We have to investigate when the equality-sign can occur in (19). That is the case if and only if

I C is a complete subgraph of order r_1,

II each vertex of C is connected with exactly $(k-2)$ vertices of B_2.

Hence we can define the associate of a vertex of C as before and, analogously, it can be seen that the associates of different vertices of C are distinct from one another. But of course not all vertices of B_2 are now associates in general; with a suitable notation we can arrange that P_1 should be the associate of P_k, P_2 that of $P_{k+1},\dots,P_{r_1}$ that of P_{k-1+r_1}; if $r_1<k-1$, then $P_{r_1+1},\dots,P_{k-1}$ are *not* associates now. Now we form $(k-1)$ classes, the first class consisting of P_k and P_1, the second of P_{k+1} and $P_2,\dots,$ the r_1-th of P_{k+r_1-1} and P_{r_1}, and if $r_1<k-1$, each of the remaining $(k-1-r_1)$ classes consisting of the single vertices $P_{r_1+1},\dots,P_{k-1}$ respectively. In order to identify this graph G with $D(k-1+r_1,k)$ we only have to show that two vertices of different classes are always connected and two of the same class never. The second assertion follows immediately from the construction of the classes and from the notion of the associate. To see the first assertion it is sufficient to remark that, according to the construction, *all* pairs of vertices are connected by an edge except the ones in C with their associates. Hence the proof is completed.

ON THE SEPARABILITY OF TOPOLOGICAL SPACES

A SUPPLEMENT TO A PAPER OF R. SIKORSKI

BY

L. DUBIKAJTIS (TORUŃ)

R. Sikorski considers in a paper[1]) six properties of a topological space, marked as

$$(*) \qquad (B),\ (\overline{M}),\ (\underline{M}),\ (I),\ (D),\ (S).$$

It is known[2]) that the following implications are true:

$$(**) \qquad \begin{array}{ccccc} (\overline{M}) & \to & (I) & \to & (S) \\ \uparrow & & \uparrow & & \uparrow \\ (B) & \to & (\underline{M}) & \to & (D) \end{array}$$

The author's intention is to prove that these implications are the only true logical connections between the properties (*). Of course, in order to prove this it suffices to show that for each conjunction

$$(*_*^*) \qquad (P_1)(P_2)(P_3)\dots(P_6)$$

((P_k) being one of the properties (*) or its negation), which is not false owing to the implications (**), there is a topological space for which this conjunction is true.

Sikorski considers the following nine conjunctions:

(1) $(I)'(D)$,	(2) $(\overline{M})'(\underline{M})$,	(3) $(\overline{M})(D)'$,
(4) $(B)'(\overline{M})(\underline{M})$,	(5) $(\overline{M})(\underline{M})'(D)$,	(6) $(I)'(D)'(S)$,
(7) $(\overline{M})'(\underline{M})'(I)(D)$,	(8) (B),	(9) $(S)'$,

regarding them as all the cases not contradictory to (**).

The last remark is not true. A conjunction not contradictory to (**) was omitted by Sikorski.

Let $B,\overline{M},\underline{M},I,D,S$ be the classes of all topological spaces for which the respective properties (*) are true. From the implications (**) there follow certain inclusions between the classes $B,\dots,S$.

Consider the following diagram where the largest square represents the class of all topological spaces, and the remaining squares represent the classes $B,\dots,S$.

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[1]) R. Sikorski, *On the separability of topological spaces*, Colloquium Mathematicum 1 (1948), p. 279-284.

[2]) See E. Marczewski, *Séparabilité et multiplication cartésienne des espaces topologiques*, Fundamenta Mathematicae 34 (1947), p. 127-143.

Each of the 10 parts into which the class of all topological spaces is divided corresponds to one of the conjunctions $(*_*{*})$. The falsity of these conjunctions does not follow from $(**)$ since all the inclusions implied by $(**)$ are allowed for in the diagram. The parts marked by numbers (1)-(9) correspond to the respective cases (1)-(9) considered by Sikorski, whereas the part (0), corresponding to the conjunction

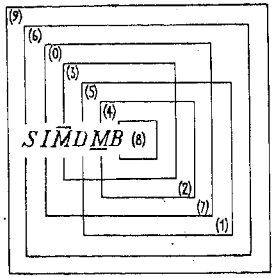

Fig. 1

$$(0) \qquad (\overline{M})'(I)(D)',$$

has been left out by him.

To supplement the proof it is necessary to give an instance of a space possessing the property (0).

R. Sikorski has proved existence of a topological space $Y_1 \hat{+} Y_2$, possessing those and only those of the properties $(*)$ which are possessed by both spaces Y_1, Y_2.

One can easily verify, that if Y_1 fulfils conjunction (2) and Y_2 fulfils conjunction (3) (and the existence of such spaces is proved by Sikorski), $Y_1 \hat{+} Y_2$ possess the property (0). Thus Sikorski's proof is completed.

ON THE REDUCIBILITY OF DECISION PROBLEMS

BY

A. JANICZAK †

EDITED BY A. GRZEGORCZYK*

The notion of general reducibility by means of machines, first mentioned briefly by Turing[1]), was defined precisely by Post[2]). Let "$X \to Y$" denote that the decision problem of the set X can be reduced to the decision problem of the set Y by means of Turing's machine.

In order to examine this relation we introduce the following definitions:

A function f is *computable with respect to a function* g if f belongs to the smallest class which:

1⁰ contains the functions: g, $x+1$, $x+y$, $Ex = x - [\sqrt{x}]^2$;

2⁰ is closed under the operations of substitution, of identification of variables, and of the effective minimum.

By the operation of *effective minimum* we mean the operation which leads from a function h to the function j defined as follows:

$$j(u) = (\min x)[h(u,x)=0] = \textit{the smallest } x \textit{ such that } h(u,x)=0$$

provided that the function h satisfies the condition

$$\prod_u \sum_x h(u,x) = 0$$

(if h does not satisfy this condition, then the operation $(\min x)$ is undefined).

Let e_X be the characteristic function of the set X. The function e_X assumes only the values $0, 1$, and

$$\prod_z e_X(z) = 1 \cdot \equiv \cdot z \in X.$$

Using these notions we can prove the

THEOREM 1. *$X \to Y$ if and only if the function e_X is computable with respect to the function e_Y.*

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* This paper and the one immediately following are posthumous works of an author who died prematurely in Warsaw on July 5, 1951. The papers were prepared for publication from the notes left by the author.

[1]) See A. M. Turing, *Systems of logic based on ordinals*, Proc. of the London Math. Soc. 45 (1939), p. 173.

[2]) See E. L. Post, *Recursively enumerable sets of positive integers and their decision problems*, Bulletin of the Am. Math. Soc. 50 (1944), p. 311.