1. Introduction. Let $(M, g)$ be a Riemannian or pseudo-Riemannian manifold.

We denote its curvature tensor, Ricci tensor, scalar curvature and Weyl conformal curvature tensor by $R$, $S$, $K$ and $C$ respectively, while $\nabla$ stands for covariant differentiation with respect to $g$.

Nomizu and Ozeki proved the following remarkable result [11]:

**Theorem A.** In a Riemannian manifold, if $\nabla_t R = 0$ for some $t \geq 1$, then $\nabla R = 0$.

Tanno extended this theorem as follows:

**Theorem B** (see [17], Theorem 2). Let $(M, g)$ be a Riemannian manifold.

(a) If $\nabla_t S = 0$ for some $t \geq 1$, then $\nabla S = 0$.
(b) If $\nabla_t C = 0$ for some $t \geq 1$, then $\nabla C = 0$.
(c) If $\nabla_t K = 0$ for some $t \geq 1$, then $K = \text{constant}$.
(d) If $\nabla_t P = 0$ for some $t \geq 1$, then $\nabla P = 0$ and $\nabla R = 0$, where $P$ denotes the Weyl projective curvature tensor of $(M, g)$.

Moreover, investigating Riemannian manifolds with conformally related metrics, Nickerson proved

**Theorem C** (see [10], Theorem 4.1). A conformally recurrent manifold with $C \neq 0$ cannot be conformal to a Riemannian locally symmetric one.

In connection with the above theorems, an interesting question arises whether these results are valid for pseudo-Riemannian manifolds.

Unfortunately, for a 4-dimensional indefinite metric Kalgorodov has proved [8] that Theorem A fails in general.

The present paper deals with examples (Examples 1 and 2) of certain $n$-dimensional ($n \geq 4$) metrics which show that neither Theorems A, B (except case (c), which will be treated in a subsequent paper) nor Nickerson’s Theorem C remain true for indefinite metrics.
We shall also prove (Corollary 7) the existence of non-recurrent Ricci-recurrent simple conformally recurrent metrics which are not conformal to any essentially conformally symmetric one.

Throughout this paper, all manifolds under consideration are assumed to be connected and of class $C^\infty$.

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2. Preliminaries. In the sequel we shall need the following definitions and lemmas:

An $n$-dimensional ($n \geq 4$) Riemannian or pseudo-Riemannian manifold is called conformally symmetric [2] if its Weyl conformal curvature tensor
\begin{equation}
C_{hijk} = R_{hijk} - \frac{1}{n-2} (g_{ij}S_{hk} - g_{ik}S_{hj} + g_{hk}S_{ij} - g_{hj}S_{ik})
\end{equation}

\begin{equation}
+ \frac{K}{(n-1)(n-2)} (g_{ij}g_{hk} - g_{ih}g_{jk})
\end{equation}
is parallel, i.e. if $\nabla C = 0$.

Clearly, the class of conformally symmetric manifolds contains all locally symmetric ones as well as all conformally flat manifolds of dimension $n \geq 4$.

The existence of essentially conformally symmetric manifolds, i.e. conformally symmetric manifolds which are neither conformally flat nor locally symmetric, will be shown in Section 3 (see also [3] and [4]). Such manifolds cannot have definite metrics [5].

Let $M$ be a manifold with a (possibly indefinite) metric $g$. A smooth tensor field $T$ on $M$ will be called recurrent if
\begin{equation}
T_{i_1 \ldots i_q}T_{j_1 \ldots j_q, l} = T_{i_1 \ldots i_q, l}T_{j_1 \ldots j_q},
\end{equation}
where the comma denotes (here and in the sequel) covariant differentiation with respect to $g$.

Every parallel tensor field is therefore recurrent.

Condition (2) states that at each point $x \in M$ such that $T(x) \neq 0$ there exists a (unique) covariant vector $a$ (called the recurrence vector of $T$) which satisfies
\begin{equation}
T_{i_1 \ldots i_q, l} = a_l T_{i_1 \ldots i_q}.
\end{equation}

The above definition of recurrency differs slightly from the classical one, i.e. that given by (3). Obviously, both definitions are equivalent on the subset of $M$ where $T$ does not vanish.

A Riemannian or pseudo-Riemannian manifold $(M, g)$ will be called recurrent [18] (Ricci-recurrent [12]) if its curvature tensor (Ricci tensor) is recurrent.
Following Adati and Miyazawa [1], an \( n \)-dimensional \((n \geq 4)\) manifold with a possibly indefinite metric will be called \textit{conformally recurrent} if its Weyl conformal curvature tensor is recurrent.

Clearly, the class of conformally recurrent manifolds contains all conformally symmetric ones as well as all recurrent manifolds of dimension \( n \geq 4 \).

A conformally recurrent manifold \((M, g)\) is said to be \textit{simple} if its metric is locally conformal to a non-conformally flat conformally symmetric one, i.e., if for each point \( x \in M \) there exist a neighbourhood \( U \) of \( x \) and a function \( p \) on \( U \) such that \( \bar{g} = (\exp 2p)g \) is a non-conformally flat conformally symmetric metric.

Obviously, every non-conformally flat conformally symmetric manifold is necessarily simple conformally recurrent.

Simple conformally recurrent manifolds can be characterized as follows:

\textbf{Lemma 1} (see [14], Theorem 1). A conformally recurrent manifold is simple conformally recurrent if and only if (i) \( C \neq 0 \) everywhere (which, in view of (2), implies

\[ C_{hi,jk,l} = a_l C_{hi,jk} \]

for some vector field \( a_j \), the recurrence vector of \( C \)), (ii) the recurrence vector is locally a gradient \((a_{i,j} = a_{j,i})\), and (iii) the Ricci tensor \( S \) is a Codazzi tensor \((S_{ij,l} = S_{il,j})\).

The existence of non-simple conformally recurrent metrics with \( C \neq 0 \) has been established in [15].

In the general case, we have

\textbf{Lemma 2} (see [13], Theorem 1). Let \((M, g)\) be conformally recurrent. If \( M \) admits a function \( p \) such that \((M, \bar{g})\) with \( \bar{g} = (\exp 2p)g \) is conformally recurrent, then

\[ p_i C_{h,ijk} + p_j C_{h,ikl} + p_k C_{h,ilj} = 0 \]

everywhere on \( M \), \( p_j = \partial_j p \).

(h) At each point \( x \in M \) such that \( C(x) \neq 0 \) we have \( \overline{a}_j = a_j - 4p_j \) and \( p_i p_i = 0 \), \( \overline{a}_j \) and \( a_j \) being the recurrence vectors of \( \overline{C} \) and \( C \) respectively.

\textbf{Lemma 3} (see [13], Theorem 2). Let \((M, g)\) be conformally recurrent. If \( p \) is a function on \( M \) satisfying condition (e), then \((M, \bar{g})\) with \( \bar{g} = (\exp 2p)g \) is conformally recurrent.

\textbf{Lemma 4} (see [13], Theorem 3). Let \((M, g)\) and \((M, \bar{g})\) be conformally symmetric. If \( \bar{g} = (\exp 2p)g \) and \( p \) is a non-constant function on \( M \), then both \((M, g)\) and \((M, \bar{g})\) are conformally flat.

The following lemma is a generalization of a result of Matsumoto [9]:
Lemma 5. Let \((M, g)\) be a Riemannian or pseudo-Riemannian manifold with \(\dim M \geq 3\). Then for each \(t \geq 1\),
\[
\nabla^t P = 0 \quad \text{if and only if} \quad \nabla^t R = 0.
\]
Proof. Suppose that \(\nabla^t P = 0\). Then, by the definition of \(P\), we have
\[
R_{h_1j_1\ldots q_t} = \frac{1}{n-1} (g_{h_1} S_{i_1j_1\ldots q_t} - g_{q_t} S_{i_tj_1\ldots q_t}),
\]
whence,
\[
S_{h_1j_1\ldots q_t} = 0.
\]
But the last equation, in view of \(S_{r_1r_2\ldots q_t} = \frac{1}{2} K_{r_1r_2\ldots q_t}\), implies \(\frac{1}{n} K_{r_1r_2\ldots q_t} = \frac{1}{n} K_{s_1s_2\ldots q_t}\). Hence, \(S_{h_1j_1\ldots q_t} = 0\) and, consequently, \(\nabla^t R = 0\). The converse implication is trivial. This completes the proof.

Remark 1. Lemma 5 seems to belong to the folklore. We have included its proof for completeness.

Lemma 6. Let \(g_{ij} = (\exp 2p)g_{ij}\). Then we have ([7], pp. 89–90):
\[
\begin{align*}
S_{ij} & = S_{ij} + (n-2)(p_{i,j} - p_{j}p_{i}) + (p_{r}^{r} + (n-2)p_{r}^{r})g_{ij}, \\
C_{ij}^{h} & = C_{ij}^{h},
\end{align*}
\]
where \(p^{h} = g^{hr}p_{r}\).

3. Basic examples. The following definitions will be convenient:
Let \((M, g)\) be a pseudo-Riemannian manifold. If its curvature (Ricci) tensor satisfies \(\nabla^t R = 0\) (\(\nabla^t S = 0\)) for some \(t \geq 2\) and \(\nabla^{t-1} R\) (\(\nabla^{t-1} S\)) does not vanish everywhere, then \((M, g)\) is called \(t\)-symmetric (Ricci \(t\)-symmetric). Similarly, if for the Weyl conformal (projective) curvature tensor the condition \(\nabla^t C = 0\) (\(\nabla^t P = 0\)) holds for some \(t \geq 2\) and \(\nabla^{t-1} C\) (\(\nabla^{t-1} P\)) does not vanish everywhere, then \((M, g)\) is said to be conformally \(t\)-symmetric (projectively \(t\)-symmetric).
In this section each Latin index runs over \(1, 2, \ldots, n\), and each Greek index over \(2, 3, \ldots, n-1\). Moreover, the comma (as well as \(\nabla\)) denotes covariant differentiation with respect to \(g\).

Example 1. Let \(M\) denote the Euclidean \(n\)-space \((n \geq 4)\) endowed with the indefinite metric \(g_{ij}\) given by
\[
\begin{align*}
g_{ij} dx^{i} dx^{j} & = Q(dx^{1})^{2} + k_{\lambda\mu} dx^{\lambda} dx^{\mu} + 2dx^{1} dx^{n}, \\
Q & = (Ak_{\lambda\mu} + c_{\lambda\mu})x^{\lambda} x^{\mu},
\end{align*}
\]
where \([k_{\lambda\mu}]\) is an arbitrary symmetric non-singular constant matrix, \([C_{\lambda\mu}]\) is an arbitrary symmetric non-zero constant matrix satisfying \(k^{\alpha\beta}c_{\alpha\beta} = 0\) with \([k_{\lambda\mu}] = [k_{\lambda\mu}]^{-1}\), and \(A\) is an arbitrary smooth non-constant function of \(x^1\) only. Then:

(i) \(M\) is essentially conformally symmetric.

(ii) \(M\) is Ricci-recurrent and its scalar curvature vanishes everywhere.

(iii) \(M\) is not recurrent, but for each \(x \in M\) such that \((\nabla R)(x) \neq 0\) there exists a vector \(b\) which satisfies \(R_{hijk,lm} = b_mR_{hijk,l}\). The last condition states that \(\nabla R\) is recurrent.

(iv) If

\[
A = \sum_{l=0}^{t-1} q_i(x^1)^l
\]

where \(t \geq 2\), \(q_i = \text{const.} (i = 0, 1, \ldots, t - 1)\) and \(q_{t-1} \neq 0\), then \(M\) is \(t\)-symmetric and Ricci \(t\)-symmetric.

**Proof.** One can easily check that in the metric (8) the only Christoffel symbols not identically zero are

\[
\begin{align*}
\{ &\lambda_1 1 1\} = -\frac{1}{2} k^{\lambda\omega}Q_{\omega}, \\
\{ &n_1 1 1\} = \frac{1}{2} Q_1, \\
\{ &n_1 1 \lambda\} = \frac{1}{2} Q_{\lambda}
\end{align*}
\]

where the dot denotes partial differentiation with respect to coordinates.

Moreover, in view of the formula

\[
R_{hijk} = \frac{1}{2}(g_{hk,ij} + g_{ij,hk} - g_{hj,ik} - g_{ik,hj})
\]

it follows that the only components \(R_{hijk}\) not identically zero are ([16], p. 179)

\[
R_{1\lambda\mu} = \frac{1}{2} Q_{\lambda\mu}.
\]

It can also be found that

\[
S_{11} = \frac{1}{2} k^{\alpha\beta}Q_{\alpha\beta}
\]

and that all other components of \(S\) are identically zero.

By an elementary computation, we can easily show that the only components of \(C, VS, \nabla R\) and \(\nabla C\) not identically zero are [14]

\[
\begin{align*}
C_{1\lambda\mu} &= \frac{1}{2} \left( Q_{\lambda\mu} - \frac{1}{n-2} k_{\lambda\mu}(k^{\alpha\beta}Q_{\alpha\beta}) \right), \\
S_{11, j} &= \frac{1}{2} k^{\alpha\beta}Q_{\alpha\beta j} ,
\end{align*}
\]

\[
\begin{align*}
R_{1\lambda\mu, j} &= \frac{1}{2} Q_{\lambda\mu j}, \\
C_{1\lambda\mu, j} &= \frac{1}{2} \left( Q_{\lambda\mu j} - \frac{1}{n-2} k_{\lambda\mu}(k^{\alpha\beta}Q_{\alpha\beta j}) \right).
\end{align*}
\]
Substituting (9) into (12), (13) and (14), we easily obtain

\begin{align}
S_{11} &= (n-2)A, \quad R_{1\lambda\mu} = Ak\lambda\mu + c\lambda\mu, \quad C_{1\lambda\mu} = c\lambda\mu, \\
S_{11,j} &= (n-2)A, \quad R_{1\lambda\mu1,j} = A_j k\lambda\mu, \quad C_{1\lambda\mu1,j} = 0,
\end{align}

which, since \(g^{11} = 0\), implies (i) and (ii).

Moreover, using (11), \(R_{1\lambda\mu1,j} = A'\delta^1_j k\lambda\mu\) and \(S_{11,j} = (n-2)A'\delta^1_j\), one can easily check that the only components of \(\nabla^t R\) and \(\nabla^t S\) not identically vanishing are

\begin{align}
R_{1\lambda\mu1,q_1\ldots q_t} &= A^{(t)} \delta^1_{q_1} \delta^1_{q_2} \ldots \delta^1_{q_t} k\lambda\mu, \\
S_{11,q_1\ldots q_t} &= (n-2)A^{(t)} \delta^1_{q_1} \delta^1_{q_2} \ldots \delta^1_{q_t},
\end{align}

where the prime \((t)\) resp. indicates the ordinary derivative (of order \(t\) resp.) with respect to \(x^1\).

Assume now that (10) holds. Then, in view of (16), we get \(\nabla^t R = 0\). Since, by (10) and (16), \(\nabla^{t-1} R\) does not vanish, \(M\) is \(t\)-symmetric. Moreover, (16) yields \(\nabla^t S = 0\), which, together with (10) and (16), shows that \(M\) is also Ricci \(t\)-symmetric.

This completes the proof of (iv).

Suppose that \(M\) is recurrent. Then, because of (15) and (2) (with \(R\) instead of \(T\)), we obtain \(c\alpha\beta k\lambda\mu = c\lambda\mu k\alpha\beta\), which, since \(k\alpha\beta c\alpha\beta = 0\) by assumption, implies \(c\lambda\mu = 0\), a contradiction. Thus, \(M\) cannot be recurrent. The second part of (iii) is an immediate consequence of \(R_{1\lambda\mu1,lm} = \frac{1}{T} A^{(t)} \delta^1_k R_{1\lambda\mu1,t}\). This completes the proof.

Hence, we have

**Corollary 1.** For each \(n \geq 4\) and for each \(t \geq 2\), there exist \(n\)-dimensional essentially conformally symmetric non-recurrent Ricci-recurrent metrics which are \(t\)-symmetric and Ricci \(t\)-symmetric.

**Remark 2.** It is easy to prove that for the metric (8), we have

\[
\text{index of } [g_{ij}] = \text{index of } [k_{\lambda\mu}] + 1,
\]

the index of a symmetric matrix being understood as the number of negative entries in its diagonal form (for details see Remark 1 of [6]).

**Remark 3.** Obviously, if \(Q = Ak\lambda\mu x^\lambda x^\mu\) \((c\lambda\mu = 0)\) and \([k_{\lambda\mu}]\) has the properties stated in Example 1, then (15) yields

\begin{align}
R_{1\lambda\mu1} &= Ak\lambda\mu, \quad S_{11} = (n-2)A, \quad C_{1\lambda\mu1} = 0, \\
S_{11,j} &= (n-2)A'\delta^1_j, \quad R_{1\lambda\mu1,t} = A'\delta^1_l k\lambda\mu.
\end{align}

Thus, in view of (10) and (16), we have
Corollary 2. For each \( n \geq 4 \) and for each \( t \geq 2 \), there exist \( n \)-dimensional conformally flat recurrent metrics which are \( t \)-symmetric and Ricci \( t \)-symmetric.

Since a parallel tensor vanishes if it vanishes at some point, Lemma 5 yields

Corollary 3. A pseudo-Riemannian manifold of dimension \( n \geq 3 \) is projectively \( t \)-symmetric if and only if it is \( t \)-symmetric.

Moreover, in view of Corollary 1, we get

Corollary 4. For each \( n \geq 4 \) and for each \( t \geq 2 \), there exist \( n \)-dimensional essentially conformally symmetric Ricci-recurrent metrics which are projectively \( t \)-symmetric. Such metrics are necessarily \( t \)-symmetric.

Example 2. Let \( M = \{(x^1, \ldots, x^n) \in \mathbb{R}^n : x^1 > 0 \text{ and } n \geq 4\} \) be endowed with the metric (8), where

\[
Q = (Ak_{\lambda\mu} + Bc_{\lambda\mu})x^\lambda x^\mu.
\]

Assume moreover that \([k_{\lambda\mu}]\) and \([c_{\lambda\mu}]\) have the properties described in Example 1, and \( A, B \) are smooth functions of \( x^1 \) only such that \( A \) does not identically vanish, \( B \neq \text{const} \), \( B \neq 0 \) everywhere and \( A \neq cB \) (\( c = \text{const} \)). Then:

(i) \( M \) is simple conformally recurrent.

(ii) \( M \) is Ricci-recurrent, non-recurrent and its scalar curvature vanishes.

(iii) If \( B = a(x^1)^{t-1} \), where \( t \geq 2 \) and \( a = \text{const.} \neq 0 \), then \( \nabla^t C = 0 \) although \( \nabla^{t-1} C \neq 0 \) everywhere.

(iv) If \( B \) is as above and

\[
A = \frac{(t-1)(t+3)}{16(x^1)^2},
\]

then \((M, g)\) admits a conformal change of metric \( g \rightarrow \bar{g} = (\exp 2p)g \) such that \((M, \bar{g})\) is locally symmetric.

Proof. Substituting (17) into (12), (13), and (14) we easily obtain

\[
S_{11} = (n-2)A, \quad R_{1\lambda\mu} = Ak_{\lambda\mu} + Bc_{\lambda\mu}, \quad C_{1\lambda\mu} = Bc_{\lambda\mu},
\]

\[
S_{11,1} = (n-2)A_t, \quad R_{1\lambda\mu,1} = A_t k_{\lambda\mu} + B_t c_{\lambda\mu}, \quad C_{1\lambda\mu,1} = B_t c_{\lambda\mu},
\]

which, because of \( C_{1\lambda\mu,1} = (\log |B|)'\delta^1_\lambda C_{1\lambda\mu} = a_t C_{1\lambda\mu} \), shows that \( M \) is conformally recurrent and its recurrence vector is given by \( a_j = (\log |B|)'\delta^1_j \).

Hence, in view of (18) and Lemma 1, \( M \) is simple conformally recurrent. Moreover, equations (18) and \( g^{11} = 0 \) show that \( M \) is Ricci-recurrent and that its scalar curvature vanishes everywhere.
Assume now that $M$ is recurrent. Then, because of (2) and (18), we get
$$(BA' - AB') \delta^1_{\lambda_\mu} = 0.$$ But this implies $A' - (B'/B)A = 0$ and, consequently, we must have $A = cB$ ($c = \text{const}.$), a contradiction. Hence, $M$ cannot be recurrent.

Using (11), (18) and $C_{1\lambda\mu_1...\lambda_l} = B'(\delta^1_{\lambda_1} \delta^1_{\lambda_2} \ldots \delta^1_{\lambda_l} \lambda_\mu)$ one can now easily check that the only components of $\nabla^C$ not identically vanishing are
$$C_{1\lambda\mu_1...\lambda_l} = B'(\delta^1_{\lambda_1} \delta^1_{\lambda_2} \ldots \delta^1_{\lambda_l} \lambda_\mu),$$ which, since $B = a(x^1)^{t-1}$ by assumption, completes the proof of (iii).

From (18) it follows that any smooth function of $x^1$ only (and in particular $p = \frac{1}{4}(t - 1) \log x^1$) satisfies condition (e) of Lemma 2.

Thus, by Lemma 3, $(M, \overline{g})$ with $\overline{g} = (\exp 2p)g = (x^1)^{(t-1)/2}g$ is conformally recurrent.

On the other hand, the recurrence vector of $(M, g)$ is given by $a_j = \frac{t-1}{x^1} \delta^1_j$, which, in view of Lemma 2, shows that $\overline{g}_j = 0$.

Hence, $(M, \overline{g})$ is conformally symmetric. It remains therefore to prove that the Ricci tensor of $(M, \overline{g})$ is parallel.

Since $p_i = \partial_i p = 0$ ($i = 2, \ldots, n$), $g^{11} = 0$ and $S_{ij} = (n - 2)A\delta^1_i \delta^1_j$, it follows that $p^r S_{ri}$ as well as $\Delta_1 p = p^r p_r$ and $\Delta_2 p = p^r \gamma$ vanish everywhere. Thus, equations (5) and (6) imply
$$\overline{S}_{ij:k} = S_{ij,k} - 2p_k S_{ij} - p_i S_{jk} - p_j S_{ik} + (n - 2)p_{i,j,k}$$
$$+ 4(n - 2)p_i p_j p_k - 2(n - 2)(p_i p_{j,k} + p_j p_{i,k} + p_k p_{i,j}),$$
where the semicolon denotes covariant differentiation with respect to $\overline{g}$. Moreover, using (11) and $p_i = 0$ ($i = 2, \ldots, n$) again, one can easily check that the only component of $\nabla \overline{g}$ not identically vanishing is
$$\overline{S}_{11:1} = S_{11,1} - 4p_1 S_{11} + (n - 2)p_{1,11} + 4(n - 2)p_1^3 - 6(n - 2)p_1 p_{1,1}.$$ But the last expression, in view of (11) and $S_{11} = (n - 2)A$, takes the form
$$\overline{S}_{11:1} = (n - 2)(p_1^{'''} - 6p_1^{''} + 4(p_1')^3 - 4A p_1' + A').$$ Using now the definitions of $p$ and $A$ one can easily verify that $\overline{g}$ is parallel. This completes the proof.

Since in the above metric $R_{1\lambda\mu_1...\lambda_l} = (A^{(t)} K_{\lambda_\mu} + B^{(t)} \xi_{\lambda_\mu}) \delta^1_{\lambda_1} \delta^1_{\lambda_2} \ldots \delta^1_{\lambda_l}$, Example 2 yields

**Corollary 5.** For each $n \geq 4$ and for each $t \geq 2$, there exist $n$-dimensional non-recurrent simple conformally recurrent Ricci-recurrent non-\(t\)-symmetric metrics which are conformally $t$-symmetric and conformal to metrics with parallel curvature tensor.

**Remark 4.** Assume that $(M, g)$ has the properties described in Example 2. If $A = \frac{1}{4}(x^1)^{-2}$ and $B = (x^1)^{-2}$, then, as one can easily verify, $(M, g)$
is recurrent. Moreover, setting \( p = -\frac{1}{2} \log x^1 \), we get \( \sigma_j = 0 \) and \( \mathcal{S}_{11,1} = 0 \). Thus, we have

**Corollary 6.** For each \( n \geq 4 \), there exist \( n \)-dimensional non-conformally flat recurrent metrics which are conformal to metrics with parallel curvature tensor (cf. [10], Corollary 4.2).

**Remark 5.** Let \( p = \frac{1}{4} (t-1) \log x^1 \). Denote by \( g \) the metric described in (iv) of Example 2. Then \( (M, \overline{g}) \), where \( \overline{g} = (\exp 2p)g \), is locally symmetric. Assume that \( q \) is a smooth function on \( M \) such that \( (M, \overline{g}_1) \) with \( \overline{g}_1 = (\exp 2q)g \) is conformally symmetric. Then, by Lemma 4, the condition \( \overline{g} = (\exp 2(p - q))\overline{g}_1 \) implies \( q = p + c \), where \( c = \text{const} \). Hence, by (19), \( (M, \overline{g}_1) \) is locally symmetric too. This yields

**Corollary 7.** For each \( n \geq 4 \) and for each \( t \geq 2 \), there exist \( n \)-dimensional non-recurrent simple conformally recurrent Ricci-recurrent non-\( t \)-symmetric metrics which are conformally \( t \)-symmetric and not conformal to any essentially conformally symmetric metric.

**Remark 6.** Nickerson’s result (Theorem C) is a consequence of Lemma 2. Indeed, the definition of a conformally recurrent manifold used in Nickerson’s paper is given by (4) with \( a_j \neq 0 \) at some point. Since the considered manifold is not conformally flat by assumption, (4) yields \( C \neq 0 \) everywhere. Assume now that \( (M, \overline{g}) \) with \( \overline{g} = (\exp 2p)g \) is conformally symmetric. Then, by Lemma 2, we must have \( pr^p = 0 \), which, since \( \sigma_j = a_j - 4p_j \) and the metric is positive definite, leads immediately to the assertion.

**REFERENCES**


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