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A NOTE ON GEODESIC MAPPINGS OF PSEUDOSYMMETRIC RIEMANNIAN MANIFOLDS

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1. Introduction. Let (M,g) be a connected n-dimensional , $n \geq 3$, semi-Riemannian smooth manifold. We denote by ∇, $\widetilde{R}$, R, S and κ the Levi-Cività connection, the curvature tensor, the Riemann–Christoffel curvature tensor, the Ricci tensor and the scalar curvature of (M,g), respectively. We define on M the tensor fields $R.R$ and $Q(g,R)$ by the formulas

$$\begin{aligned}
&(R.R)(X_1,X_2,X_3,X_4;X,Y)\\
&\quad= -R(\widetilde{R}(X,Y)X_1,X_2,X_3,X_4) - R(X_1,\widetilde{R}(X,Y)X_2,X_3,X_4)\\
&\qquad - R(X_1,X_2,\widetilde{R}(X,Y)X_3,X_4) - R(X_1,X_2,X_3,\widetilde{R}(X,Y)X_4)\,,\\
&Q(g,R)(X_1,X_2,X_3,X_4;X,Y)\\
&\quad= R((X\wedge Y)X_1,X_2,X_3,X_4) + R(X_1,(X\wedge Y)X_2,X_3,X_4)\\
&\qquad + R(X_1,X_2,(X\wedge Y)X_3,X_4) + R(X_1,X_2,X_3,(X\wedge Y)X_4)\,,
\end{aligned}$$

where

$$(X\wedge Y)Z = g(Y,Z)X - g(X,Z)Y\,,$$

$X,Y,Z,X_1,\ldots,X_4 \in \Xi(M)$, $\Xi(M)$ being the Lie algebra of vector fields on M.

A semi-Riemannian manifold (M,g) is said to be *pseudosymmetric* ([4]) if at every point of M the following condition is satisfied:

(∗) the tensors $R.R$ and $Q(g,R)$ are linearly dependent.

A semi-Riemannian manifold (M,g) is pseudosymmetric if and only if

$$R.R = LQ(g,R) \tag{1}$$

on the set $U_R = \{x \in M \mid Z(R) \neq 0 \text{ at } x\}$, where L is a function on U_R and

$$Z(R) = R - \frac{\kappa}{n(n-1)}G$$

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with G defined by

$$G(X_1, X_2, X_3, X_4) = g((X_1 \wedge X_2)X_3, X_4)\,,$$

$X_1, \ldots, X_4 \in \Xi(M)$.

If $R.R = 0$ on M, then the manifold (M, g) is called *semisymmetric* ([9]). The local and global structure of Riemannian semisymmetric manifolds was described in [9] and [10]. The class of pseudosymmetric manifolds is essentially wider than the class of semisymmetric manifolds (cf. [3], [4], [2], [7]). The study of totally umbilical submanifolds of semisymmetric manifolds as well as the consideration of geodesic mappings onto semisymmetric manifolds lead to the concept of pseudosymmetric manifolds (see [1], [5], [6], [8]).

In the survey paper [8] the following theorem is presented.

THEOREM 1.1 ([8], Theorem 3). *If (M, g) is a pseudosymmetric semi-Riemannian manifold admitting a non-trivial geodesic mapping f onto a manifold $(\overline{M}, \overline{g})$ then $(\overline{M}, \overline{g})$ is also a pseudosymmetric manifold.*

Unfortunately, the proof of this theorem is not published. On the other hand, Theorem 1.1 is very important in the study of pseudosymmetric manifolds. In this paper we give a proof of this theorem.

2. Preliminaries. Let (M, g), $n = \dim M \geq 3$, be a semi-Riemannian manifold covered by a system of coordinate neighbourhoods $\{V; x^j\}$. We denote by Γ^h_{ij}, g_{ij}, R^h_{ijk}, R_{hijk} and S_{ij} the local components of the Levi-Cività connection ∇ and the local components of the tensors g, $\widetilde{R}$, R and S, respectively. Further, we denote by

$$\begin{aligned}(2)\quad (R.R)_{hijklm} &= \nabla_m \nabla_l R_{hijk} - \nabla_l \nabla_m R_{hijk} \\ &= -R_{rijk}{R^r}_{hlm} - R_{hrjk}{R^r}_{ilm} - R_{hirk}{R^r}_{jlm} - R_{hijr}{R^r}_{klm}\,,\end{aligned}$$

$$\begin{aligned}(3)\quad Q(g, R)_{hijklm} &= g_{hm}R_{lijk} + g_{im}R_{hljk} + g_{jm}R_{hilk} + g_{km}R_{hijl} \\ &\quad - g_{hl}R_{mijk} - g_{il}R_{hmjk} - g_{jl}R_{himk} - g_{kl}R_{hijm}\,,\end{aligned}$$

the local components of the tensors $R.R$ and $Q(g, R)$, respectively.

For a $(0, 2)$-tensor field A on (M, g) one defines the endomorphism $X \wedge_A Y$ of $\Xi(M)$ by the formula

$$(X \wedge_A Y)Z = A(Y, Z)X - A(X, Z)Y$$

where $X, Y, Z \in \Xi(M)$. In particular, we have

$$X \wedge_g Y = X \wedge Y\,.$$

Further, for a $(0, k)$-tensor field T on (M, g), $k \geq 1$, we define the tensor field $Q(A, T)$ by the formula

$$Q(A, T)(X_1, \ldots, X_k; X, Y) = T((X \wedge_A Y)X_1, X_2, \ldots, X_k)$$

$$+ T(X_1, (X \wedge_A Y)X_2, \ldots, X_k) + \ldots + T(X_1, \ldots, X_{k-1}, (X \wedge_A Y)X_k)\,,$$

where $X, Y, X_1, \ldots, X_k \in \Xi(M)$. Evidently, putting in the above formula $A = g$, $T = R$ we obtain the tensor field $Q(g, R)$.

Let (M, g) and $(\overline{M}, \overline{g})$ be two n-dimensional semi-Riemannian manifolds. A diffeomorphism $f : M \to \overline{M}$ which maps geodesic lines into geodesic lines is called a *geodesic mapping*. It is known that in a common coordinate system $\{x^1, \ldots, x^n\}$, Christoffel symbols and curvature tensors of (M, g) and $(\overline{M}, \overline{g})$ are related by

$$\overline{\Gamma}^h_{ij} = \Gamma^h_{ij} + \delta^h_i \psi_j + \delta^h_j \psi_i\,, \tag{4}$$

$$\overline{R}^h{}_{ijk} = R^h{}_{ijk} + \delta^h_j \psi_{ik} - \delta^h_k \psi_{ij}\,, \tag{5}$$

where

$$\psi_{ij} = \nabla_j \psi_i - \psi_i \psi_j\,, \tag{6}$$

$$\psi_i = \frac{1}{2(n+1)} \frac{\partial}{\partial x^i} \left(\log \left| \frac{\det \overline{g}}{\det g} \right| \right)\,. \tag{7}$$

In the sequel such a geodesic mapping of (M, g) onto $(\overline{M}, \overline{g})$ will be denoted by $f : (M, g) \xrightarrow{\psi} (\overline{M}, \overline{g})$ and the manifolds (M, g) and $(\overline{M}, \overline{g})$ will be called *geodesically related*. A geodesic mapping $f : (M, g) \xrightarrow{\psi} (\overline{M}, \overline{g})$ is called *non-trivial* on M if the covector field ψ with the local components ψ_i is non-zero.

Remark. If $f : (M, g) \xrightarrow{\psi} (\overline{M}, \overline{g})$ is a geodesic mapping, then $f(U_R) = U_{\overline{R}}$. We can prove this using the fact that the Weyl projective curvature tensor W, defined by

$$W(X, Y)Z = \widetilde{R}(X, Y)Z - \frac{1}{n-1}(S(Y, Z)X - S(X, Z)Y)\,,$$

$X, Y, Z \in \Xi(M)$, is invariant under geodesic mappings and that W vanishes at a point of M if and only if $Z(R)$ vanishes at this point.

LEMMA 2.1. *Let $f : (M, g) \xrightarrow{\psi} (\overline{M}, \overline{g})$ be a geodesic mapping of a pseudosymmetric manifold (M, g) onto a manifold $(\overline{M}, \overline{g})$ and let the condition $R.R = LQ(g, R)$ be satisfied on U_R. Then in a common coordinate system $\{x^1, \ldots, x^n\}$ on U_R and $U_{\overline{R}}$*

$$\begin{aligned}(\overline{R}.\overline{R})_{hijklm} &= \frac{1}{n}\eta(\overline{g}_{hl}\overline{R}_{mijk} - \overline{g}_{hm}\overline{R}_{lijk}) \\ &+ B_{il}\overline{R}_{hmjk} - B_{im}\overline{R}_{hljk} + B_{jl}\overline{R}_{himk} - B_{jm}\overline{R}_{hilk} + B_{lk}\overline{R}_{hijm} - B_{km}\overline{R}_{hijl} \\ &+ F_{il}\overline{G}_{hmjk} - F_{im}\overline{G}_{hljk} - F_{jl}\overline{G}_{himk} - F_{jm}\overline{G}_{hilk} + F_{lk}\overline{G}_{hijm} - F_{km}\overline{G}_{hijl}\,,\end{aligned} \tag{8}$$

where

$$B_{ij} = -Lg_{ij} + \psi_{ij}\,,$$

$$F_{ij} = -\frac{1}{n}A_{ij} - \frac{1}{n}L\overline{g}^{rs}(\psi_{rs}g_{ij} - g_{rs}\psi_{ij}),$$
$$A_{ij} = \psi_{ir}\overline{S}^r{}_j - \psi_{rs}\overline{R}^r{}_{ij}{}^s\,,$$
$$\eta = \overline{g}^{rs}(-g_{rs}L + \psi_{rs})\,.$$

P r o o f. Using the Ricci identity and (5) we obtain

$$\begin{aligned}
&\overline{\nabla}_m\overline{\nabla}_l\overline{R}^s{}_{ijk} - \overline{\nabla}_l\overline{\nabla}_m\overline{R}^s{}_{ijk} = \nabla_m\nabla_l R^s{}_{ijk} - \nabla_l\nabla_m R^s{}_{ijk}\\
&\quad + \psi_{il}R^s{}_{mjk} - \psi_{im}R^s{}_{ljk} + \psi_{jl}R^s{}_{imk} - \psi_{jm}R^s{}_{ilk} + \psi_{kl}R^s{}_{ijm} - \psi_{km}R^s{}_{ijl}\\
&\quad + \delta^s_k(\psi_{ir}R^r{}_{jlm} + \psi_{jr}R^r{}_{ilm}) - \delta^s_j(\psi_{ir}R^r{}_{klm} + \psi_{kr}R^r{}_{ilm})\\
&\quad + \delta^s_l\psi_{mr}R^r{}_{ijk} - \delta^s_m\psi_{lr}R^r{}_{ijk}\,,
\end{aligned}$$

which, by making use of (5), $(R.R)^s{}_{ijklm} = LQ(g,R)^s{}_{ijklm}$ and (3), turns into

$$\begin{aligned}
&\overline{\nabla}_m\overline{\nabla}_l\overline{R}^h{}_{ijk} - \overline{\nabla}_l\overline{\nabla}_m\overline{R}^h{}_{ijk} = -L(\delta^h_l R_{mijk} - \delta^h_m R_{lijk})\\
&\quad + \delta^h_l E_{mijk} - \delta^h_m E_{lijk} + \delta^h_k(E_{ijlm} + E_{jilm}) - \delta^h_j(E_{iklm} + E_{kilm})\\
&\quad + \psi_{il}\overline{R}^h{}_{mjk} - \psi_{im}\overline{R}^h{}_{ljk} + \psi_{jl}\overline{R}^h{}_{imk} - \psi_{jm}\overline{R}^h{}_{ilk} + \psi_{kl}\overline{R}^h{}_{ijm} - \psi_{km}\overline{R}^h{}_{ijl}\\
&\quad - L(g_{il}(\overline{R}^h{}_{mjk} + \delta^h_k\psi_{mj} - \delta^h_j\psi_{mk}) - g_{im}(\overline{R}^h{}_{ljk} + \delta^h_k\psi_{lj} - \delta^h_j\psi_{lk})\\
&\quad + g_{jl}(\overline{R}^h{}_{imk} + \delta^h_k\psi_{im} - \delta^h_m\psi_{ik}) - g_{jm}(\overline{R}^h{}_{ilk} + \delta^h_k\psi_{il} - \delta^h_l\psi_{ik})\\
&\quad + g_{kl}(\overline{R}^h{}_{ijm} + \delta^h_m\psi_{ij} - \delta^h_j\psi_{im}) - g_{km}(\overline{R}^h{}_{ijl} + \delta^h_l\psi_{ij} - \delta^h_j\psi_{il}))\,,
\end{aligned}$$

where

$$E_{mijk} = \psi_{mr}\overline{R}^r{}_{ijk}\,.$$

But this, by contraction with $\overline{g}_{hs}$, gives

$$\begin{aligned}
(9)\qquad &(\overline{R}.\overline{R})_{hijklm} = -L(\overline{g}_{hl}R_{mijk} - \overline{g}_{hm}R_{lijk})\\
&\quad + \overline{g}_{hl}E_{mijk} - \overline{g}_{hm}E_{lijk} + \overline{g}_{hk}(E_{ijlm} + E_{jilm}) - \overline{g}_{hj}(E_{iklm} + E_{kilm})\\
&\quad + \psi_{il}\overline{R}_{hmjk} - \psi_{im}\overline{R}_{hljk} + \psi_{jl}\overline{R}_{himk} - \psi_{jm}\overline{R}_{hilk} + \psi_{kl}\overline{R}_{hijm} - \psi_{km}\overline{R}_{hijl}\\
&\quad - L(g_{il}(\overline{R}_{hmjk} + \overline{g}_{hk}\psi_{mj} - \overline{g}_{hj}\psi_{mk}) - g_{im}(\overline{R}_{hljk} + \overline{g}_{hk}\psi_{lj} - \overline{g}_{hj}\psi_{lk})\\
&\quad + g_{jl}(\overline{R}_{himk} + \overline{g}_{hk}\psi_{im} - \overline{g}_{hm}\psi_{ik}) - g_{jm}(\overline{R}_{hilk} + \overline{g}_{hk}\psi_{il} - \overline{g}_{hl}\psi_{ik})\\
&\quad + g_{kl}(\overline{R}_{hijm} + \overline{g}_{hm}\psi_{ij} - \overline{g}_{hj}\psi_{im}) - g_{km}(\overline{R}_{hijl} + \overline{g}_{hl}\psi_{ij} - \overline{g}_{hj}\psi_{il}))\,.
\end{aligned}$$

Symmetrizing (9) in h, i we obtain

$$\begin{aligned}
&\overline{g}_{hl}E_{mijk} - \overline{g}_{hm}E_{lijk} + \overline{g}_{il}E_{mhjk} - \overline{g}_{im}E_{lhjk}\\
&\quad + \overline{g}_{hk}(E_{ijlm} + E_{jilm}) + \overline{g}_{ik}(E_{hjlm} + E_{jhlm})\\
&\quad - \overline{g}_{hj}(E_{iklm} + E_{kilm}) - \overline{g}_{ij}(E_{hklm} + E_{khlm})
\end{aligned}$$

$$
\begin{aligned}
&+ \psi_{il}\overline{R}_{hmjk} - \psi_{im}\overline{R}_{hljk} + \psi_{hl}\overline{R}_{imjk} - \psi_{hm}\overline{R}_{iljk} \\
&- L(\overline{g}_{il}R_{mhjk} + \overline{g}_{hl}R_{mijk} - \overline{g}_{hm}R_{lijk} - \overline{g}_{im}R_{lhjk}) \\
&- L(g_{il}(\overline{R}_{hmjk} + \overline{g}_{hk}\psi_{mj} - \overline{g}_{hj}\psi_{mk}) + g_{hl}(\overline{R}_{imjk} + \overline{g}_{ik}\psi_{mj} - \overline{g}_{ij}\psi_{mk}) \\
&- g_{im}(\overline{R}_{hljk} + \overline{g}_{hk}\psi_{lj} - \overline{g}_{hj}\psi_{lk}) - g_{hm}(\overline{R}_{iljk} + \overline{g}_{ik}\psi_{lj} - \overline{g}_{ij}\psi_{lk}) \\
&+ g_{jl}(\overline{g}_{hk}\psi_{im} - \overline{g}_{im}\psi_{hk} + \overline{g}_{ik}\psi_{hm} - \overline{g}_{hm}\psi_{ik}) \\
&- g_{jm}(\overline{g}_{hk}\psi_{il} - \overline{g}_{il}\psi_{hk} + \overline{g}_{ik}\psi_{hl} - \overline{g}_{hl}\psi_{ik}) \\
&+ g_{kl}(\overline{g}_{hm}\psi_{ij} - \overline{g}_{ij}\psi_{hm} + \overline{g}_{im}\psi_{hj} - \overline{g}_{hj}\psi_{im}) \\
&- g_{km}(\overline{g}_{hl}\psi_{ij} - \overline{g}_{ij}\psi_{hl} + \overline{g}_{il}\psi_{hj} - \overline{g}_{hj}\psi_{il})) = 0\,.
\end{aligned}
$$

Contracting this with $\overline{g}^{hl}$ and using the identity

$$
\begin{aligned}
(\overline{R}.g)_{imjk} = &- g_{is}\overline{R}^{s}{}_{mjk} - g_{ms}\overline{R}^{s}{}_{ijk} = -g_{is}R^{s}{}_{mjk} - g_{ms}R^{s}{}_{ijk} \\
&- g_{is}(\delta^s_j\psi_{mk} - \delta^s_k\psi_{mj}) - g_{ms}(\delta^s_j\psi_{ik} - \delta^s_k\psi_{ij})
\end{aligned}
$$

we get

$$
\begin{aligned}
(10)\quad &(n+1)E_{mijk} + E_{jikm} + E_{kimj} \\
&+ \overline{g}_{ik}A_{jm} - \overline{g}_{ij}A_{km} + \eta\overline{R}_{imjk} - nLR_{mijk} \\
&- L(n(g_{jm}\psi_{ik} - g_{km}\psi_{ij}) + \overline{g}^{rs}g_{rs}(\overline{g}_{ik}\psi_{mj} - \overline{g}_{ij}\psi_{mk}) \\
&+ \overline{g}^{rs}\psi_{rs}(g_{km}\overline{g}_{ij} - g_{jm}\overline{g}_{ik}) + \overline{g}_{ij}D_{km} + \overline{g}_{ik}D_{mj} + \overline{g}_{im}D_{jk}) = 0\,,
\end{aligned}
$$

where

$$D_{ij} = g_{jr}\overline{g}^{rs}\psi_{si} - g_{ir}\overline{g}^{rs}\psi_{sj}\,.$$

Next, permuting (10) cyclically in the indices m, j, k, we obtain

$$
\begin{aligned}
(11)\qquad (n+3)(E_{mijk} + E_{kimj} + E_{jikm}) + \overline{g}_{ik}\widetilde{A}_{jm} + \overline{g}_{im}\widetilde{A}_{kj} + \overline{g}_{ij}\widetilde{A}_{mk} \\
-3L(\overline{g}_{ij}D_{km} + \overline{g}_{ik}D_{mj} + \overline{g}_{im}D_{jk}) = 0\,,
\end{aligned}
$$

which, by contraction with $\overline{g}^{ij}$, yields

$$(12)\qquad (2n+1)\widetilde{A}_{mk} = -3(n-2)LD_{mk}\,,$$

where

$$\widetilde{A}_{ij} = A_{ij} - A_{ji}\,.$$

On the other hand, (10), together with (11), implies

$$
\begin{aligned}
(13)\qquad &nE_{mijk} + \overline{g}_{ik}A_{jm} - \overline{g}_{ij}A_{km} \\
&- \frac{1}{n+3}(\overline{g}_{ik}\widetilde{A}_{jm} + \overline{g}_{im}\widetilde{A}_{kj} + \overline{g}_{ij}\widetilde{A}_{mk}) \\
&- \frac{n}{n+3}L(\overline{g}_{ik}D_{mj} + \overline{g}_{im}D_{jk} + \overline{g}_{ij}D_{km}) \\
&+ \eta\overline{R}_{imjk} - nLR_{mijk} \\
&- L(n(g_{jm}\psi_{ik} - g_{km}\psi_{ij}) + \overline{g}^{rs}g_{rs}(\overline{g}_{ik}\psi_{mj} - \overline{g}_{ij}\psi_{mk})
\end{aligned}
$$

$$+ \overline{g}^{rs}\psi_{rs}(g_{km}\overline{g}_{ij} - g_{im}\overline{g}_{jk})) = 0\,.$$

Contracting this with $\overline{g}^{ij}$ and antisymmetrizing the resulting equality we find

$$(2n+1)(n+1)\widetilde{A}_{mk} = -n(3n-1)\,LD_{mk}\,,$$

which, by (12), gives

$$\widetilde{A}_{mk} = LD_{mk} = 0\,.$$

Now (13) turns into

$$\begin{aligned} E_{mijk} = {} & \frac{1}{n}(\overline{g}_{ij}A_{km} - \overline{g}_{ik}A_{jm}) + \frac{1}{n}\eta\overline{R}_{mijk} + LR_{mijk} \\ & + L\bigg((g_{jm}\psi_{ik} - g_{km}\psi_{ij}) + \frac{1}{n}\overline{g}^{rs}g_{rs}(\overline{g}_{ik}\psi_{mj} - \overline{g}_{ij}\psi_{mk}) \\ & \qquad + \frac{1}{n}\overline{g}^{rs}\psi_{rs}(g_{km}\overline{g}_{ij} - g_{im}\overline{g}_{ik})\bigg)\,. \end{aligned}$$

Finally, substituting this in (9), we obtain our assertion.

3. Main result

PROPOSITION 3.1. *Let* $f : (M,g)\overset{\psi}{\longrightarrow}(\overline{M},\overline{g})$ *be a geodesic mapping of a pseudosymmetric manifold* (M,g), $\dim M \geq 3$, *onto a manifold* $(\overline{M},\overline{g})$. *Then the manifold* $(U_{\overline{R}},\overline{g})$ *is also pseudosymmetric.*

Proof. Assume that the condition (1) holds on U_R. Moreover, let $\{x^1,\ldots,x^n\}$ be a common coordinate system on U_R and $U_{\overline{R}}$. Antisymmetrizing (8) in h,i and symmetrizing the resulting equality in the pairs h,i and j,k we find

$$\begin{aligned} (14)\qquad 4(\overline{R}.\overline{R})_{hijklm} = {} & -\frac{1}{n}\eta Q(\overline{g},\overline{R})_{hijklm} \\ & - 3Q(B,\overline{R})_{hijklm} - 3Q(F,\overline{G})_{hijklm}\,. \end{aligned}$$

On the other hand, symmetrizing (8) in h,i we obtain

$$\begin{aligned} (15)\qquad 0 = {} & \widetilde{B}_{il}\overline{R}_{hmjk} - \widetilde{B}_{im}\overline{R}_{hljk} + \widetilde{B}_{hl}\overline{R}_{imjk} - \widetilde{B}_{hm}\overline{R}_{iljk} \\ & + F_{il}\overline{G}_{hmjk} - F_{im}\overline{G}_{hljk} + F_{hl}\overline{G}_{imjk} - F_{hm}\overline{G}_{iljk}\,, \end{aligned}$$

where

$$\widetilde{B}_{il} = B_{il} - \frac{1}{n}\eta\overline{g}_{il}\,.$$

We now prove that the tensor $\widetilde{B}$ with the local components $\widetilde{B}_{ij}$ is a zero tensor. Suppose that $\widetilde{B}$ is non-zero at a point. Moreover, let V be a vector at this point with local components V^i such that $V^iV^j\widetilde{B}_{ij} = \varepsilon$, $\varepsilon = \pm 1$.

Transvecting (15) with V^i and V^l and antisymmetrizing the resulting equality in h, m we obtain

$$\overline{R}_{hmjk} = -\varepsilon V^s V^r F_{sr} \overline{G}_{hmjk} ,$$

which easily gives $Z(\overline{R}) = 0$, a contradiction.

Thus we see that (15) reduces to

$$F_{il} \overline{G}_{hmjk} - F_{im} \overline{G}_{hijk} + F_{hl} \overline{G}_{imjk} - F_{hm} \overline{G}_{iljk} = 0 ,$$

which, by contraction with $\overline{g}^{hk}$ and $\overline{g}^{mi}$, yields $F_{il} = 0$. Now (14) completes the proof of our proposition.

Let $f : (M, g) \xrightarrow{\psi} (\overline{M}, \overline{g})$ be a geodesic mapping of a manifold (M, g) onto a manifold $(\overline{M}, \overline{g})$. We note that if the tensor $Z(R)$ vanishes at a point $x \in M$ then the tensor $Z(\overline{R})$ also vanishes at the point $f(x) \in \overline{M}$. This remark, together with Proposition 3.1, implies the assertion of Theorem 1.1.

REFERENCES

[1] A. Adamów and R. Deszcz, *On totally umbilical submanifolds of some class of Riemannian manifolds*, Demonstratio Math. 16 (1983), 39–59.

[2] J. Deprez, R. Deszcz and L. Verstraelen, *Examples of pseudosymmetric conformally flat warped products*, Chinese J. Math. 17 (1989), 51–65.

[3] R. Deszcz, *On pseudosymmetric warped product manifolds*, J. Geom., to appear.

[4] R. Deszcz and W. Grycak, *On some class of warped product manifolds*, Bull. Inst. Math. Acad. Sinica 15 (1987), 311–322.

[5] R. Deszcz and M. Hotloś, *On geodesic mappings in pseudosymmetric manifolds*, ibid. 16 (1988), 251–262.

[6] —, —, *Notes on pseudosymmetric manifolds admitting special geodesic mappings*, Soochow J. Math. 15 (1989), 19–27.

[7] R. Deszcz, L. Verstraelen and L. Vrancken, *On the symmetry of warped product spacetimes*, Gen. Relativity Gravitation, in print.

[8] J. Mikesh, *Geodesic mappings of special Riemannian spaces*, in: Topics in Differential Geometry (Hajduszoboszló 1984), Colloq. Math. Soc. János Bolyai 46, Vol. II, North-Holland, Amsterdam 1988, 793–813.

[9] Z. I. Szabó, *Structure theorems on Riemannian spaces satisfying $R(X, Y) \cdot R = 0$. I. The local version*, J. Differential Geom. 17 (1982), 531–582.

[10] —, *Structure theorems on Riemannian spaces satisfying $R(X, Y) \cdot R = 0$. II. Global versions*, Geom. Dedicata 19 (1985), 65–108.

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