LEMMA. $Z_1, Z_2 \in \varphi S \Rightarrow Z_1 \subset Z_2 \vee Z_2 \subset Z_1$.

Proof. Let Z be the intersection of all T, $Z \in \varphi S$, such that T contains every element V of φS for which there exists in φS an element W. By (3), $Z \in \varphi S$.

The set

(7) $$\mathscr{S} = \{R \mid (R \in \varphi S \,\&\, R \supset Z) \vee R \in \varphi Z\}$$

is one of the $\mathscr{X}$'s in (2) for S as Z. For, intersections of sets of elements of $\mathscr{S}$ are again in $\mathscr{S}$ and if $R \in \mathscr{S}$, then $R^+ \in \mathscr{S}$ (in case $R \supset Z \,\&\, R \neq Z$, $\gamma R \notin Z$ since otherwise R^+ would be one of the V's, with $W = Z$, not contained in Z: a contradiction).

Hence $\varphi S \subset \mathscr{S}$. On the other hand, by (4), $\mathscr{S} \subset \varphi S$, so $\mathscr{S} = \varphi S$.

Suppose now $Z \neq \emptyset$. Then γZ must be an element of a set V and $V \neq Z$ by definition of Z, so $V \not\subset Z^+$. But by $\varphi S = \mathscr{S}$ we have $V \in \varphi Z$ and (5), (6) imply $V \subset Z^+$. Hence $Z = \emptyset$ and the lemma is proved.

We define a mapping $\Phi: S \to \varphi S$ by

(8) $$\Phi s = \bigcap_{\substack{Z \in \varphi S \\ s \in Z}} Z.$$

Then $s \in \Phi s \neq \emptyset$, and by (1) and (3), $\Phi s \in \varphi S$. $(\Phi s)^+$ is a proper subset of Φs; so, by (8), $\gamma \Phi s = s$ and Φ is 1-1.

Finally we define the relation $\leqslant$ by

(9) $$s_1 \leqslant s_2 \Longleftrightarrow \Phi s_1 \subset \Phi s_2.$$

Using the lemma and the fact that Φ is 1-1, we see immediately that $\leqslant$ is a relation of total ordering.

Let Z be a non-void subset of S and let T be the intersection of all elements of φS containing (as subsets of S) all Φs for $s \in Z$. γT must be an element of some Φs_0, $s_0 \in Z$. But then $T = \Phi s_0$, for otherwise T would be incomparable to Φs_0, contrary to the lemma. Hence $s_0 \leqslant s$ for all $s \in Z$ and $\leqslant$ is a well-ordering.

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REMARKS ON DYADIC SPACES

BY

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Let $D = \{0, 1\}$ denote the two point discrete space. For any cardinal number $\mathfrak{m}$ by the $\mathfrak{m}$-Cantor set we mean the Cartesian product $D^{\mathfrak{m}}$ of $\mathfrak{m}$ copies of D. The $\aleph_0$-Cantor set is a well-known Cantor perfect set on the real line. It is known (see e. g. [9], vol. II, p. 13) that every compact metrizable space is a continuous image of $D^{\aleph_0}$. In [1] P. S. Alexandroff defined a dyadic space as a compact space which, for some cardinal number $\mathfrak{m}$, is a continuous image of $D^{\mathfrak{m}}$, and has raised the problem of whether every compact space is dyadic. This problem was solved in [10] by E. Marczewski, who has shown that every family of non-empty, pairwise disjoint, open sets in $D^{\mathfrak{m}}$ (and then in any dyadic space) is countable, and remarked that the one-point compactifications of high power discrete space are therefore never dyadic (for proofs see [8], p. 166). The class of dyadic spaces was investigated by Šanin [13], Esenin-Volpin [7] and, recently by Efimov [6], [6a].

In this note we give simple proofs of two known theorems (1 and 2) and we establish two theorems (3 and 4) which seem to be new. In section 1 theorems 1-4 are formulated and the proofs of theorems 1 and 2 are given. Section 2 contains purely topological proofs of theorems 3 and 4 and two examples in connection with theorem 3. In section 3 we give proofs of theorems 3 and 4 by using the "function space method", based on the fact that the functor $C(\cdot)$ establishes the contravariant isomorphism of the category of compact spaces with homeomorphic embeddings and continuous mappings onto as morphisms, to the category of Banach algebras of all continuous real-valued functions on compact spaces with homomorphisms onto and isomorphic embeddings as morphisms.

By *space* we always mean a completely regular space. By E, I, N and D, we shall denote the real line, the closed interval $0 \leqslant x \leqslant 1$, the set of positive integers with discrete topology and the two point discrete space, respectively. $D^{\mathfrak{m}}$ and $I^{\mathfrak{m}}$ denote the Cartesian product of $\mathfrak{m}$ copies of D and I, respectively. The Čech-Stone compactification of a space X is denoted by βX. It is characterized among the compactifications of X

(to within a homeomorphism keeping X pointwise fixed) by the fact that every continuous function $f: X \to I$ has a continuous extension over βX. This fact implies that every continuous function $f: X \to Z$ into a compact space Z has a continuous extension over βX.

1. Let $\{X_s\}_{s\in S}$ be a family of spaces, Y a space, and let $f: \underset{s\in S}{\boldsymbol{P}} X_s \to Y$ be a continuous function.

If for some $S_0 \subset S$ we have $f(x) = f(y)$ for all $x = \{x_s\}$ and $y = \{y_s\}$ in $\underset{s\in S}{\boldsymbol{P}} X_s$ such that $x_s = y_s$ for $s \in S_0$, then we shall say that f depends only on coordinates belonging to S_0. If f depends only on coordinates belonging to some S_0 of cardinality at most $\mathfrak{m}$, then we say that f depends only on $\mathfrak{m}$ coordinates.

We begin with two well-known lemmas (see e.g. [3]):

LEMMA 1. *Every real-valued function defined on the Cartesian product $\underset{s\in S}{\boldsymbol{P}} X_s$ of compact spaces depends only on $\aleph_0$ coordinates.*

Proof. Denote by C the family of all real-valued continuous functions on $\underset{s\in S}{\boldsymbol{P}} X_s$ depending only on $\aleph_0$ coordinates. For very $f \in C$ let $S(f)$ be a countable subset of S such that f depends only on coordinates in $S(f)$. It is easy to see that for every $f, g \in C$ the functions $f+g$ and $f\cdot g$ depend only on coordinates in $S(f) \cup S(g)$ and that any real-valued function f on $\underset{s\in S}{\boldsymbol{P}} X_s$ which is a uniform limit of the sequence $\{f_i\}$ of functions belonging to C depends only on coordinates in $\bigcup_{i=1}^{\infty} S(f_i)$. Since the family C contains all the constant functions and separates points of $\underset{s\in S}{\boldsymbol{P}} X_s$, the lemma follows from the Stone-Weierstrass Theorem (see e. g. [5], p. 242).

COROLLARY. *If X is a dyadic space, then, for every real-valued continuous function g on X, there exists a compact metrizable subspace $X_g \subset X$ such that $g(X) = g(X_g)$.*

Indeed, the function $f = gh$, where h is a mapping of some $D^{\mathfrak{m}} = \underset{s\in S}{\boldsymbol{P}} D_s$ onto X, depends only on coordinates belonging to some countable $S_0 \subset S$. Since a continuous image of a compact metrizable space is metrizable, we may take for X_g the space $h(\underset{s\in S}{\boldsymbol{P}} D_s^*)$, where $D_s^* = D$ for $s \in S_0$ and $D_s^* = \{0\}$ for $s \in S \setminus S_0$.

LEMMA 2. *Every function f defined on the Cartesian product $\underset{s\in S}{\boldsymbol{P}} X_s$ of compact spaces and with values in a space Y of weight* ([1]) $\mathfrak{m} \geqslant \aleph_0$ *depends only on $\mathfrak{m}$ coordinates.*

([1]) The *weight* of a space X is the minimal cardinality of bases of X.

Proof. By Tychonoff embedding theorem, Y can be regarded as a subspace of $I^{\mathfrak{m}}$ and f as a family of $\mathfrak{m}$ real-valued functions. Since each of these functions depends only on $\aleph_0$ coordinates, the function f depends on $\mathfrak{m}\cdot\aleph_0 = \mathfrak{m}$ coordinates.

Since the weight of a continuous image $f(X)$ of a compact space X is not greater than the weight of X, we have

COROLLARY. *If X is a dyadic space then, for every continuous mapping $g: X \to Y$, into the space Y of weight $\mathfrak{m}$, there exists a compact subspace $X_g \subset X$ of weight $\mathfrak{m}$ such that $g(X) = g(X_g)$.*

THEOREM 1 (Šanin [13]). *If X is a dyadic space of weight $\mathfrak{m}$, then there exists a continuous mapping of the $\mathfrak{m}$-Cantor set $D^{\mathfrak{m}}$ onto X.*

Proof. By definition there exists, for some cardinal $\mathfrak{n}$, a mapping g of the $\mathfrak{n}$-Cantor set $D^{\mathfrak{n}} = \underset{s\in S}{\boldsymbol{P}} D_s$ onto X. In virtue of Lemma 2 there is a subset $S_0 \subset S$ of cardinality at most $\mathfrak{m}$ such that g depends only on coordinates in S_0. The subspace $D_*^{\mathfrak{m}} = \underset{s\in S}{\boldsymbol{P}} D_s^* \subset D^{\mathfrak{n}}$, where $D_s^* = D$ if $s \in S_0$ and $D_s^* = \{0\}$ if $s \in S \setminus S_0$, is homeomorphic with the $\mathfrak{m}$-Cantor set and it is easy to see that $g_* = g|D_*^{\mathfrak{m}}$ maps $D_*^{\mathfrak{m}}$ onto X.

LEMMA 3. *For every closed G_δ set X in $D^{\mathfrak{m}}$ there exists a countable set $S_0 \subset S$ and a closed subset $X_0 \subset \underset{s\in S_0}{\boldsymbol{P}} D_s$ such that $X = X_0 \times \underset{s\in S\setminus S_0}{\boldsymbol{P}} D_s$.*

Proof. In virtue of a result by Vedenisoff [16] there exists a continuous real-valued function $f: D^{\mathfrak{m}} = \underset{s\in S}{\boldsymbol{P}} D_s \to I$ such that $X = f^{-1}(0)$. By Lemma 1, f depends only on coordinates in a countable set $S_0 \subset S$ and $f = f_1 p$, where $p: D^{\mathfrak{m}} = \underset{s\in S}{\boldsymbol{P}} D_s \to \underset{s\in S_0}{\boldsymbol{P}} D_s$ is the projection and $f_1: \underset{s\in S_0}{\boldsymbol{P}} D_s \to I$. We have then $X = X_0 \times \underset{s\in S\setminus S_0}{\boldsymbol{P}} D_s$, where $X_0 = f_1^{-1}(0)$.

COROLLARY 1. *If $\mathfrak{m}$ is a cardinal number greater than $\aleph_0$, then every closed G_δ set in $D^{\mathfrak{m}}$ is homeomorphic with $D^{\mathfrak{m}}$.*

Indeed, we have shown that $X = X_0 \times D^{\mathfrak{m}}$; by a clasical result (see [9], vol. II, p. 58) $X_0 \times D^{\aleph_0}$, as a compact metrizable space not containing isolated points, is homeomorphic with $D^{\aleph_0}$. We have then

$$X = X_0 \times D^{\mathfrak{m}} = X_0 \times D^{\aleph_0} \times D^{\mathfrak{m}} = D^{\aleph_0} \times D^{\mathfrak{m}} = D^{\mathfrak{m}}.$$

COROLLARY 2. *For every X which is a closed G_δ set in $D^{\mathfrak{m}}$ there exists a retraction of $D^{\mathfrak{m}}$ onto X.*

Indeed, we have shown that $X = X_0 \times \underset{s\in S\setminus S_0}{\boldsymbol{P}} D_s$, where X_0 is a closed subset of $\aleph_0$-Cantor set $\underset{s\in S_0}{\boldsymbol{P}} D_s$, and hence (see [9], vol. I, p. 169) a retract of $\underset{s\in S_0}{\boldsymbol{P}} D_s$. This means that there exists a mapping $r: \underset{s\in S_0}{\boldsymbol{P}} D_s \to X_0$ such that $r(x) = x$ for $x \in X_0$. It is easy to see that X is a continuous image of

$D^{\mathfrak{m}}$ by the retraction $g\colon D^{\mathfrak{m}} = \underset{s\in S_0}{\boldsymbol{P}} D_s \times \underset{s\in S\setminus S_0}{\boldsymbol{P}} D_s \to X_0 \times \underset{s\in S\setminus S_0}{\boldsymbol{P}} D_s = X$, where $g(x, y) = \big(r(x), y\big)$ for every $x \in \underset{s\in S_0}{\boldsymbol{P}} D_s$ and $y \in \underset{s\in S\setminus S_0}{\boldsymbol{P}} D_s$.

From Corollary 2 follows

THEOREM 2 (B. Efimov [6]). *Every space X which can be embedded as a closed G_δ in a dyadic space is itself a dyadic space.*

In sections 2 and 3 we shall prove the following theorems (²):

THEOREM 3. *If the Čech-Stone compactification βX of a space X is dyadic, then X is pseudocompact* (³).

THEOREM 4. *There is no infinite extremally disconnected* (⁴) *dyadic compact space* (⁵).

2. The following lemma shows that to prove Theorem 3 it is sufficient to show that βE is not dyadic. Indeed, if the space X can be continuously mapped onto a dense subset of Y then βY is an image of βX.

LEMMA 4. *For every non-pseudocompact space X there exists a continuous function $f\colon X \to E$ such that $f(X)$ is a dense subset of E.*

Proof. We may confine our attention to X which are non-bounded subsets of E. In this case X contains an infinite closed subset $X_0 = \{x_1, x_2, \ldots\}$ homeomorphic with a countable discret space. Let $\{w_1, w_2, \ldots\}$ be the sequence of all rational numbers. By Tietze extension theorem the function $f_0: X_0 \to E$ defined by the condition $f_0(x_i) = w_i$, for $i = 1, 2, \ldots$, can be extended to a function $f\colon X \to E$. It is easy to see that $f(X)$ is dense in E.

Proof of theorem 3. Let g be an extension over βE of the function $f\colon E \to I$ defined by

$$f(x) = \frac{1}{2}\left(\frac{x}{1+|x|}\right) + \frac{1}{2}.$$

(²) M. Katětov has remarked that Theorems 3 and 4 follow from his (unpublished)

THEOREM. *Every non-isolated point of a dyadic space is a limit point of a sequence of distinct points.*

(³) By *pseudocompact space* we mean a space X such that every real-valued function of X is bounded. For normal spaces pseudocompactness coincides with countable compactness.

M. Katětov has first remarked that βN is not dyadic. His (unpublished) proof involves properties of the space A_7, constructed by Alexandroff and Urysohn in [2], which is compact, first countable but non metrizable, and thus, by a theorem due to Esenin-Volpin [7], not dyadic. A_7 contains a countable dense subset, hence it is a continuous image of βN, and βN cannot be dyadic. We have not succeeded in adapting the elegant reasoning of Katětov so as to get the proof of Theorem 3.

(⁴) A space X is called *extremally disconnected* if the closure of every open set in X is open; it is easy to see that X is extremally disconnected if and only if any two disjoint open sets in X have disjoint closures.

(⁵) This theorem has been conjectured by P. S. Alexandroff.

It is evident that $g(\beta E \setminus E) = \{0, 1\}$ and that $g = f$ being one-to-one on E there is no proper compact subspace $X_g \subset \beta E$ such that $g(X_g) = g(\beta E) = I$. Since βE is not metrizable, we obtain, by Corollary of Lemma 1, that βE is not dyadic.

From Theorem 3 follows

COROLLARY. *The Čech-Stone compactification βX of a metric space X is dyadic if and only if βX is a metrizable space* (⁶).

Proof of theorem 4 follows, by Theorem 3, from

LEMMA 5. *Every infinite extremally disconnected compact space contains, as open subset, a homeomorph of βX_0, for some normal and non-pseudocompact X_0.*

Furthermore, X_0 can be represented as a countable union of pairwise disjoint non-empty closed-open subsets.

Proof. Since X is infinite and extremally disconnected, there exist in X closed-open sets $V_1, V_2, \ldots$ such that

$$V_i \neq 0 \quad \text{and} \quad V_i \cap V_j = 0 \quad \text{for } i \neq j.$$

$X_0 = \bigcup_{i=1}^{\infty} V_i$ is a non-pseudocompact space and, as an F_σ in normal space X, it is normal. Its closure $\overline{X}_0$ is a closed-open subset of X. To show that $\overline{X}_0$ is homeomorphic with βX_0 it is sufficient to prove that every function $f\colon X_0 \to I$ has an extension $f\colon \overline{X}_0 \to I$ (⁷).

For any disjoint closed sets $A, B \subset I$ the counter-images $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint and closed in X_0. In view of the normality of X_0 there exist two open (in X_0, and thus in X) sets U, V such that

$$f^{-1}(A) \subset U, \quad f^{-1}(B) \subset V \quad \text{and} \quad U \cap V = 0.$$

Since X is extremally disconnected, $\overline{U} \cap \overline{V} = 0$ and $\overline{f^{-1}(A)} \cap \overline{f^{-1}(B)} = 0$. By a theorem due to Taimanov [15] this fact implies that there exists an extension $\bar{f}$ of f to $\overline{X}_0 \subset X$.

COROLLARY 1. *Every infinite extremally disconnected compact space contains a homeomorph of βN.*

Indeed, X_0 being normal and non-pseudocompact contains a closed subset homeomorphic with N. The closure of N in βX_0 is by Urysohn extension theorem homeomorphic with βN.

COROLLARY 2 (Gęba and Semadeni [4]). *Every infinite extremally disconnected compact space X can be continuously mapped onto βN.*

Indeed, let $X_0 = \bigcup_{i=1}^{\infty} V_i$, where $V_i \neq 0$, $V_i \cap V_j = 0$ for $i \neq j$ and V_i is open in X_0, be a subset of X such that $\overline{X}_0$ is open in X and homeo-

(⁶) See Appendix at the end of this paper.

(⁷) This is known, see [5], p. 23, problem 1H.6. We present here another proof.

morphic with βX_0. The mapping $f\colon X_0 \to N$ such that $f(V_i) = i$ for $i = 1, 2, \ldots$ can be extended over $\bar{X}_0$ and over the whole X; every such extension obviously maps X onto βN ([8]).

EXAMPLE 1. *There exists a normal (non-compact) pseudocompact space X such that βX is not dyadic.*

Let X be a space of all ordinals less than or equal to the first uncountable ordinal Ω with order topology. It is known (see e. g. [5], p. 74) that X is a normal and pseudocompact space and that $\beta X = X \cup \{\Omega\}$. Every non limit ordinal is an isolated point of βX. It follows then that βX contains $\aleph_1$ pairwise disjoint non-empty open sets and that βX is not dyadic.

EXAMPLE 2. *There exists a non-compact space X (pseudocompact by Theorem 3) such that βX is dyadic.*

Let X be the subspace of $I^c = \mathop{\boldsymbol{P}}_{s \in S} I_s$ composed of such $\{x_s\}$ that $x_s = 0$ for all but a countable number of $s \in S$. It is shown by Corson in [3] that I^c is the Čech-Stone compactification of X. Since I is a continuous image of the Cantor perfect set, I^c is an image of D^c, i. e. βX is dyadic.

3. If X is a compact space, then $C(X)$ denotes the Banach algebra of all continuous real-valued functions on X with the norm $\|f\| = \sup_{s \in S} |f(s)|$. Restating the definition of the dyadic space in the terms of the dual category of algebras of continuous functions we get

PROPOSITION. *A compact space X is dyadic if and only if the algebra $C(X)$ can be isomorphically embedded into an algebra $C(D^{\mathfrak{m}})$.*

This proposition is a consequence of the general fact (see e. g. [5], p. 141) that in order that there exists a continuous mapping from a compact space X into a compact space Y it is necessary and sufficient that the algebra $C(Y)$ can be isomorphically embedded into the algebra $C(X)$.

LEMMA 6. *Let X be a dyadic space and let A be a linear closed subspace of $C(X)$. If A is isomorphic (= linearly homeomorphic) with the Banach space c of all convergent real sequences, then there exists a continuous linear projection from $C(X)$ onto A.*

Proof. Since X is a dyadic space, $C(X)$ may be considered as a subspace of $C(D^{\mathfrak{m}})$ for some $\mathfrak{m} \geqslant \aleph_0$. Thus it is sufficient to show that there exists a linear continuous projection P from $C(D^{\mathfrak{m}})$ onto A; indeed, the restriction of P to $C(X)$ is the required projection.

([8]) R. Sikorski has remarked that by a similar way one can extend Theorem 4 and Corollary 2 to the case of zero-dimensional compact Hausdorff spaces with the property that every open set which is a countable sum of closed-open sets has the open closure.

By the assumption, A is separable, i. e. contains a countable dense subset $f_1, f_2, \ldots$ Each of the functions f_i depends, by Lemma 1, only on coordinates in some countable set $S_i \subset S$. It is easy to see that every function $f \in A$ depends only on coordinates in $S_0 = \bigcup_{i=1}^{\infty} S_i$. Let B be a subalgebra of $C(D^{\mathfrak{m}})$ consisting of all functions depending only on coordinates in S_0. Then $B \supset A$ and B is isomorphic, as a ring, to $C(D^{\aleph_0})$. Since B is separable and A is a Banach space isomorphic with c, by a result of Sobczyk [14] (see also [11], p. 217), there exists a linear continuous projection P_1 from B onto A. For any $f \in C(D^{\mathfrak{m}})$ let $P_2 f = g$, where $g(\{x_s\}) = f(\{x'_s\})$ and $x'_s = x_s$ for $s \in S_0$, $x'_s = 0$ for $s \notin S_0$. It is easily seen that P_2 is a linear continuous projection from $C(D^{\mathfrak{m}})$ onto B. Finally we put $P = P_1 P_2$.

THEOREM 5. *If X is a dyadic space, then no closed linear subspace of $C(X)$ is isomorphic to the space m of all bounded real sequences.*

Proof. Suppose, on the contrary, that there is a dyadic space X such that there exists a subspace B of $C(X)$ isomorphic with m. Let A be a subspace of B isomorphic with c (for example) the subspace corresponding to the set of all convergent sequences in m. Then, by Lemma 6, there exists a continuous linear projection P from $C(X)$ onto A. Thus the restriction of P to the space B would be a linear continuous projection from B onto A. But this contradicts the result of Philips [12] that there is no linear continuous projection from m onto its subspace isomorphic with c.

Proof of theorem 3. Suppose that X is a non pseudocompact space. There exists a continuous real-valued function $f\colon X \to E$ such that $A = f(X)$ is not bounded. The space A contains, as a closed subset, a space M homeomorphic with N. Let B be a subspace of the space $C^*(A)$ (of all bounded real-valued functions on A) composed of all broken lines with vertices in M. It is easily seen that B is a linear subspace of $C^*(A)$ isomorphic with m. But $C^*(A)$ is a linear subspace of $C^*(X) = C(\beta X)$ and, by Theorem 5, X is not dyadic.

Proof of theorem 4. By a result of Gęba and Semadeni [4] (which is the dual restatement of Corollary 2 to our Lemma 5) if X is an extremally disconnected infinite compact space, then the space $C(X)$ contains a subspace isomorphic with m. Hence, by Theorem 5, X is not dyadic.

Appendix. B. Efimov has kindly communicated to us that the following theorem (which presents a generalization of our Corollary of Theorem 3) is a consequence of his recent results [6a]:

THEOREM. *A compactification rX of a metric space X (i. e. a compact space containing X as a dense subspace) is dyadic if and only if rX is a metrizable space.*

We present here another proof of this theorem.

LEMMA. *For every compactification rX of a separable metric space X there exists a metrizable compactification $r'X$ of the space X, smaller than rX, i. e. such that for some $g\colon rX \to r'X$ we have $g(x) = x$ for every $x \in X$.*

Proof. Let $\mathfrak{B}$ be a countable basis of X and let $\mathfrak{P}$ be the set of all pairs (U, W) of elements of $\mathfrak{B}$ such that $\overline{U} \cap \overline{(rX \setminus W)} = 0$ (closure in rX). For any pair $p = (U, W) \in \mathfrak{P}$ let $f_p\colon rX \to I$ be such that $f_p(\overline{U}) = \{1\}$ and $f_p(rX \setminus W) = \{0\}$. It is easy to see that $g = \{f_p\}_{p \in \mathfrak{P}}\colon rX \to I^{\aleph_0}$ is the homeomorphic embedding and that $r'X = g(rX)$ is the required compactification.

Proof of the theorem. It is sufficient to show that if rX is dyadic then rX is metrizable.

It is well known that every non separable metric space contains, for some $\varepsilon > 0$, an uncountable family of disjoint ε-spheres. Since in rX every family of open, pairwise disjoint, non-empty sets is countable, X is separable. Let $r'X$ and $g\colon rX \to r'X$ be as in Lemma. Because $g(rX \setminus X) = r'X \setminus X$ (see [5], p. 92) if $g(X_g) = g(rX)$ for some $X_g \subset rX$ then $X \subset X_g$. It follows then, by Corollary of Lemma 2, that rX is metrizable.

Let us note that by the same method one can prove that the weight of any dyadic compactification rX of a space X is equal to the weight of X.

COROLLARY. *Every dispersed (not containing perfect subspaces) dyadic space X is metrizable.*

Indeed, such X is a compactification of the (discrete) set of its isolated points.

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