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A FIXED POINT FREE MAPPING OF A CONNECTED PLANE SET

BY

F. B. JONES (RIVERSIDE, CALIF.)

J. L. Kelley pointed out in [2] that the question of the possible existence of a fixed point free, periodic, continuous mapping of a connected and non-cutting subset of the plane into itself was still unanswered. Contained in the proof of a special case of his Conjecture A is the suggestion that such a mapping might possibly be of period 2. Such a mapping of period 2 does, in fact, exist.

THEOREM. *If M is a plane pseudo-arc and K is a composant of M, then there exists a fixed point free homeomorphism of period 2 of the connected point set $M-K$ onto itself. Furthermore, if K contains a point accessible from the complement of M, then the complement of $M-K$ is strongly (= continuum-wise) connected.*

Proof. The construction of a pseudo-arc M from p to q given in Moise's thesis [3] as the intersection of $\overline{C}_n^*$ where, for each n, C_n is a finite collection of open plane sets whose closures form a chain of a specific type, may be carried out in a fashion symmetrical with respect to the two points p and q as indicated in his Figure 1. In particular, if for each n, $C_n = \{C_{in}\}$ $(i = 1, \ldots, i_n)$ is the natural ordering of C_n from p to q and $C_n = \{C'_{in}\}$ $(i = 1, \ldots, i_n)$ is the reverse ordering (i. e., the natural ordering from q to p), then $\overline{C}_{jm}$ intersects $\overline{C}_{kn}$ if and only if $\overline{C}'_{jm}$ intersects $\overline{C}'_{kn}$.

Now let x be a point of M and for each n, let i be the smallest integer such that $\overline{C}_{in}$ contains x. So $x = \bigcap \overline{C}_{in}$. Define the function f on M so that $f(x) = \bigcap \overline{C}'_{in}$. Obviously $f(p) = q$, $f(q) = p$ and $f(x) = x$ if and only if $x = \bigcap \overline{C}_{in} = \bigcap \overline{C}'_{in}$. Clearly f is continuous, 1-1, and of period 2 with only one fixed point, namely, the point o determined by the middle elements of the chains.

Let o' be a point of the composant K of M. Since M is homogeneous [1], there is a homeomorphism h of M onto M such that $h(o) = o'$. The homeomorphism hfh^{-1} leaves o' fixed and hence $hfh^{-1}(K) = K$. So hfh^{-1} is of period 2 on $M-K$ and has no fixed point.

Obviously $M-K$ is connected and if K contained a point accessible from the complement of M, then $M-K$ would be strongly connected.

It may be well to point out in summary that since all pseudo-arcs are homeomorphic [3], no generality has been lost, and, since no pseudo-arc separates the plane and each contains a point accessible from its complement, it follows that each plane pseudo-arc (regardless of how it is embedded in the plane) contains a connected subset which does not cut the plane and on which there exists a homeomorphism of period 2 with no fixed point.

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THE UNIVERSITY OF CALIFORNIA, RIVERSIDE

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AN INTERSECTION PROPERTY OF SETS WITH POSITIVE MEASURE

BY

P. ERDÖS, H. KESTELMAN, AND C. A. ROGERS (LONDON)

1. If $A_1, A_2, \ldots$ are Lebesgue-measurable sets of real numbers in the interval $I = [0, 1]$ with measures satisfying

$$\mu(A_r) > \eta > 0, \quad r = 1, 2, \ldots,$$

the set

$$\bigcap_{n\geqslant 1} \bigcup_{r\geqslant n} A_r$$

is measurable with measure at least η. So it is certainly possible to choose a sequence $n_1 < n_2 < \ldots$ such that the intersection $\bigcap_{r=1}^{r=\infty} A_{n_r}$ is non-empty. But (see the example in § 2) there may be no such sequence for which the intersection has positive measure. However, we show that the subsequence can be chosen to ensure that the intersection is uncountable. More precisely, we prove (see §§ 3 and 4)

THEOREM 1. *Suppose η is a positive number and $A_1, A_2, \ldots$ are Lebesgue-measurable subsets of the interval* $[0, 1]$ *with* $\limsup \mu(A_r) \geqslant \eta$. *Then there is a Borel set S with $\mu(S) \geqslant \eta$, and a sequence $q_1 < q_2 < \ldots$ such that every point of S is a point of condensation of the set*

$$\bigcup_{j\geqslant 1} \bigcap_{r\geqslant j} A_{q_r},$$

so that every open set containing points of S also contains a perfect subset of $A_{q_j} \cap A_{q_{j+1}} \cap \ldots$ for some j.

We arrange our proof so that it can be trivially generalized (see § 5).

It is natural to ask if, under the conditions of Theorem 1, one can say anything about Hausdorff measures of the set

$$\bigcap_{j\geqslant 1} A_{q_j}$$