contrary that there exists a subalgebra A_s, A_1 say, which is not isomorphic to the real field. We examine the transformation T_u of algebra A defined by the formula

$$T_u\Big(\sum_{r=1}^{n} x_r\Big) = ux_1 + \sum_{r=2}^{n} x_r,$$

where u denotes any element belonging to $S \cap A_1$. The transformations T_u are isometries preserving the unit sphere S. In fact, in virtue of Corollary of Lemma 10 the minimal norm in each of the algebras R, C, Q, is multiplicative; so, since $u \in S$, we have $\|ux_1\| = \|u\|\cdot\|x_1\| = \|x_1\|$. Hence, by Theorem 3, we obtain the equation

$$\Big\| T_u\Big(\sum_{r=1}^{n} x_r\Big)\Big\| = \Big\| ux_1 + \sum_{r=2}^{n} x_r \Big\| = \max_{1\leqslant r\leqslant n} \|x_r\| = \Big\| \sum_{r=1}^{n} x_r \Big\|.$$

Since A_1 is a division algebra, different transformations T_u correspond to different elements $u \in S \cap A_1$. Since $\dim A_1 \geqslant 2$, there exist infinitely many elements $u \in S \cap A_1$. Accordingly, there exist infinitely many isometries that transform the unit sphere S onto itself, contrary to the assumption. Theorem 4 is thus proved.

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A PROOF OF THE WELL-ORDERING THEOREM

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The usual proofs of the well-ordering theorem proceed by induction. It is also well known how to avoid induction and ordinal numbers in the proof. However, the resulting arguments are rather lengthy. Here we present a proof of this kind which we believe is still very short.

Let S be a non-void set and let $\mathscr{P}$ stand for "power-set of." By the axiom of choice there exists a mapping $\gamma\colon (\mathscr{P}S - \{\emptyset\}) \to S$ such that $\gamma Z \in Z$ for every $Z \in \mathscr{P}S - \{\emptyset\}$. Let Z^+ denote $Z - \{\gamma Z\}$.

We define a mapping $f\colon \mathscr{P}^2 S \to \mathscr{P}^2 S$ by

(1) $$f\mathscr{Z} = \{\bigcap_{Z\in\mathscr{U}} Z \mid \emptyset \neq \mathscr{U} \subset \mathscr{Z}\} \cup \{Z^+ \mid \emptyset \neq Z \in \mathscr{Z}\},$$

i. e. for $\mathscr{Z} \in \mathscr{P}^2 S$, $f\mathscr{Z}$ consists of all intersections of non-void sets of elements of $\mathscr{Z}$ (considered as subsets of S) as well as of all subsets of S obtained from elements Z ($Z \neq \emptyset$) of $\mathscr{Z}$ by removing from them their element γZ.

Next, we define a mapping $\varphi\colon \mathscr{P}S \to \mathscr{P}^2 S$ by

(2) $$\varphi Z = \bigcap_{\mathscr{Z} \supset \{Z\} \cup f\mathscr{Z}} \mathscr{Z}.$$

By (1), $\varphi Z \subset f\varphi Z$. Conversely, by (2), $f\varphi Z = f \cap \mathscr{Z} \subset \bigcap f\mathscr{Z} \subset \bigcap \mathscr{Z} = \varphi Z$, hence

(3) $$f\varphi Z = \varphi Z.$$

By (2),

(4) $$Z_2 \in \varphi Z_1 \Rightarrow \varphi Z_2 \subset \varphi Z_1.$$

As $\varphi Z \cap \{V \mid V \subset Z\}$ is one of the $\mathscr{Z}$'s in (2),

(5) $$V \in \varphi Z \Rightarrow V \subset Z.$$

By (4) and (5), $\varphi Z^+ \subset \varphi Z - \{Z\}$. On the other hand, $\{Z\} \cup \varphi Z^+$ is one of the $\mathscr{Z}$'s in (2), hence

(6) $$\varphi Z^+ = \varphi Z - \{Z\}.$$

LEMMA. $Z_1, Z_2 \in \varphi S \Rightarrow Z_1 \subset Z_2 \vee Z_2 \subset Z_1$.

Proof. Let Z be the intersection of all T, $Z \in \varphi S$, such that T contains every element V of φS for which there exists in φS an element W. By (3), $Z \in \varphi S$.

The set

$$\mathscr{S} = \{R \mid (R \in \varphi S \,\&\, R \supset Z) \vee R \in \varphi Z\} \tag{7}$$

is one of the $\mathscr{Z}$'s in (2) for S as Z. For, intersections of sets of elements of $\mathscr{S}$ are again in $\mathscr{S}$ and if $R \in \mathscr{S}$, then $R^+ \in \mathscr{S}$ (in case $R \supset Z \,\&\, R \neq Z$, $\gamma R \notin Z$ since otherwise R^+ would be one of the V's, with $W = Z$, not contained in Z: a contradiction).

Hence $\varphi S \subset \mathscr{S}$. On the other hand, by (4), $\mathscr{S} \subset \varphi S$, so $\mathscr{S} = \varphi S$.

Suppose now $Z \neq \emptyset$. Then γZ must be an element of a set V and $V \neq Z$ by definition of Z, so $V \not\subset Z^+$. But by $\varphi S = \mathscr{S}$ we have $V \in \varphi Z$ and (5), (6) imply $V \subset Z^+$. Hence $Z = \emptyset$ and the lemma is proved.

We define a mapping $\Phi: S \to \varphi S$ by

$$\Phi s = \bigcap_{\substack{Z \in \varphi S \\ s \in Z}} Z. \tag{8}$$

Then $s \in \Phi s \neq \emptyset$, and by (1) and (3), $\Phi s \in \varphi S$. $(\Phi s)^+$ is a proper subset of Φs; so, by (8), $\gamma \Phi s = s$ and Φ is 1-1.

Finally we define the relation $\leqslant$ by

$$s_1 \leqslant s_2 \Longleftrightarrow \Phi s_1 \subset \Phi s_2. \tag{9}$$

Using the lemma and the fact that Φ is 1-1, we see immediately that $\leqslant$ is a relation of total ordering.

Let Z be a non-void subset of S and let T be the intersection of all elements of φS containing (as subsets of S) all Φs for $s \in Z$. γT must be an element of some Φs_0, $s_0 \in Z$. But then $T = \Phi s_0$, for otherwise T would be incomparable to Φs_0, contrary to the lemma. Hence $s_0 \leqslant s$ for all $s \in Z$ and $\leqslant$ is a well-ordering.

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REMARKS ON DYADIC SPACES

BY

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Let $D = \{0, 1\}$ denote the two point discrete space. For any cardinal number $\mathfrak{m}$ by the $\mathfrak{m}$-Cantor set we mean the Cartesian product $D^{\mathfrak{m}}$ of $\mathfrak{m}$ copies of D. The $\aleph_0$-Cantor set is a well-known Cantor perfect set on the real line. It is known (see e. g. [9], vol. II, p. 13) that every compact metrizable space is a continuous image of $D^{\aleph_0}$. In [1] P. S. Alexandroff defined a dyadic space as a compact space which, for some cardinal number $\mathfrak{m}$, is a continuous image of $D^{\mathfrak{m}}$, and has raised the problem of whether every compact space is dyadic. This problem was solved in [10] by E. Marczewski, who has shown that every family of non-empty, pairwise disjoint, open sets in $D^{\mathfrak{m}}$ (and then in any dyadic space) is countable, and remarked that the one-point compactifications of high power discrete space are therefore never dyadic (for proofs see [8], p. 166). The class of dyadic spaces was investigated by Šanin [13], Esenin-Volpin [7] and, recently by Efimov [6], [6a].

In this note we give simple proofs of two known theorems (1 and 2) and we establish two theorems (3 and 4) which seem to be new. In section 1 theorems 1-4 are formulated and the proofs of theorems 1 and 2 are given. Section 2 contains purely topological proofs of theorems 3 and 4 and two examples in connection with theorem 3. In section 3 we give proofs of theorems 3 and 4 by using the "function space method", based on the fact that the functor $C(\cdot)$ establishes the contravariant isomorphism of the category of compact spaces with homeomorphic embeddings and continuous mappings onto as morphisms, to the category of Banach algebras of all continuous real-valued functions on compact spaces with homomorphisms onto and isomorphic embeddings as morphisms.

By *space* we always mean a completely regular space. By E, I, N and D, we shall denote the real line, the closed interval $0 \leqslant x \leqslant 1$, the set of positive integers with discrete topology and the two point discrete space, respectively. $D^{\mathfrak{m}}$ and $I^{\mathfrak{m}}$ denote the Cartesian product of $\mathfrak{m}$ copies of D and I, respectively. The Čech-Stone compactification of a space X is denoted by βX. It is characterized among the compactifications of X