y\|_{\Phi} \leq 1\}$.*

LEMMA 3. *Let $x \in S(\ell^{\Phi})$ and let $x^* \in \mathrm{Grad}(x)$ be regular. Then $\operatorname{supp} x^* \subset A_x = \{i \in \mathbb{N} : |x_i| = a_i\}$ whenever $I_{\Phi}(x) < \sup\{I_{\Phi}(y) : \|y\|_{\Phi} \leq 1\}$.*

LEMMA 4. (i) *Let Φ and $x \in S(\ell^{\Phi})$ be such that $A_x = \{i \in \mathbb{N} : |x_i| = a_i\} \neq \emptyset$. Let $(\lambda_i)_{i \in A_x}$ be a family of nonnegative numbers such that $\sum_{i \in A_x} \lambda_i = 1$. Then the functional x^* defined by the formula*

$$x^*(y) = \sum_{i \in A_x} \lambda_i y_i / x_i \qquad (\forall y = (y_i) \in \ell^{\Phi})$$

is a support functional at x.

(ii) *If additionally $A_x^{\infty} = \{i \in A_x : |x_i| = b_i\}$ and $\partial \Phi_i(x_i) = \{\pm\infty\}$ for some $i \in A_x^{\infty}$, then every regular functional $x^* \in \mathrm{Grad}(x)$ has its support in A_x^{∞} and it is of the form*

$$x^*(y) = \sum_{i \in A_x^{\infty}} \lambda_i y_i / x_i \qquad (\forall y = (y_i) \in \ell^{\Phi}),$$

where $\lambda_i \geq 0$ for any $i \in A_x^{\infty}$ and $\sum_{i \in A_x^{\infty}} \lambda_i = 1$.

LEMMA 5. *Let $x \in S(\ell^{\Phi})$, suppose $I_{\Phi}(x) = \sup\{I_{\Phi}(y) : \|y\|_{\Phi} \leq 1, \operatorname{supp} y \subset \operatorname{supp} x\}$ and let $x^* \in (\ell^{\Phi})^*$ be defined by the formula*

$$x^*(y) = \Big(\sum_{i=1}^{\infty} z_i y_i\Big) \Big/ \Big(\sum_{i=1}^{\infty} z_i x_i\Big) \qquad (\forall y = (y_i) \in \ell^{\Phi}), \tag{1}$$

where $z_i \in \partial\Phi_i(x_i)$ *and* $\Phi_i^-(|x_i|) < \infty$ *for any* $i \in \mathbb{N}$, *and* $0 < \sum_{i=1}^\infty z_i x_i < \infty$. *Then* x^* *is a support functional at* x.

If $|x_i| < b_i$ *for any* $i \in \mathbb{N}$ *then any* $x^* \in \mathrm{Grad}(x)$ *is represented by formula* (1).

The proof proceeds in the same way as the proof of Theorem 1.9 of [4].

LEMMA 6. *Let* $x \in S(\ell^\Phi)$ *be such that* $\Phi_i^-(|x_i|) < \infty$ *for any* $i \in \operatorname{supp} x$, *let* $x^* \in (\ell^\Phi)^*$ *be regular and* $A = \operatorname{supp} x^*$. *Then* $x^* \in \mathrm{Grad}(x)$ *if and only if*

(i) $I_\Phi(x) = \sup\{I_\Phi(y) : \|y\|_\Phi \le 1, \operatorname{supp} y \subset A\}$,
(ii) $x^*(y) = (\sum_{i\in A} d_i y_i)/(\sum_{i\in A} d_i x_i)$ $(\forall y = (y_i) \in \ell^\Phi)$, *where*
(iii) $d_i \in \partial\Phi_i(x_i)$ *for any* $i \in A$ *and* $0 < \sum_{i\in A} d_i x_i < \infty$.

Main results. We start with the following result.

PROPOSITION 1. *Assume that* $x \in S(\ell^\Phi)$ *and* $d(x, \mathrm{h}^\Phi) = 1$, *i.e.*

$$\sum_{i=m}^{\infty} \Phi_i(\lambda x_i) = \infty \qquad \text{for any } \lambda > 1 \text{ and any } m \in \mathbb{N}\,. \tag{2}$$

Then $x = y + z$, *where* $\operatorname{supp} y \cap \operatorname{supp} z = \emptyset$ *and* $y, z \in S(\ell^\Phi)$.

Proof. Take a sequence $(\lambda_i)_{i=1}^\infty$ of positive reals such that $\lambda_1 > \lambda_2 > \ldots$ and $\lambda_i \to 1$ as $i \to \infty$. Define $m_1 = 1$. There is an $n_1 \in \mathbb{N}$ such that

$$\sum_{i=1}^{n_1-1} \Phi_i(\lambda_1 x_i) \ge 1\,.$$

Since in view of (2) we have $\sum_{i=n_1}^\infty \Phi_i(\lambda_1 x_i) = \infty$, there is an $m_2 > n_1$ such that $\sum_{i=n_1}^{m_2-1} \Phi_i(\lambda_1 x_i) \ge 1$. We have $\sum_{i=m_2}^\infty \Phi_i(\lambda_2 x_i) = \infty$, so again we can find natural numbers m_3, n_2 such that $m_3 > n_2 > m_2$ and

$$\sum_{i=m_2}^{n_2-1} \Phi_i(\lambda_2 x_i) \ge 1 \quad \text{and} \quad \sum_{i=n_2}^{m_3-1} \Phi_i(\lambda_2 x_i) \ge 1\,.$$

Continuing this process we find sequences (m_k) and (n_k) of natural numbers such that

$$n_{k+1} > m_{k+1} > n_k > m_k \qquad (k = 1, 2, \ldots)\,, \tag{3}$$

$$\sum_{i=m_k}^{n_k-1} \Phi_i(\lambda_k x_i) \ge 1\,, \tag{4}$$

$$\sum_{i=n_k}^{m_{k+1}-1} \Phi_i(\lambda_k x_i) \ge 1\,. \tag{5}$$

Define

$$y=\sum_{k=1}^{\infty}\sum_{i=m_k}^{n_k-1}x_ie_i\,,\qquad z=\sum_{k=1}^{\infty}\sum_{i=n_k}^{m_{k+1}-1}x_ie_i\,.$$

We have $I_\Phi(y)\le I_\Phi(x)\le 1$ and $I_\Phi(z)\le I_\Phi(x)\le 1$. Moreover, for any $\lambda>1$, we can find $k\in\mathbb{N}$ such that $\lambda\ge\lambda_k$. Hence, by (4) and (5) we get

$$I_\Phi(\lambda y)\ge I_\Phi(\lambda_k y)\ge\sum_{i=m_k}^{n_k-1}\Phi_i(\lambda_k x_i)\ge 1\,,$$

$$I_\Phi(\lambda z)\ge I_\Phi(\lambda_k z)\ge\sum_{i=n_k}^{m_{k+1}-1}\Phi_i(\lambda_k x_i)\ge 1\,.$$

These inequalities prove that $\|y\|_\Phi=\|z\|_\Phi=1$. The other assertions are obvious, so the proof is finished.

PROPOSITION 2. *Let* $x\in S(\ell^\Phi)$ *and* $\mathrm{Card}(\{i\in\mathbb{N}:|x_i|=a_i\})\ge 2$. *Then there are* $y,z\in S(\ell^\Phi)$ *such that* $\mathrm{supp}\,y\cap\mathrm{supp}\,z=\emptyset$ *and* $x=y+z$.

Proof. By the assumption there are $j,k\in\mathbb{N}$, $j\ne k$, such that $|x_j|=a_j$, $|x_k|=a_k$. Defining

$$y=x_ke_k\,,\qquad z=x_je_j+\sum_{i\ne j,i\ne k}x_ie_i\,,$$

we have $\mathrm{supp}\,y\cap\mathrm{supp}\,z=\emptyset$, $I_\Phi(y)\le I_\Phi(x)\le 1$ and $I_\Phi(z)\le I_\Phi(x)\le 1$. Moreover, $I_\Phi(\lambda y)>1$ and $I_\Phi(\lambda z)>1$ for any $\lambda>1$, which yields $\|y\|_\Phi=\|z\|_\Phi=1$.

Now, we are in a position to prove the main result of this paper. We first formulate, for any $x\in S(\ell^\Phi)$, two pairs of excluding cases. The first pair is

I. $|x_i|<b_i$ for any $i\in\mathbb{N}$,
II. $|x_i|=b_i$ for some $i\in\mathbb{N}$.

The second one is

(i) $I_\Phi(x)=\alpha$, where $\alpha=\sup\{I_\Phi(y):\|y\|_\Phi=1,\ \mathrm{supp}\,y\subset\mathrm{supp}\,x\}$,
(ii) $I_\Phi(x)<\alpha$.

THEOREM 1. *Let* $x\in S(\ell^\Phi)$. *Then*

1. *If* x *satisfies condition* I, *then* x *is smooth if and only if*:
   (a) $d(x,h^\Phi)<1$, *i.e.* $I_\Phi(\lambda x)<\infty$ *for some* $\lambda>1$,
   (b) $x_i\in\mathrm{Smooth}(\Phi)$ *for any* $i\in\mathbb{N}$ *or* $\mathrm{Card}(\{i\in\mathbb{N}:\partial\Phi_i(x_i)\ne\{0\}\}=1$.
2. *If* x *satisfies conditions* II *and* (i), *then* x *is smooth if and only if*:
   (a) *there is only one* $i_0\in\mathbb{N}$ *such that* $|x_{i_0}|=a_{i_0}$,

(b) *$\Phi^-(a_{i_0}) = \infty$ or $\partial\Phi_i(x_i) = \{0\}$ for any $i \in \mathbb{N}$, $i \neq i_0$, or $v \notin \ell^{\Phi^*}$ for any $v = (v_i)$ such that $v_i \in \partial\Phi_i(x_i)$ for $i = 1, 2, \ldots$,*

(c) *$d(x, \mathrm{h}^\Phi) < 1$.*

3. *If x satisfies conditions* II *and* (ii), *then x is smooth if and only if*:

(a) *there is only one $i_0 \in \mathbb{N}$ such that $|x_{i_0}| = a_{i_0}$,*

(b) *$d(x, \mathrm{h}^\Phi) < 1$.*

Proof. 1. Assume that $x \in S(\ell^\Phi)$ satisfies (a) and (b). By (a) and Lemma 1 every support functional x^* at x is regular. Therefore, in view of Lemmas 5 and 6, Card(Grad(x)) = 1 (the only support functional at x is given by (1)), i.e. x is smooth.

We now prove that (a) and (b) are necessary for x to be smooth. Assume first that (a) is not satisfied, i.e. $d(x, \mathrm{h}^\Phi) = 1$. Then in view of Proposition 1 there are $y, z \in S(\ell^\Phi)$ with disjoint supports such that $x = y + z$ and $y - z \in S(\ell^\Phi)$. By the Hahn–Banach theorem there exist $y^* \in \mathrm{Grad}(y)$, $z^* \in \mathrm{Grad}(z)$, i.e.

$$\|y^*\| = y^*(y) = 1 \quad \text{and} \quad \|z^*\| = z^*(z) = 1\,.$$

We have $y^*(y \pm z) \le \|y^*\|\|y \pm z\| = 1$, whence $y^*(y) \pm y^*(z) \le 1$, i.e. $1 \pm y^*(z) \le 1$, which yields $y^*(z) = 0$. In the same way we obtain $z^*(y) = 0$. This means that $y^* \neq z^*$. We also have

$$y^*(x) = y^*(y+z) = y^*(y) + y^*(z) = y^*(y) = 1\,,$$
$$z^*(x) = z^*(y+z) = z^*(y) + z^*(z) = z^*(z) = 1\,.$$

This means that $y^*, z^* \in \mathrm{Grad}(x)$, i.e. x is not smooth.

Assume now that (a) is satisfied but (b) is not. By (a) any $x^* \in \mathrm{Grad}(x)$ is regular. Since (b) is not satisfied, formula (1) of Lemma 5 defines at least two different functionals, i.e. x is not smooth.

2. By Lemma 1 condition (c) implies that any $x^* \in \mathrm{Grad}(x)$ is regular. Next, (b) and Lemmas 4, 5 and 6 imply that $\operatorname{supp} x^* = \{i_0\}$. Now, Lemma 4 implies that the functional x^* defined by

$$x^*(y) = y_{i_0}/x_{i_0} \qquad (\forall y = (y_i) \in \ell^\Phi)$$

is the only support functional at x, i.e. x is smooth.

Now, we prove the necessity of (a)–(c). If (a) or (c) is not satisfied, then by Propositions 1 and 2 there are $y, z \in S(\ell^\Phi)$ such that $x = y + z$ and $\|y - z\|_\Phi = 1$. Now, we can repeat the proof of condition (a) in 1 to deduce that x is not smooth. Assume that (a) and (c) are satisfied but (b) is not. Since $\Phi^-(a_{i_0}) < \infty$, $\operatorname{supp} x^*$ need not coincide with $\{i_0\}$. By the assumption there is a sequence $v = (v_i) \in \ell^{\Phi^*}$ with $v_i \in \partial\Phi_i(x_i)$ for any $i \in \mathbb{N}$ such that $v_{i_1} \neq 0$ for some $i_1 \neq i_0$. In view of Lemma 4(i), the functional

$$x^*(y) = y_{i_0}/x_{i_0} \qquad (\forall y = (y_i) \in \ell^\Phi)$$

belongs to $\mathrm{Grad}(x)$. Now, by Lemma 5 the functional

$$y^*(y) = \Big(\sum_{i=1}^{\infty} v_i y_i\Big) \Big/ \Big(\sum_{i=1}^{\infty} v_i x_i\Big) \qquad (\forall y = (y_i) \in \ell^{\Phi})$$

is also a support functional at x, different from x^* (because $\partial\Phi_{i_1}(x_{i_1}) \neq \{0\}$). Thus, x is not smooth.

3. By (b) any $x^* \in \mathrm{Grad}(x)$ is regular. Next, $\operatorname{supp} x^* = \{i_0\}$ by Lemma 3 and (a). Now, Lemma 4 shows that the functional

$$x^*(y) = y_{i_0}/x_{i_0} \qquad (\forall y = (y_i) \in \ell^{\Phi})$$

is the only support functional at x.

We now prove the necessity. For (b), it can be proved in the same way as in cases 1 and 2. So, assume (b) is satisfied but (a) is not. By (b) any $x^* \in \mathrm{Grad}(x)$ is regular. Since (a) is not satisfied there are distinct $i_0, i_1 \in \mathbb{N}$ such that $|x_{i_0}| = a_{i_0}$ and $|x_{i_1}| = a_{i_1}$. This implies that the functionals

$$x_0^*(y) = y_{i_0}/x_{i_0}\,, \qquad x_1^*(y) = y_{i_1}/x_{i_1} \qquad (\forall y = (y_i) \in \ell^{\Phi})$$

are two different elements of $\mathrm{Grad}(x)$.

Remark 1. The criterion for smoothness of $x \in S(\mathsf{h}^{\Phi})$ is almost the same as for smoothness of $x \in S(\ell^{\Phi})$. The only difference is that the condition $d(x, \mathsf{h}^{\Phi}) < 1$ need not be assumed because it is always satisfied for $x \in S(\mathsf{h}^{\Phi})$.

Note. Criteria for smoothness of Musielak–Orlicz sequence spaces ℓ^{Φ} were given in [9] (under some restrictions on Φ) and in [4] (in the general case).

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