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UNIFORMLY COMPLETELY RAMSEY SETS

BY

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Galvin and Prikry defined completely Ramsey sets and showed that the class of completely Ramsey sets forms a σ-algebra containing open sets. However, they used two definitions of completely Ramsey. We show that they are not equivalent as they remarked. One of these definitions is a more uniform property than the other. We call it the uniformly completely Ramsey property. We show that some of the results of Ellentuck, Silver, Brown and Aniszczyk concerning completely Ramsey sets also hold for uniformly completely Ramsey sets. We also investigate the relationships between uniformly completely Ramsey sets, universally measurable sets, sets with the Baire property in the restricted sense and Marczewski sets.

1. Introduction. Lebesgue measurability may be defined as follows: $M \in L$ iff M is the union of two sets, one of which is F_σ and the other is of measure zero. The Baire property in the wide sense (i.e. $M \in B_w$ iff M is the union of two sets, one of which is G_δ and the other is of first category) is a topological analogue of L. The class of completely Ramsey sets, *CR*, has meaning only in $[\omega]^\omega$. The class *CR* was first defined by Galvin and Prikry in [GP], where they proved that the Borel sets are completely Ramsey. This theorem was extended to analytic sets by Silver in [S], and Silver's proof was greatly simplified by Ellentuck in [E] and independently by Louveau in [L]. *CR* may be viewed as a combinatorial analogue of the classes L and B_w.

A set $M \subseteq [\omega]^\omega$ is *Ramsey*, or for short $M \in R$, iff there exists $U \in [\omega]^\omega$ such that $[U]^\omega \subseteq M$ or $[U]^\omega \subseteq M^c$. The class of Ramsey sets does not form a σ-algebra.

We say that $[F, U]$ is an *E-set* if F is a finite subset of ω, U is an infinite subset of ω, and $[F, U] = \{x : x \text{ is in } [\omega] \text{ and } x = F \cup V \text{ where } V \in [U] \text{ and } \min V > \max F\}$. Without loss of generality, we will assume that for an E-set $[F, U]$, $\min U > \max F$. Also, we write $[U] = [\emptyset, U]$. $M \subseteq [\omega]$ is *completely Ramsey*, or for short M is *CR*, provided that for every E-set $[F, U]$ there is an E-set $[F, V]$ such that $[F, V] \subseteq [F, U]$ and $[F, V] \subseteq M$ or $[F, V] \subseteq M^c$. M is CR_0 provided that if $[F, U]$ is an

E-set, then there is an E-set $[F,V] \subseteq [F,U]$ such that $[F,V] \subseteq M^c$. This is equivalent to M being *hereditarily CR*, i.e. every subset of M is CR.

Let $\Phi : [\omega] \to 2^\omega$ be the characteristic function. $\Phi([\omega])$ is 2^ω minus the countable set D, where D is the set of all points of 2^ω that have only finitely many 1's. We use the product topology on 2^ω, and define a topology on $[\omega]$ by just taking the preimage under Φ of open sets in 2^ω. Using this map, $[\omega]$ may be viewed as a subset of 2^ω. We also define the following set function which we will use frequently. For each U in $[\omega]$, let $\Phi^*(U) = \prod A_i$ where $A_i = \{0,1\}$ if $i \in U$, otherwise $A_i = \{0\}$. Note that $\Phi([U]) = \Phi^*(U) \setminus D$. Sometimes we will abuse the definition of Ramsey sets and completely Ramsey sets and call a set $M \subseteq 2^\omega$ Ramsey and respectively, completely Ramsey. By this, we mean that $\Phi^{-1}(M)$ is Ramsey and respectively, completely Ramsey, in $[\omega]$.

M is *universally measurable*, denoted by $M \in U$, iff M is measurable with respect to the completion of every non-atomic Borel measure on the space. Equivalently, $M \in U$ iff the image of M under every homeomorphism is Lebesgue measurable. M has the *Baire property in the restricted sense*, denoted by $M \in B_r$, iff M has the Baire property in the wide sense relative to every perfect subset of the space. The classes U and B_r are in some sense "uniformly" L and B_w, respectively. M is a *Marczewski set*, denoted by $M \in (s)$, iff for every perfect set P, there is a perfect set $Q \subseteq P$ such that $Q \subseteq M$ or $Q \subseteq M^c$. Galvin and Prikry in [GP] gave two definitions of completely Ramsey, one of which we stated earlier. They suggested that they are not equivalent. Theorem 3 shows that indeed they are not equivalent, and that one of them is a more uniform property, analogous to U and B_r. We call this property uniformly completely Ramsey. A set $M \subseteq 2^\omega$ (if $M \subseteq [\omega]$, embed it into 2^ω using Φ) is *uniformly completely Ramsey*, denoted by $M \in UCR$, iff for every continuous function $f : 2^\omega \to 2^\omega$, $f^{-1}(M)$ is Ramsey. Or, more precisely, $\Phi^{-1}(f^{-1}(M))$ is Ramsey. M is UCR_0 iff M is *hereditarily UCR*, i.e. every subset of M is UCR. M is U_0, and AFC (always first category) iff M is hereditarily U and B_r, respectively.

In Section 2, we will investigate the relationships between U, B_r, (s), UCR and S, the smallest σ-algebra containing open sets and closed under operation A. In Section 3, we state some facts about CR sets and state some open questions. Before we prove our results, we state some well known theorems concerning completely Ramsey sets.

THEOREM [Galvin–Prikry]. *Each of CR and UCR forms a σ-algebra containing open sets. Each of CR_0 and UCR_0 forms a σ-ideal.*

Proof. Refer to [GP].

THEOREM [Silver]. *Analytic sets are CR.*

Proof. Refer to [S].

THEOREM [Silver]. *Under MA, the union of less than continuum many CR-sets is CR.*

Proof. Refer to [S].

A collection G of sets is called an *A-system* provided that $G = \{M_f\}_{f\in\omega^{<\omega}}$ and that if $f, g \in \omega^{<\omega}$ and $f \subseteq g$, then $M_f \supseteq M_g$. If $f \in \omega^\omega$, then $f|k$ is the restriction of f to $\{0, 1, \ldots, k\}$. If G is an A-system, then the statement that M is the result of operation A on G means that $M = \bigcup_{f\in\omega^\omega} \bigcap_{k=0}^\infty M_{f|k}$. A collection H of sets is *closed under operation A* provided that if G is an A-system such that every member of G is in H, then the result of operation A on G is in H.

THEOREM. *If $M \in S$, the smallest σ-algebra containing opens sets and closed under operation A, then M is completely Ramsey.*

Proof. Refer to [E] or [L].

2. Relationships between S, (s), U, B_r, and UCR.

First, we prove that there is a set which is CR_0 but not UCR.

LEMMA 1. *If $U \in [\omega]$ and $M = \{V \in [\omega] : U \subseteq V\}$, then M is CR_0.*

Proof. Let $U \in [\omega]$ and $M = \{V \in [\omega] : U \subseteq V\}$. Let $[F, T]$ be an E-set. Assume that $[F, T]$ intersects M. Then $U \cap T$ is an infinite set. Let T' be an infinite subset of T such that $U \backslash (\{1, 2, \ldots, \max F\} \cup T') \neq \emptyset$. Let $p \in U \backslash (\{1, 2, \ldots, \max F\} \cup T')$. Then $[F, T'] \subseteq [F, T]$ and $[F, T'] \subseteq M^c$ because no element of $[F, T']$ contains p whereas every element of M does. Therefore, M is CR_0. ■

LEMMA 2. *If $M \subseteq [\omega]$ and M contains a perfect set, then M contains a set which is not UCR.*

Proof. Suppose $M \subseteq [\omega]$ is perfect. Let C be a Cantor set contained in M. Now, let $h : 2^\omega \to \Phi(C)$ be a homeomorphism onto $\Phi(C)$. Let B be a Bernstein set in $\Phi(C)$. Then $\Phi^{-1}(h^{-1}(B))$ is Bernstein in $[\omega]$ and therefore is not Ramsey. So, B is not UCR. ■

A set which is completely Ramsey does not have to be (s). To see this consider a Bernstein subset B of $M = \{V \in [\omega] : V$ contains positive odd integers$\}$. B is CR_0 by Lemma 1, but it is not (s). However, for the uniformly completely Ramsey sets, we have the following theorem.

THEOREM 3. *$UCR \Rightarrow CR$, $UCR_0 \Rightarrow CR_0$, and $CR_0 \not\Rightarrow UCR$, $UCR \Rightarrow (s)$, $UCR_0 \Rightarrow (s_0)$.*

P r o o f. Refer to [GP] for the proofs of the first two claims. To prove the third claim, consider the set $M = \{X \in [\omega] : X$ contains all the positive odd integers$\}$. Since M is perfect in $[\omega]$, M contains a set which is not UCR by Lemma 2, but M is CR_0 by Lemma 1. So, there exists a set which is CR_0 but not UCR.

Let us now show that $UCR \subseteq (s)$. Let M be a subset of 2^ω that is UCR. Let P be a perfect set contained in 2^ω. Let $f : 2^\omega \to 2^\omega$ be a homeomorphism onto P. Since M is UCR, $\Phi^{-1}(f^{-1}(M))$ is R. Let $U \in [\omega]$ such that $[U] \subseteq \Phi^{-1}(f^{-1}(M))$ or $[U] \subseteq (\Phi^{-1}(f^{-1}(M)))^c$. Let K be a Cantor set which is a subset of $[U]$. Then $f(\Phi(K))$ is a Cantor set which is a subset of P such that $f(\Phi(K)) \subseteq M$ or $f(\Phi(K)) \subseteq M^c$. Thus, M is Marczewski. If M is UCR_0, then by what we just proved, M is (s). And by Lemma 2, M is totally imperfect. Since all totally imperfect (s) sets are (s_0), M is (s_0). ■

LEMMA 4. *$M \subseteq 2^\omega$ is UCR if and only if for every continuous $f : 2^\omega \to 2^\omega$, $\Phi^{-1}(f^{-1}(M))$ is CR.*

P r o o f. This lemma follows from the Galvin–Prikry theorem which states that $UCR \subseteq CR$.

THEOREM 5. *UCR is closed under operation A so S is also a subclass of UCR.*

P r o o f. Let $T = \{M_f\}_{f\in\omega^{<\omega}}$ be an A-system of sets such that each element of T is UCR. Let
$$M = \bigcup_{f\in\omega^\omega} \bigcap_{k\in\omega} M_{f|k}\,.$$
We want to show that M is UCR. Let $f : 2^\omega \to 2^\omega$ be a continuous function. Then
$$\Phi^{-1}(f^{-1}(M)) = \bigcup_{n\in\omega} \bigcap_{k\in\omega} \Phi^{-1}(f^{-1}(M_{n|k}))\,.$$
By Lemma 4, $\Phi^{-1}(f^{-1}(M_{n|k}))$ is CR, and since the class CR is closed under operation A, $\Phi^{-1}(f^{-1}(M))$ is CR. Thus, $\Phi^{-1}(f^{-1}(M))$ is R for every continuous $f : 2^\omega \to 2^\omega$. So, M is UCR. ■

LEMMA 6. *Suppose $f : 2^\omega \to 2^\omega$ is a continuous function and $M \subseteq 2^\omega$. If there is $U \in [\omega]$ such that $f(\Phi^*(U)) \cap M$ is UCR, then there is $V \in [U]$ such that $[V] \subseteq \Phi^{-1}(f^{-1}(M))$ or $[V] \subseteq \Phi^{-1}(f^{-1}(M))^c$.*

P r o o f. Let h be the increasing function from ω onto U. Now, let $H : 2^\omega \to 2^\omega$ be a homeomorphism such that if $(x_0, x_1, x_2, \ldots) \in 2^\omega$, then $H((x_0, x_1, x_2, \ldots)) = (y_0, y_1, y_2, \ldots)$ where $y_n = 0$ when $n \notin U$ and if $n \in U$, then $y_n = x_{h(n)}$. $H(f)$ is a continuous function from 2^ω into 2^ω, and

$M \cap [H(f(2^\omega))]$ is *UCR*. So, there is $V' \in [\omega]$ such that

$$[V'] \subseteq \Phi^{-1}(H \circ f)^{-1}(M) \quad \text{or} \quad [V'] \subseteq ((\Phi^{-1}(H \circ f)^{-1}(M)))^c .$$

Let $V = h(V')$. Then we have $V \in [U]$ and $V \subseteq \Phi^{-1}(f^{-1}(M))$ or $V \subseteq (\Phi^{-1}(f^{-1}(M)))^c$. ∎

A set M is (s_0) iff M is (s) and totally imperfect. We have a similar theorem for the uniformly completely Ramsey sets.

THEOREM 7. *Suppose $M \subseteq 2^\omega$ is UCR. Then M is UCR_0 iff M is totally imperfect.*

Proof. Suppose M is UCR_0. Then M is totally imperfect by Lemma 2.

Assume that M is *UCR* and totally imperfect. We want to show that M is hereditarily *UCR*. So, let $K \subseteq M$, and $f : 2^\omega \to 2^\omega$. We want to show that $\Phi^{-1}(f^{-1}(K))$ is R. Since M is *UCR*, there is $U \in [\omega]$ such that

$$[U] \subseteq \Phi^{-1}(f^{-1}(M)) \quad \text{or} \quad [U] \subseteq (\Phi^{-1}(f^{-1}(M)))^c .$$

If $[U] \subseteq (\Phi^{-1}(f^{-1}(M)))^c$, then we would have $[U] \subseteq (\Phi^{-1}(f^{-1}(K)))^c$, and so $\Phi^{-1}(f^{-1}(K))$ would be R. So, let us assume that $[U] \subseteq \Phi^{-1}(f^{-1}(M))$. Since M is totally imperfect and $f(\Phi^*(U))$ is compact, $f(\Phi^*(U)) \cap M$ is countable. So, $f(\Phi^*(U)) \cap K$ is countable and therefore *UCR*. Then, by Lemma 6, there is $V \in [U]$ such that

$$[V] \subseteq \Phi^{-1}(f^{-1}(K)) \quad \text{or} \quad [V] \subseteq (\Phi^{-1}(f^{-1}(K)))^c .$$

So, we have shown that for every $K \subseteq M$ and every $f : 2^\omega \to 2^\omega$, $\Phi^{-1}(f^{-1}(K))$ is R. Therefore, M is hereditarily *UCR*. ∎

Note that, by Lemma 1, the corresponding statement for the class *CR* is not true. However, we should not expect it to be true because *CR* is similar to L and B_w, and the corresponding statements for the classes L and B_w are false.

The natural metric on 2^ω is the following: If $x = (x_1, x_2, x_3, \ldots)$ and $y = (y_1, y_2, y_3, \ldots)$ are two points in 2^ω, then $d(x, y) = \sum 2^{-i}|x_i - y_i|$. The Lebesgue measure μ on 2^ω is the product measure on 2^ω. The Lebesgue measure μ on $[\omega]$ is the measure induced on $[\omega]$ by the homeomorphism Φ. The statement that M is a *Lusin* (*Sierpiński*) *set* in $[\omega]$ means that if K is a first category set (measure zero set), then $M \cap K$ is countable. The statement that M is a *c-Lusin* (*c-Sierpi/nski*) *set* in $[\omega]$ means that if K is a first category set (measure zero set), then $M \cap K$ has cardinality less than that of the continuum. Refer to [M] for more information regarding Lusin sets and Sierpiński sets.

LEMMA 8. *Let $f : 2^\omega \to 2^\omega$ be a continuous function. Then there is $U \in [\omega]$ such that $f(\Phi^*(U))$ is of measure zero and nowhere dense in 2^ω.*

P r o o f. We use the metric on 2^ω as defined earlier. Since f is continuous on a compact metric space, f is uniformly continuous. Let $\varepsilon_n = (n+1)^{-1} \times 2^{-n-1}$ for $n \in \omega$. By uniform continuity, for each $n \in \omega$, let $m(n)$ be such that if $x, y \in 2^\omega$ and $d(x,y) < 2^{-m(n)+1}$, then $d(f(x), f(y)) < \varepsilon_n$ and $m(n+1) > m(n)$. Now, let $U = \{m(n) : n \in \omega\}$. First, we want to show that $f(\Phi^*(U))$ has measure zero. Let $\varepsilon > 0$. Let n be a positive integer such that $(n+1)^{-2} < \varepsilon$. Let $T = \{h \in 2^{m(n)}\colon$ if $j \notin \{m(0), m(1), \ldots, m(n)\}$, then $h(j) = 0\}$. Then $|T| = 2^{n+1}$. Now, for each $h \in T$, let $O_h = h \times \{0,1\} \times \{0,1\} \times \ldots$ Then O_h is an open set in 2^ω, and if x and y are in O_h, then $d(x,y) < 2^{m(n)-1}$. So, $d(f(x), f(y)) < \varepsilon_n$ for every x and y in O_h. From this it follows that $\mu(f(O_h)) < \varepsilon_n$. But $f(\Phi^*(U)) = \bigcup_{h \in T} f(O_h)$. Since there are not more than 2^{n+1} elements in T,

$$\mu(f(\Phi^*(U))) = \mu\Big(\bigcup_{h\in T} f(O_h)\Big) \le \sum_{h \in T} \mu(f(O_h)) \le 2^{n+1}\varepsilon_n < \varepsilon\,.$$

So, we have shown that $f(\Phi^*(U))$ has measure zero, and since every open subset of 2^ω has positive measure, $f(\Phi^*(U))$ is nowhere dense. ■

THEOREM 9. *Lusin sets and Sierpiński sets are* UCR_0.

P r o o f. Let $M \subseteq 2^\omega$ be a Lusin or a Sierpiński set. We want to show that M is *UCR*. Let $f : 2^\omega \to 2^\omega$ be a continuous function. By Lemma 8, there is $U \in [\omega]$ such that $f(\Phi^*(U))$ has measure zero and is of first category. So, $M \cap f(\Phi^*(U))$ is countable. Since countable sets are *UCR*, by Lemma 6, there is $V \in [U]$ such that

$$[V] \subseteq \Phi^{-1}(f^{-1}(M)) \quad \text{or} \quad [V] \subseteq (\Phi^{-1}(f^{-1}(M)))^c\,.$$

Therefore, $\Phi^{-1}(f^{-1}(M))$ is R and we conclude that M is *UCR*. Since Lusin and Sierpiński sets are hereditarily Lusin and Sierpiński, respectively, M is UCR_0. ■

COROLLARY 10. *Under CH*, *UCR does not imply L or* B_w.

P r o o f. Under the assumption of the continuum hypothesis, there are Lusin and Sierpiński sets of the cardinality of the continuum. Let A_1 and A_2 be Lusin and Sierpiński sets, respectively. Then $A_1 \cup A_2$ is *UCR* by Theorem 9, but $A_1 \cup A_2$ is neither L nor B_w. ■

Under the assumption of MA, there are no Lusin nor Sierpiński sets. We modify our Theorem 9 so we can show that c-Lusin and c-Sierpiński sets are *UCR*. This way we will get a UCR_0 set of cardinality of the continuum under MA. It will also show us where the difficulties lie in trying to construct, in ZFC, a UCR_0 set of the cardinality that of the continuum.

LEMMA 11. *Under MA*, *less than the continuum union of UCR sets is UCR.*

Proof. Let G be a collection of UCR sets such that $|G|$ is less that continuum. Let $f : 2^\omega \to 2^\omega$ be a continuous map. Then, for each $g \in G$, $f^{-1}(g)$ is CR by Lemma 4. We have $\Phi^{-1}(f^{-1}(\bigcup_{g\in G} g)) = \bigcup_{g\in G} \Phi^{-1}(f^{-1}(g))$. $\bigcup_{g\in G} \Phi^{-1}(f^{-1}(g))$ is CR by the theorem of Silver that states that the union of less than continuum many CR sets is CR [S]. So, we have shown that $\Phi^{-1}(f^{-1}(\bigcup G))$ is R, and therefore, $\bigcup G$ is CR_1. ∎

THEOREM 12. *Under MA, c-Lusin and c-Sierpiński sets are* UCR_0.

Proof. We need MA to prove this theorem, whereas we proved Theorem 9 in ZFC. The reason for needing MA for this part is that we do not know in ZFC whether every set of cardinality less than that of the continuum is UCR_0. This may possibly be true.

Let M be a c-Lusin (c-Sierpiński) set. Let $f : 2^\omega \to 2^\omega$ be a continuous function. By Lemma 8, there is $U \in [\omega]$ such that $f(\Phi^*(U))$ is of measure zero and first category. So, $f(\Phi^*(U)) \cap M$ has cardinality less than that of the continuum. But, by Lemma 11, $f(\Phi^*(U)) \cap M$ is UCR. So, by Lemma 6, there exists a $V \in [U]$ such that $[V] \subseteq \Phi^{-1}(f^{-1}(M))$ or $[V] \subseteq [\Phi^{-1}(f^{-1}(M))]^c$. Therefore, $\Phi^{-1}(f^{-1}(M))$ is R, and M is UCR_0.

COROLLARY 13. *Under MA, there is a* UCR_0 *set of cardinality that of the continuum.*

3. Some facts about *CR* sets and questions. Aniszczyk *et al.* in [AFP] showed that under MA there is a U_0 set which is not R. Brown in [B] showed that under CH there is an (s_0) set which is not R. It also follows from Example 6 in [B] that assuming CH there is an AFC set which is not R. We show that under MA there is an AFC set which is not R. We present our argument here because it uses a technique different from that of Brown.

THEOREM 14. *Under MA, there is an AFC set which is not R, therefore not CR or UCR.*

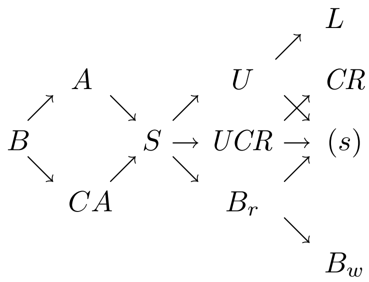
Proof. For each $U \in [\omega]$, let μ_U be a measure defined on $\Phi^*(U)$ in the following fashion. Let h be the increasing function from ω onto U. Let $H : 2^\omega \to \Phi^*(U)$ be defined in the following manner. If $(x_1, x_2, x_3, \ldots) \in 2^\omega$, then $H((x_1, x_2, x_3, \ldots)) = (y_1, y_2, y_3, \ldots)$, where $y_n = 0$ if $n \notin U$, otherwise $y_n = x_m$ where m is $h^{-1}(n)$. Let μ_U be the measure on $\Phi^*(U)$ induced by the measure on 2^ω under the homeomorphism H. Now, let $\{G_\alpha\}_{\alpha<c}$ be a well-ordering of those G_δ subsets of 2^ω that satisfy the condition that for each $U \in [\omega]$, $\mu_U(G_\alpha \cap \Phi^*(U)) = 0$. Let $\{A_\alpha\}_{\alpha<c}$ be a well-ordering of $[\omega]$. Now for each $\alpha < c$, let p_α and q_α be such that $p_\alpha \neq q_\alpha$ and $\{p_\alpha, q_\alpha\} \subseteq \Phi^*(A_\alpha) \setminus \Phi^{-1}((\bigcup_{\beta\le\alpha} G_\beta) \cup (\bigcup_{\beta<\alpha}\{p_\beta, q_\beta\}))$. Under the assumption of MA, the choice of such p_α and q_α is possible. Now, let $M = \{p_\alpha : \alpha < c\}$. Then it is clear that M is not R.

Now, we want to show that M is AFC. Let P be a perfect set in 2^ω. First, we will construct a dense G_δ subset G of P such that $\mu_U(G\cap \Phi^*(U)) = 0$ for each $U\in[\omega]$. Let $p_0, p_1, p_2, \ldots$ be a countable dense subset of P, where $p_n = (p_{n,0}, p_{n,1}, p_{n,2}, \ldots)$. For each $(k,l)\in\omega\times\omega$, let

$$U_{k,l} = \{p_{k,0}\}\times\{p_{k,1}\}\times\ldots\times\{p_{k,l}\}\times\{0,1\}\times\{0,1\}\times\ldots$$

Now, let $O_n = \bigcup_{i=0}^{\infty} U_{i,n+i}$. Let $G = P\cap(\bigcap_{i=0}^{\infty} O_n)$. Then G is G_δ in P and has the desired property that $\mu_U(G) = 0$ for each $U\in[\omega]$. Then $M\cap P = M\cap((P\backslash G)\cup G) = (M\cap(P\backslash G))\cup(M\cap G)$. Now, $M\cap(P\backslash G)$ is of first category in P, and $M\cap G$ is of first category because the cardinality of $M\cap G$ is less than that of the continuum by the construction and MA implies that if a set has cardinality less than the continuum then it is of first category. So, we see that $M\cap P$ is first category. Therefore, M is AFC in 2^ω but not R. ■

We summarize the results of this note in the following diagram.



Question 1. Is there, in ZFC, a UCR_0 set of the cardinality of the continuum?

Question 2. Ellentuck in [E] showed that the set of all E-sets forms a basis for a topology on $[\omega]$, and the class of B_w sets forms in this topology is precisely the class of CR sets. Can the class of UCR sets be characterized in this topology as perhaps B_r sets? Or may be as (s) sets?

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