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ON A SUMMATION FORMULA OF E. COHEN

BY

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The following theorem is well known:

If the series $\sum_{n=1}^{\infty} g(n)/n$ *is absolutely convergent and* $f(n) = \sum_{d|n} g(d)$, *then*

$$\lim_{x\to\infty} \frac{1}{x} \sum_{n\leqslant x} f(n) = \sum_{n=1}^{\infty} g(n)/n.$$

Recently E. Cohen [2] proved the following generalization of this theorem:

If the series $\sum_{n=1}^{\infty} g(n)/n$ *is absolutely convergent and* $g_s(n) = \sum_{d|n} g(d)\tau_s(n/d)$ (*where* $\tau_s(n)$ *is defined by* $\tau_1(n) = 1$, $\tau_{s+1}(n) = \sum_{d|n}\tau_s(d)$), *then*

$$\lim_{x\to\infty} \frac{1}{x\log^{s-1}x} \sum_{n\leqslant x} g_s(n) = \frac{1}{(s-1)!} \sum_{n=1}^{\infty} \frac{g(n)}{n} \qquad (s = 1, 2, \ldots).$$

In this note we give a simple proof of the theorem of E. Cohen, based on the remark that if $\|a_{n,k}\|$ is an infinite matrix satisfying the conditions

(i) $|a_{n,k}| \leqslant M$ with some M independent of k and n,

(ii) for every n the sequence $\{a_{n,k}\}_{k=1}^{\infty}$ is convergent to, say, a_n,

then from $\sum_{m=1}^{\infty} |c_m| < \infty$ follows

$$\lim_{k\to\infty} \sum_{n=1}^{\infty} a_{n,k} c_n = \sum_{n=1}^{\infty} a_n c_n.$$

The following formula is well-known and can be easily proved by induction:

$$(*) \qquad \lim_{x\to\infty} \frac{1}{x\log^{s-1}x} \sum_{m\leqslant x} \tau_s(m) = 1/(s-1)!.$$

Now

$$\sum_{k\leqslant x} g_s(k) = \sum_{k\leqslant x}\sum_{d|k} g(d)\tau_s(k/d)$$

$$= \sum_{d\leqslant x} g(d) \sum_{\substack{k\leqslant x\\ k|d}} \tau_s(k/d) = \sum_{d\leqslant x} \frac{g(d)}{d} \sum_{l\leqslant x/d} d\tau_s(l).$$

Let us put

$$a_{n,k} = \frac{1}{k\log^{s-1}k} \sum_{l\leqslant k/n} n\tau_s(l)$$

if $k \geqslant n$, and $a_{n,k} = 0$ if $k < n$. Moreover, let $c_n = g(n)/n$. Condition (ii) follows from (*), and (i) follows from

$$|a_{n,k}| = \frac{1}{k\log^{s-1}k} \sum_{l\leqslant k/n} n\tau_s(l)$$

$$= \frac{n}{k\log^{s-1}k}\left(\frac{1}{(s-1)!}\,\frac{k}{n}\log^{s-1}\left(\frac{k}{n}\right) + o\left(\frac{k}{n}\log^{s-1}\frac{k}{n}\right)\right)$$

$$= \frac{1}{(s-1)!}\left(1-\frac{\log n}{\log k}\right)^{s-1} + o\left(\left(1-\frac{\log n}{\log k}\right)^{s-1}\right) = \frac{1}{(s-1)!} + O(1) = O(1)$$

in the case $k \geqslant n$ and is obvious in the case $k < n$. As $a_n = 1/(s-1)!$ for all n, the theorem follows.

It should be remarked that the same method leads to a similar theorem (in case $s = 1$) regarding the unitary convolution (see [1]), namely:

If the series $\sum\limits_{n=1}^{\infty} g(n)/n$ *is absolutely convergent and* $f(n) = \sum\limits_{\substack{d|n\\ (d,n/d)=1}} g(d)$, *then*

$$\lim_{x\to\infty} \frac{1}{x} \sum_{n\leqslant x} f(n) = \sum_{n=1}^{\infty} \frac{g(n)\varphi(n)}{n^2}.$$

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A SIMPLE REMARK ON MATRICES

BY

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This paper deals with a certain representation of matrices which can be useful as a tool for treating the projective group. The author is sure that the results are known to many persons, but he doubts if they have ever been published.

1. Consider the group $GL(n+1, K)$ of matrices of the form

$$A = \begin{pmatrix} a_{00} & a_{01} & \dots & a_{0n} \\ a_{10} & a_{11} & \dots & a_{1n} \\ \cdot & \cdot & \cdot & \cdot \\ a_{n0} & a_{n1} & \dots & a_{nn} \end{pmatrix}$$

over any field K of characteristic 0.

It is a matter of elementary calculations to notice that the set G of matrices with constant sums of rows, i. e. satisfying the condition

$$\sum_{i=0}^{n} a_{ij} = \lambda \text{ does not depend on } j,$$

as well as the set G_0 of matrices satisfying the condition

$$\sum_{i=0}^{n} a_{ij} = 1$$

are subgroups of $GL(n+1)$.

The subgroup G_0 is isomorphic with the subgroup of matrices of the form

$$\begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ a_{10} & a_{11} & a_{12} & \dots & a_{1n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{n0} & a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix}.$$