AN EXAMPLE OF THE GAME OF BANACH AND MAZUR

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In this note an example of the game of Banach and Mazur is given in which the existence of the winning strategy for the player A depends on the presence of only one point in the set Z [1]).

Let us denote by S_n the segment

$$\left[\frac{2^{n-1}-1}{2^{n-1}}, \frac{2^n-1}{2^n}\right),$$

closed the left, and let us put

$$S = \bigcup_{n=1}^{\infty} S_{2n}.$$

We shall prove the following statements:

I. *If we add to the set S the point 1 and take the set $S \cup \{1\}$ thus obtained as the set Z in the game of Banach and Mazur, then the game will be closed to the advantage of the player A;*

II. *If we take S as the set Z in the game of Banach and Mazur, then the game will be closed to the advantage of the player B.*

As a matter of fact, let us consider the following method of play for the player A:

(i) let $a_1 = 1/2$;

(ii) if $p_n = \sum_{i=1}^{n} (a_i+b_i)$ is in a segment S_{2i}, then take for a_{n+1}, $a_{n+2}, \ldots$ so smal numbers that

$$g = \sum_{i=1}^{\infty} (a_i+b_i)$$

also lies in S_{2i} (this is possible because according to the rules of the game $a_1 > b_1 > a_2 > b_2 > \ldots$);

(iii) if p_n is in a segment S_{2i-1}, then take for a_{n+1} such a number that p_n+a_{n+1} is the left end-point of the nearest segment S_{2i} lying to the right of p_n (it follows from the definition of S that it is possible and that $a_{n+1} < (1-p_n)/2$ which implies $p_{n+1}<1$;

(iv) if $p_{n-1}+a_n > 1$, then choose any $b_n, b_{n+1}, \ldots$ in accordance with the rules of the game.

Now it is immediately seen that if the player A plays according to (i)-(iv), then either for a certain n the situation described in (ii) occurs or all numbers p_n+a_{n+1} will be the left end-points of different segments S_{2i} so that we shall have $g = 1$, which proves I.

The proof of II is analogous.

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[1]) For definitions and notations see the paper of S. Zubrzycki, *On the game of Banach and Mazur*, this volume, p. 227.