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ON REARRANGEMENT OF SERIES, IV

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This paper is an addendum to [2]. The numbers of formulae refer to [2] and are continued. Here we solve the parts, relating to (i) series and (ii) divergent series, of problem P 378 in [1].

(a) *Find a necessary and sufficient condition* (*NSC*) *that* (5) *be true for every* (i) *series*, (ii) *divergent series*, (iii) *convergent series A satisfying* (6) *and*

(7) $$\sum_{r\leqslant n<N_r} a_{N_r} = o(1) \quad \textit{as} \quad n\to\infty.$$

(b) *Solve* (a) *with* (5) *replaced by* (1). (c) *Solve* (a) *with* (6) *and* (7) *replaced by* (2).

Following are the solutions:

(a) (i) and (ii). *An NSC is* (3).

(b) (i) and (ii). *An NSC is*

(8) $$\sum_{r>n\geqslant N_r} \frac{1}{N_r} = o(1) \quad \textit{as} \quad n\to\infty.$$

(c) (i) and (ii). *An NSC is* (3). *Also* (1) *is necessarily satisfied.*

The proofs of sufficiency are easy.

To prove necessity, in (a), (i) and (ii), let (3) be false and either

(9) $$a_{N_r} = \begin{cases} 1/N_r & \text{if} \quad r>N_r, \\ 0 & \text{otherwise} \end{cases}$$

or

(10) $$a_n = \frac{1}{n\sqrt{s_n}}, \quad \text{where} \quad s_\nu = 1+\max_{n\leqslant\nu} \sum_{r>n\geqslant N_r} \frac{1}{N_r}.$$

Then we can verify that A is divergent, (6) and (7) are true and (5) false.

To prove necessity, in (b), (i) and (ii), let (8) be false and (9) true. Then we can verify that A is divergent, (6) and (7) are true and (1) false.

To prove necessity, in (c) (i) and (ii), let (3) be false and (10) true. Then we can verify that A is divergent, (2) is true and (5) false. (1) follows from the result quoted in § 1.

Details of the proofs are omitted. Arguments similar to those in § 2 of [1] can be used.

We can also now prove the

THEOREM. *Relation* (8) *is an NSC that* (1) *be true for every* (i) *series*, (ii) *divergent series* A *satisfying* (6).

For the proof of sufficiency, see the note after Theorem 5 of [1]. To prove necessity, let (8) be false and (9) true. Then we can verify that A is divergent, (6) is true and (1) false. Details are omitted.

The above theorem improves Theorem 5 (supplemented by Remark 3) of [1], where the above result was obtained under the assumption that

$$\sum_{r\leqslant n<N_r}\frac{1}{N_r}=o(1) \quad \text{as} \quad n\to\infty.$$

REFERENCES

[1] P. H. Diananda, *On rearrangement of series II*, Colloquium Mathematicum 9 (1962), p. 277-279.

[2] — *On rearrangement of series III*, ibidem 10 (1963), p. 287-288.

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