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Modelling imperfect time intervals in a two-dimensional space *

by

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Abstract: Every event has an extent in time, which is usually described by crisp time intervals. However, under some circumstances, temporal extents of events are imperfect, and therefore cannot be adequately modelled by crisp time intervals. Rough sets and fuzzy sets are two frequently used tools for representing imperfect temporal information. In this paper, we apply a two-dimensional representation of crisp time intervals, which is called the Triangular Model (TM), to investigate rough time intervals (RTIs) and fuzzy time intervals (FTIs). With this model, RTIs and FTIs, as well as their temporal relations, can be represented as graphics (i.e. discrete geometries or continuous fields) in a two-dimensional time space. Compared to the traditional linear representation of time intervals, we found that TM provides a more compact and clearer representation of imperfect time intervals and relations. Moreover, temporal queries of imperfect intervals can be graphically addressed in TM, which is closer to human intuition than mathematical expressions. As human minds are more efficient in perceiving and processing graphic representations than numerical representations, we believe TM can be applied as a valuable assistant tool for analysing and reasoning about imperfect time intervals.

Keywords: rough set, rough time interval, fuzzy set, fuzzy time interval, the Triangular Model, temporal relation.

1. Introduction

Recently, a lot of research has been done on representing and reasoning about time intervals. Most of this work focused on crisp time intervals (CTI), namely,

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time intervals that are bounded by a beginning and ending (e.g. Allen, 1983; Allen and Hayes, 1985; Ladkin, 1987; Galton, 1990; Freksa, 1992). However, under some circumstances, these CTIs cannot adequately describe temporal occupations of events and processes. For example, due to imprecise information, the beginning and end of the interval is known to be within certain ranges. However, the exact beginning and end cannot be defined. In these cases, time intervals of events can be represented by rough sets (see Pawlak, 1982, 1991) which classify the upper and lower approximations of the interval. The thus obtained time intervals are called rough time intervals (RTI). On the other hand, some events may start or end gradually and therefore their beginning and end times cannot be pinned to exact time stamps. Intervals of this kind of events can be described by fuzzy sets (see Zadeh, 1996a,b) through the quantification of the graded truth of whether a time point is a member of the interval, bringing the concept of fuzzy time interval (FTI). Currently, a lot of disciplines (e.g. archaeology, geography, psychology, and philosophy) are faced with the problem of handling imperfect temporal information, as is reflected in many contributions on representing and reasoning about RTIs (e.g. Bittner, 2002; Bassiri et al., 2009) and FTIs (e.g. De Caluwe et al., 1997, 1999; Nagypál and Motik, 2003; Ohlbach, 2004; Schockaert and De Cock, 2008; Garrido et al., 2009). However, most of the work is based on the linear concept of time, in which time intervals are modelled as linear segments on the numerical line. Kulpa (1997a, 2006) proposed an alternative representation of CTIs, in which time intervals are mapped to points in a two-dimensional space. Based on Kulpa's work, Van de Weghe (2007) named it the Triangular Model (TM) and applied it in an archaeological context. De Tré et al. (2006) made the first trial of handling imperfect time intervals by TM and applied it to a historical database. In these studies, TM has already shown its potential in delivering compact and human-friendly visualisations of time intervals. Yet, at present, the power of TM in handling FTIs and RTIs has not been fully revealed. Therefore, in this paper, we will comprehensively study the use of TM in handling RTIs and FTIs.

In the remainder of the paper, we first discuss the two types of imperfect time intervals, namely fuzzy time intervals and rough time intervals (Section 2). Further, after a brief introduction of the Triangular Model (Section 3), we will show how TM can be used to handle rough time intervals (Section 4) and fuzzy time intervals (Section 5), as well as their temporal relations. In Section 6, this new model is then applied to an archaeological case in order to illustrate its analytical power. Finally, conclusions are drawn and future work is proposed.

2. Rough and fuzzy time intervals

In philosophy, there are two opposite theories of time, namely, absolute time and relative time (Lin, 1991). An interesting overview of other theories is given by Knight and Ma (1993), who propose a consensus glossary of temporal concepts. In the absolute concept, time is totally decided by numbers on the time line and is independent of anything else. The basic time units are time intervals, which are defined by two numbers on the time line. Because the time line underlying most calendar systems can be modelled as a single numerical axis, which is isomorphic to real numbers $\mathbb{R}$, a time interval is usually understood as an interval of $\mathbb{R}$, bounded by two real numbers I^- and I^+, with $I^- < I^+$. Absolute time is a machine-oriented concept, because it is measured by clocks, and can be easily recorded and processed by computers. Therefore, the absolute concept of time is widely accepted in computer science and artificial intelligence (Shoham and Goyal, 1988; Vila, 1994). In contrary to absolute time, the relative concept claims that time is determined by events and properties of time must be defined by investigating properties of events. This concept is based on human perception of time, which is often expressed by *when*-clauses in human language. For example, in the sentence 'Katrina tornado happened *when* George W. Bush was the president', the event 'Bush was the president' indicates the period during which Katrina tornado happened. Theoretically, relative time may be expressed in terms of absolute time. In other words, an event perceived by human beings always corresponds to a specific interval in the absolute time line. For example, the period, when George W. Bush was the president corresponds to the interval between January 20, 2001 and January 20, 2009. Such matches between relative time and absolute time broadly exist in daily life. All historical events are linked to specific past time intervals and all plans in people's schedules occupy specific intervals in the future. However, difficulties in matching relative time (intervals defined by events) to absolute time (intervals defined by numbers on the time line) are likely to happen. In these cases, the temporal location of events cannot be perfectly described by a CTI in the absolute time. In order to solve these problems, scientists applied rough sets and fuzzy sets to represent time intervals of these events, bringing concepts of RTI and FTI. In the following two subsections, we will introduce the concepts of rough and fuzzy time intervals.

2.1. Rough time interval

Due to incomplete information, sometimes the temporal location of an event cannot be decisively matched to a CTI in absolute time. It may be known that the event started within a certain range and ended within a certain range. However, information of the exact beginning time and ending time is not available. Scientists applied rough sets to describe intervals of such events (e.g. Bittner, 2002; Bassiri et al., 2009), by defining upper approximations and lower approximations of intervals that are occupied by events. These intervals, described by rough sets, are called rough time intervals (RTI). Generally speaking, incomplete information may have two causes. The first one is the granularity of descriptions. In daily life, time intervals are usually described with certain partitions of the time line (e.g. year, month, days, and hours). Sometimes, these partitions may be too coarse to reflect the exact intervals of events. For instance, the sentence 'Bush started his presidency in 2001 and ended it in 2009' is correct with respect to the yearly granularity. Nevertheless, this sentence lacks information on the exact date when Bush started/finished his presidency. Bittner (2002) applied the rough set theory to represent the relations between time intervals and partitions of the time line. Cells of the partition that are definitely occupied by the time interval form the lower approximation. Cells that may be occupied by the time interval constitute the upper approximation. The second cause of incomplete information stems from the data acquisition process. In many observation activities, data are acquired at discrete time stamps. Through these snapshots, the time interval of an event can only be approximately decided. Remote sensing, for instance, relies on images or photographs taken at discrete time stamps by which one can determine the state of an object. A specific state of an object can be understood as an event, for instance, the existence of a building. From discrete images, one can determine whether a building exists at specific time stamps. However, existence of the building between two time stamps is uncertain. With these discrete observations, the interval of the existence of the building can be described by an upper approximation and a lower approximation (Bassiri et al., 2009).

The basic idea of a rough set is classifying a set R into a lower approximation $\underline{R}$ and an upper approximation $\overline{R}$ according to a subset of its attributes. Within $\underline{R}$, elements are definitely members of a target set X; while outside $\overline{R}$, elements are not members of X, with $\underline{R} \subseteq \overline{R}$. The difference between $\overline{R}$ and $\underline{R}$ forms the boundary regions. If the boundary region is nonempty, i.e. $\overline{R} \neq \underline{R}$, the set R is said to be rough; otherwise R is a crisp set. In the boundary region, elements cannot be decisively classified as members or non-members of X. Since time intervals are considered as convex subsets of real numbers ($\mathbb{R}$), an imprecise interval I can also be modelled by an upper approximation $\overline{I}$ and a lower approximation $\underline{I}$, with $\underline{I} \subseteq \overline{I}$ (Fig. 1). We call such a pair of $\overline{I}$ and $\underline{I}$ a Rough Time Interval (RTI), denoted as I^R. Both $\overline{I}$ and $\underline{I}$ are close sets because their left and right extremes are parts of themselves. In our notion, a rough time interval is not a special kind of time interval. It is essentially an imprecise description of a crisp interval which cannot be precisely described due to lack

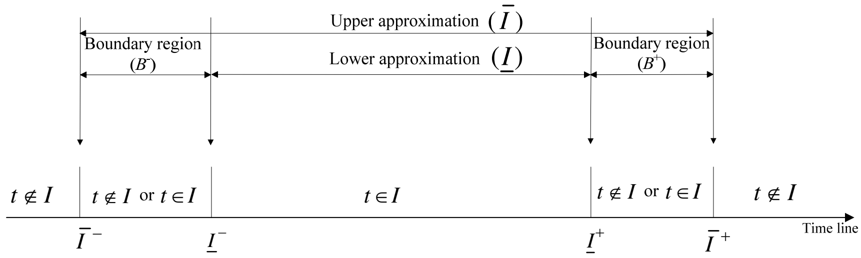

Figure 1. The linear representation of rough time intervals

of information. Since a CTI can be considered as a crisp subset of $\mathbb{R}$, an RTI can be considered as a rough subset of $\mathbb{R}$. Time points in $\underline{I}$ definitely belong to I, whereas time points out of $\overline{I}$ definitely do not belong to I. In between $\underline{I}$ and $\overline{I}$, there are two boundary intervals (B^-, B^+) with time points that cannot be decided whether they belong to I or not. $\overline{I}$ is the largest possibility of I, which is bounded by the earliest possibility of the beginning $\overline{I}^-$ and the latest possibility of the end $\overline{I}^+$. $\underline{I}$ is the shortest possibility of I, which is bounded by the latest possibility of the beginning $\underline{I}^-$ and the earliest possibility of the end $\underline{I}^+$.

2.2. Fuzzy time interval

Difficulties in matching events to intervals in absolute time may originate from the fuzzy nature of events. Some events may start or end gradually, and thus lack distinct beginning or ending time. For example, it is difficult to decide when the Industrial Revolution started and finished. Though some historians like to use the invention of the steam engine to mark its beginning, it is unnatural to understand that the revolution suddenly started when the steam engine was invented. Other examples are clauses like ‘when I was young’. It does not make sense to consider a specific day, after which one is suddenly old. Intervals of these fuzzy events cannot be adequately described by CTIs. The fuzzy set theory is a frequently used tool for modelling intervals of such fuzzy events (see Nagypál and Motik, 2003; Ohlbach, 2004; Schockaert and De Cock, 2008; Schockaert et al., 2008). It extends conventional (crisp) set theory and handles the concept of partial truth, i.e. graded truth values between 0 (completely false) and 1 (completely true) (Zadeh, 1996a,b). A fuzzy set A is modelled by a membership function $\mu_A(x)$ that maps every real number x in $\mathbb{R}$ to a real number between 0 and 1, representing the truth of whether x is a member of A, or to what extent x belongs to A. Besides fuzzy events mentioned above, fuzzy set can also express uncertainties of non-fuzzy events in the framework of possibility theory (Dubois et al., 1991, 2003; Garrido et al., 2009). But in this paper, we are not going to investigate FTIs in the possibilistic context, which will be left for future work.

Following the principles of the fuzzy set theory, time intervals of fuzzy events are modelled as fuzzy sets, which quantify the truth of whether time points on the time line are occupied by events. Intervals described by fuzzy sets are called fuzzy time intervals (FTI) and denoted as $\tilde{I}$. For an arbitrary FTI $\tilde{I}$, every time point t on the time line is mapped to $\tilde{I}$ by a membership function $\tilde{I}(t)$, which returns the truth value of whether t is a member of $\tilde{I}$. All time points t that satisfy $\tilde{I}(t) = 1$ form the core of $\tilde{I}$, denoted as Core($\tilde{I}$), while all time points satisfy $\tilde{I}(t) > 0$ form the support of $\tilde{I}$, denoted as Support($\tilde{I}$) (Fig. 2). If Core($\tilde{I}$) = Support($\tilde{I}$), $\tilde{I}$ reduces to a CTI. In order to start from the simplest situation, we assume that an FTI $\tilde{I}$ must have a non-empty core, and both Support($\tilde{I}$) and Core($\tilde{I}$) are convex intervals. Intervals in between Support($\tilde{I}$)

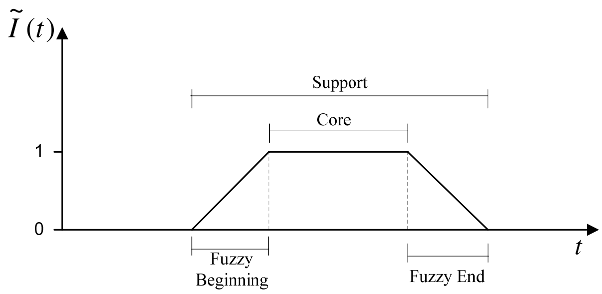

Figure 2. An FTI illustrated by a line chart

and Core($\tilde{I}$) are called fuzzy beginning and fuzzy end (Fig. 2). Unlike relations between CTIs, relations between FTI and another interval (CTI or FTI) cannot be decided by a yes or no answer, but only quantified as a truth value between 0 and 1 (Nagypál and Motik, 2003; Ohlbach, 2004; Schockaert and De Cock, 2008; Schockaert et al., 2008).

3. The Triangular Model (TM)

3.1. Representing crisp time intervals (CTI) in TM

In the classical representation, a time interval is represented as a finite linear segment on a horizontal time line (Fig. 3 top-left). From the paralleled scales on the time line, one may read the numbers of I^- and I^+ of the interval. The vertical dimension is solely used to differentiate multiple overlapping intervals, if used at all. The linear representation of time intervals is widely used in our daily life, for example Gantt charts and historical time lines. In different reasoning systems, whether an interval is open (at one side or both sides) is defined differently (Vila, 1994). TM does not intend to solve reasoning issues that concerns this controversy. Whether an interval is open does not affect its representation in TM. In this paper, we define a crisp interval as close at both sides and denote it as $[I^-, I^+]$. Differently from the classic representation, the basic concept of the Triangular Model (TM) is the construction of two lines through the extremes of an interval (Fig. 3 top-right). For each time interval I, two straight lines (L_1 and L_2) are constructed, with L_1 passing through I^- and L_2 passing through I^+. α_1 is the angle between L_1 and the horizontal axis and α_2 is the angle between L_2 and the horizontal axis, with $\alpha_1 = -\alpha_2 = \alpha$. The intersection of L_1 and L_2 is called the interval point. The beginning of the interval I^-, the end of the interval I^+ and interval point form an isosceles triangle. That is why we call this model the Triangular Model (TM). The angle α is a pre-defined constant which is identical for all interval points, in order to ensure that each time interval is mapped to a unique 2D point. In this paper, we keep consistency with previous research of TM (Kulpa, 1997a; Van de Weghe et al., 2007) and set $\alpha = 45°$. Of course, α can also be altered to other angles (between 0 and 90 degrees), for specific purposes. In TM, all time intervals are represented as such interval points in a two-dimensional space (Fig. 3 bottom). In other words, the position of an interval point completely determines both, the beginning and the end of the interval. The two-dimensional space of interval points is called the Interval Space. Because all time intervals can be considered as subsets of real numbers $\mathbb{R}$, the interval space is denoted as $I\mathbb{R}$ (Kulpa, 2006).

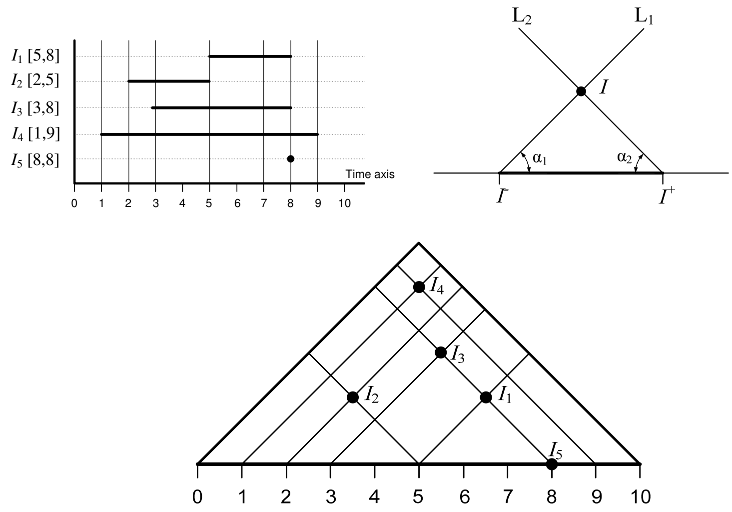

Figure 3. Transformation from the linear representation to TM. Top-left: the classic linear model. Top-right: construction of an interval point. Bottom: time intervals in TM

3.2. Representing crisp temporal relations in TM

Let the beginning I^- and end I^+ of two intervals have the following three possible relations: smaller than (<), equal to (=) and larger than (>). Then, according to Allen (1983), thirteen possible relationships between two CTIs can be defined (see Table 1). In TM, every Allen's relation corresponds to a specific zone (Kulpa, 1997a,b). Given a study period from 0 to 100, all examined intervals are located within the isosceles triangle of $I[0, 100]$. Let us consider a reference interval $I_2[33, 66]$ and several intervals (I_{1a}, I_{1b}, I_{1c}) existing before the interval I_2 (Fig. 4a). In TM, I_{1a}, I_{1b}, I_{1c} are located in the triangular zone in the left corner of the study area (Fig. 4b). Therefore, it is easy to deduce that all intervals before I_2 must be located in the black zone (Fig. 4c). Namely, this zone encloses all intervals that are before I_2. All Allen's relations with respect to a

Table 1. Thirteen Allen's Relations (Allen, 1983)

I_1 *equal* I_2	if $I_1^- = I_2^-$	$\wedge\ I_1^+ = I_2^+$	
I_1 *starts* I_2	if $I_1^- = I_2^-$	$\wedge\ I_1^+ < I_2^+$	
I_1 *started-by* I_2	if $I_1^- = I_2^-$	$\wedge\ I_2^+ < I_1^+$	
I_1 *finishes* I_2	if $I_1^+ = I_2^+$	$\wedge\ I_1^- > I_2^-$	
I_1 *finished-by* I_2	if $I_1^+ = I_2^+$	$\wedge\ I_2^- > I_1^-$	
I_1 *meets* I_2	if $I_1^+ = I_2^-$		
I_1 *met-by* I_2	if $I_2^+ = I_1^-$		
I_1 *overlaps* I_2	if $I_2^- > I_1^-$	$\wedge\ I_1^+ < I_2^+$	$\wedge\ I_1^+ > I_2^-$
I_1 *overlapped-by* I_2	if $I_1^- > I_2^-$	$\wedge\ I_1^- < I_2^+$	$\wedge\ I_2^+ < I_1^+$
I_1 *during* I_2	if $I_1^- > I_2^-$	$\wedge\ I_1^+ < I_2^+$	
I_1 *contains* I_2	if $I_2^- > I_1^-$	$\wedge\ I_2^+ < I_1^+$	
I_1 *before* I_2	if $I_1^+ < I_2^-$		
I_1 *after* I_2	if $I_2^+ < I_1^-$		

CTI can be represented by such zones in $I\mathbb{R}$ (Fig. 5). For each individual figure in Fig. 5, the reference CTI I has been chosen in the centre of the study period in order to avoid visual bias. Because all zones in Fig. 5 occupy a unique area in $I\mathbb{R}$, $I\mathbb{R}$ can be divided into thirteen zones with respect to thirteen Allen's relations (Fig. 6). We call these zones Crisp Relational Zones (CRZ). Each CRZ represents the set of CTIs that are in a specific relation with respect to the reference interval I. Such set of CTIs is denoted as $R(I)$. For example, the set of CTIs that are before I is denoted as *before*(I). According to the number of common extremes (i.e. beginning and end), Allen's relations can be categorized into two types. Type 1 intervals have no common extremes (*before*, *overlaps*, *during*, *contains*, *overlapped-by*, *after*). Type 2 intervals have common extremes (*meets*, *met-by*, *starts*, *started-by*, *finishes*, *finished-by* and *equal*). In TM, Type 1 CRZs have a point-like or linear geometry (zero-dimensional and one-dimensional); while Type 2 CRZs have a triangle-like or lozenge-like shape (two-dimensional).

4. Handling rough time intervals (RTI) in TM

4.1. Representing RTIs in TM

Different from the linear model in which an RTI is a linear segment with an uncertain beginning and an uncertain end (Fig. 7), TM represents RTIs as geometries in a two-dimensional space. In order to construct an RTI I^R in TM, four lines are projected respectively from $\overline{I}^-$, $\underline{I}^-$, $\underline{I}^+$ and $\overline{I}^+$, forming a lozenge (Fig. 8). This lozenge indicates a zone where the exact CTI I can be found. In other words, this zone represents the set of CTIs that are possibly equal to I. Therefore, the zone is called the *maybe equal* (*ME*) zone. Other relational zones of RTIs will be discussed in detail in the next section. We note that the *ME*

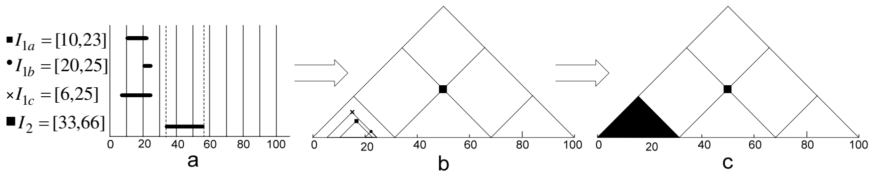

Figure 4. Temporal relations in the linear model and TM, taking *before* as an example

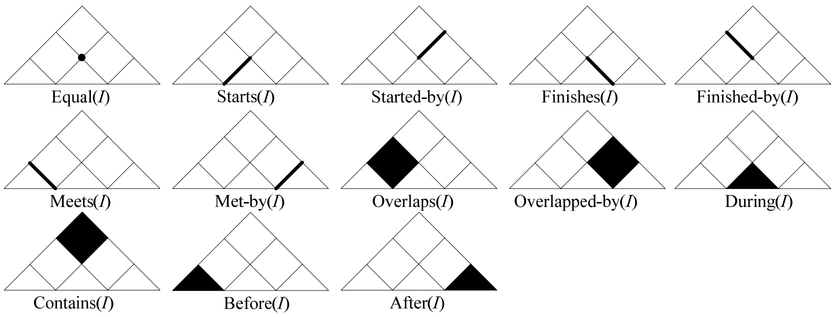

Figure 5. CRZs in individual interval spaces, representing sets of intervals in Allen relations to the reference interval I

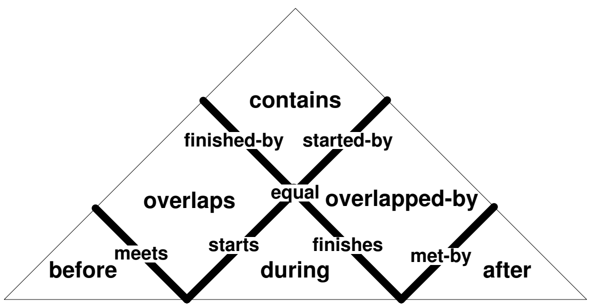

Figure 6. CRZs of an interval in TM

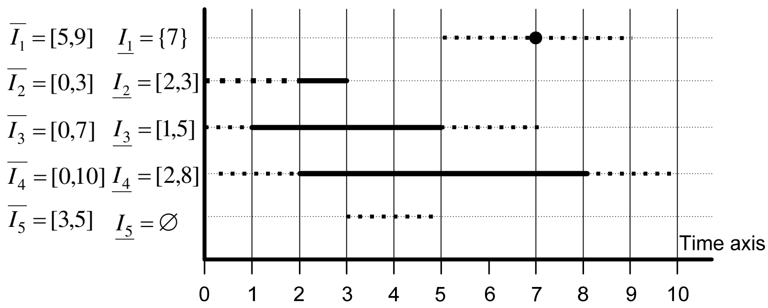

Figure 7. Rough time intervals in the linear concept. Solid lines denote $\underline{I}$, and dotted lines denote B^- and B^+ . The combination of the solid line and dotted lines forms $\underline{I}$

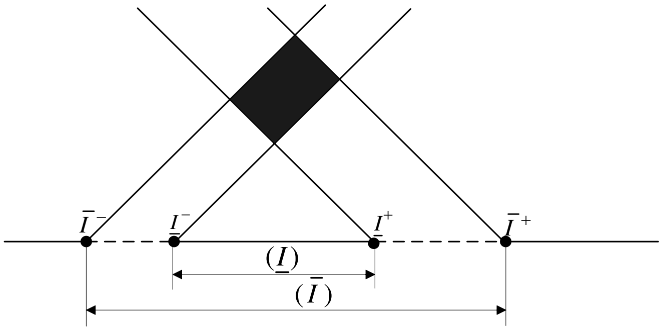

Figure 8. The construction of an RTI in TM

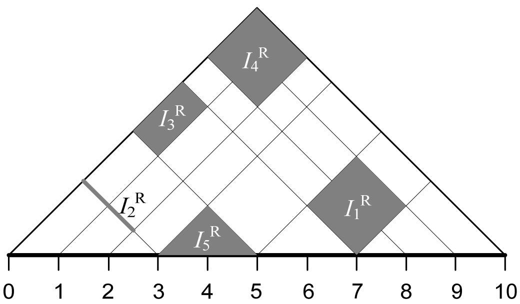

Figure 9. Using TM to represent RTIs of Fig. 7

zone is the only relational zone that contains all the information of I^R. Thus, I^R can be represented by its *ME* zone which covers all possible locations of I. In this way, I^R is represented by a 2D geometry (Fig. 9). Apart from a lozenge, the 2D geometry can take other shapes. For instance, if $\underline{I} = \emptyset$, the *ME* zone becomes a triangle on the horizontal axis (e.g. I_5 in Fig. 9) . If either $B^- = \emptyset$ or $B^+ = \emptyset$, the *ME* zone becomes a line (e.g. I_2 in Fig. 9). If both $B^- = \emptyset$ and $B^+ = \emptyset$, I^R reduces to a CTI and its *ME* zone becomes a point. In this paper, we will emphasize RTIs whose *ME* zone is a lozenge.

4.2. Rough relational zones (RRZ) of RTIs

According to the upper approximation $\overline{I}$ and the lower approximation $\underline{I}$ of an RTI, the interval space ($I\mathbb{R}$) can be divided into zones. The number of zones depends on whether the lower approximation or boundary regions are empty. Firstly we focus on RTIs with $B^+ \neq \emptyset$, $B^- \neq \emptyset$ and $\underline{I} \neq \emptyset$. With respect to this kind of RTIs, $I\mathbb{R}$ are divided into 15 zones (Fig. 10). We call these zones Rough Relational Zones (RRZ). Table 2 lists details of the 15 RRZs of Fig. 10. When comparing CRZs and RRZs, we can see that polygons in CRZs (i.e. zones of Type 1 relations) remain polygons in RRZs, whereas the point and lines in CRZs (i.e. zones of Type 2 relations) have expanded to polygons in RRZs. Two new RRZs are expanded from the beginning and end point of I. Contrary to CRZs, which express determinate relations, these expanded RRZs express more than one possible relation. For example, intervals in the *Maybe Meets* (*MM*) zone have three possible relations to I, i.e. *meets*, *before* and *overlaps* (Table 2). In RRZs that are not expanded from points and lines, only one

Table 2. Details of RRZs with respect to Fig. 10

Name of RRZ	Abbreviation	Possible Relations to *I*	Original Name in CRZ
Before	B	Before	Before
Overlaps	O	Overlaps	Overlaps
Contains	C	Contains	Contains
During	D	During	During
Overlapped-by	OB	Overlapped-by	Overlapped-by
After	A	After	After
Maybe Meets	MM	Meets, Before, Overlaps	Meets
Maybe Starts	MS	Overlap, Starts, During	Starts
Maybe Finished-by	MFB	Overlaps, Finished-by, Contains	Finished-by
Maybe Equal	ME	Contains, Started-by, Overlapped-by, Finishes, During, Starts, Overlaps, Finished-by, Equal	Equal
Maybe Started-by	MSB	Contains, Overlapped-by, Started-by	Started-by
Maybe Finishes	MF	Overlapped-by, Finishes, During	Finishes
Maybe Met-by	MMB	Overlapped-by, Met-by, After	Met-by
Rough Beginning	RB	Before, Meets, Overlaps, Starts, During	N/A
Rough End	RE	During, Finishes, Overlapped-by, Met-by, After	N/A

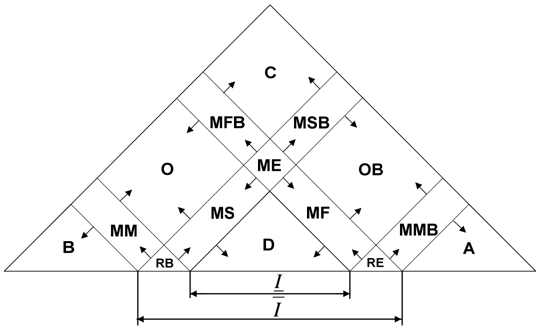

Figure 10. Rough Relational Zones of an RTI, with $B^+ \neq \emptyset$, $B^- \neq \emptyset$ and $\underline{\mathrm{I}} \neq \emptyset$

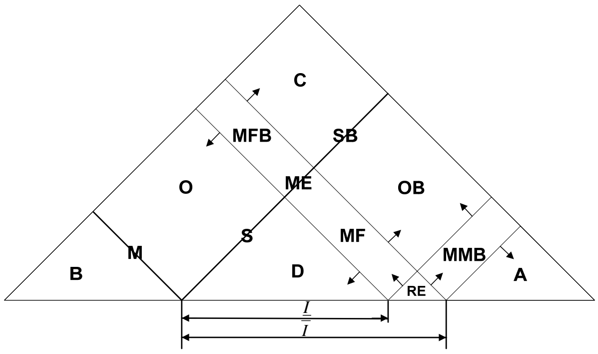

Figure 11. Rough Relational Zones of an RTI, with $B^+ = \emptyset, B^- \neq \emptyset$ and $\underline{\mathrm{I}} \neq \emptyset$

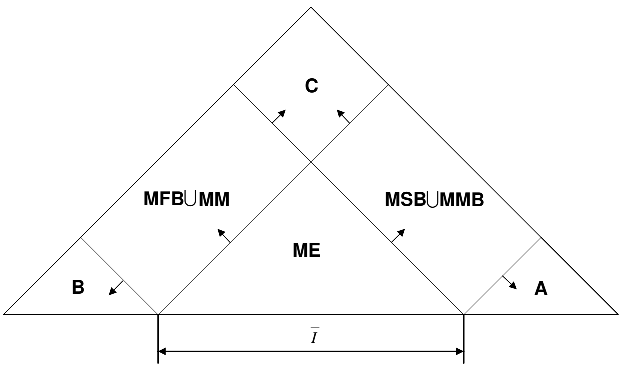

Figure 12. Rough Relational Zones of an RTI, with $\underline{I} \neq \emptyset$

relation is possible. Boundaries between RRZs do not occupy any space, but belong to the neighbouring RRZs. Arrows in Fig. 10 indicate the belongingness of boundaries. Moreover, RTIs with an empty lower approximation or an empty boundary region have different number and structure of RRZs. For example, RTIs with an empty boundary region (i.e. $B^- = \emptyset$) have 14 RRZs (Fig. 11). For this type of RTIs, some RRZs shrink to lines or points, and the RB zone does not exist any more. RTIs with an empty lower approximation ($\underline{I} = \emptyset$) have only 6 RRZs (Fig. 12).

4.3. Temporal relations between RTIs

Bassiri (2009) enumerates 68 topological relations between two RTIs (e.g. I_1^R, I_2^R) by 2×2 matrixes of relations between $\overline{I_1}$ and $\overline{I_2}$, $\overline{I_1}$ and $\underline{I_2}$, $\underline{I_1}$ and $\overline{I_2}$, $\underline{I_1}$ and $\underline{I_2}$. However, these topological relations between RTIs do not directly deliver practical meanings. Because RTIs are essentially imprecise descriptions of CTIs, the useful information comes from temporal relations between the exact CTIs behind RTIs. Therefore, in this section, we investigate how to use topological relations between RTIs to deduce relations between the exact CTIs. If two RTIs (e.g. I_1^R, I_2^R) are represented in the linear model (Fig. 13), the possible temporal relations between I_1 and I_2 and cannot be directly captured by human beings. But by TM, relations between I_1 and I_2 can be easily decided by intersecting I_1^R and RRZs of I_2^R. For example, in Fig. 14, when intersecting the lozenge of I_1^R and RRZs of I_2^R, I_1^R 'touches' four RRZs of I_2^R, namely, the *maybe meets*, *rough beginning*, *overlaps* and *maybe starts* zones. Note that, within this context, 'touch' means that two zones have common parts. By checking Table 2, we obtain the possible relations between I_1 and I_2 as the union of possible relations of these four RRZs, i.e. {*meets, before, overlaps*}$\bigcup${*meets, before, overlaps, starts ,during*}$\bigcup${*overlaps*, *starts*, *during*}$\bigcup${*overlaps*}={*meets*, *before*, *overlaps*, *starts*, *during*}. With this approach, one can easily deduce possible temporal relations between two RTIs. If I_2^R has a non-empty lower approximation and non-empty boundary regions (namely $B_2^- \neq \emptyset$, $B_2^+ \neq \emptyset$ and $\underline{I_2} \neq \emptyset$)), there are 80 topological relations between I_1 and RRZs of I_2 (Fig.$\tilde{1}$5). 21 different sets of possible relations between I_1 and I_2 can be inferred from these 80 relations (Table 3). There are 12 more topological relations than Bassiri's (2009), because Bassiri missed situations when I_1^R has an empty lower approximation and I_1^R is totally in the boundary region of I_2^R. We marked these situations with stars in Fig. 15.

4.4. Temporal relations between RTIs and CTIs

If an interval I is described by an upper approximation $\overline{I}$ and a lower approximation $\underline{I}$, the set of CTIs that are in a specific Allen's relation to I also will also have an upper approximation $\overline{R(I)}$ and a lower approximation $\underline{R(I)}$. Fig. 16 displays rough sets counterparts of Allen's relations in TM. Black zones represent lower

Table 3. Twenty one different sets of possible relations between I_1 and I_2 with respect to Figure 15

Number	Possible Relations	Number	Possible Relations	Number	Possible Relations
1	before	8	before, meets, finished-by, contains, overlaps	15	overlapped, finishes, during
2	overlapped	9	overlaps, finished-by, contains	16	contains, started-by, overlapped-by
3	contains	10	before, meets, overlaps, starts, during	17	finishes, during, overlapped-by, met-by, after
4	during	11	before, meets, overlaps, overlapped-by, starts, started-by, during, equal, finishes, finished-by, contains	18	overlapped-by, met-by, after,
5	overlapped-by	12	overlaps, starts, during ,finishes, equal, overlapped-by, started-by, finished-by, contains	19	during, finishes, overlapped-by, met-by, after, starts, equal, overlaps, started-by, finished-by, contains
6	after	13	contains, started-by, overlapped-by	20	contains, started-by, overlapped-by, met-by, after
7	before, meets, overlaps	14	overlaps, starts, during	21	All 13 Allen's relations

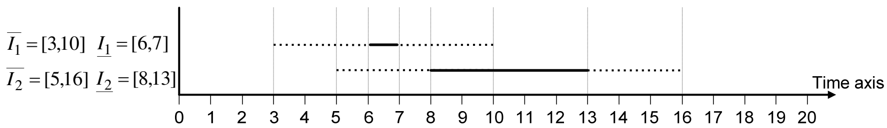

Figure 13. The linear representation of I_1^R and I_2^R

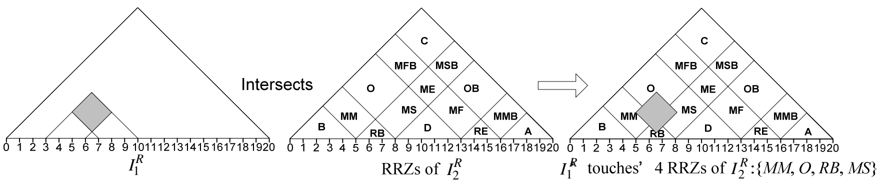

Figure 14. Using TM to deduce possible relations between I_1 and I_2

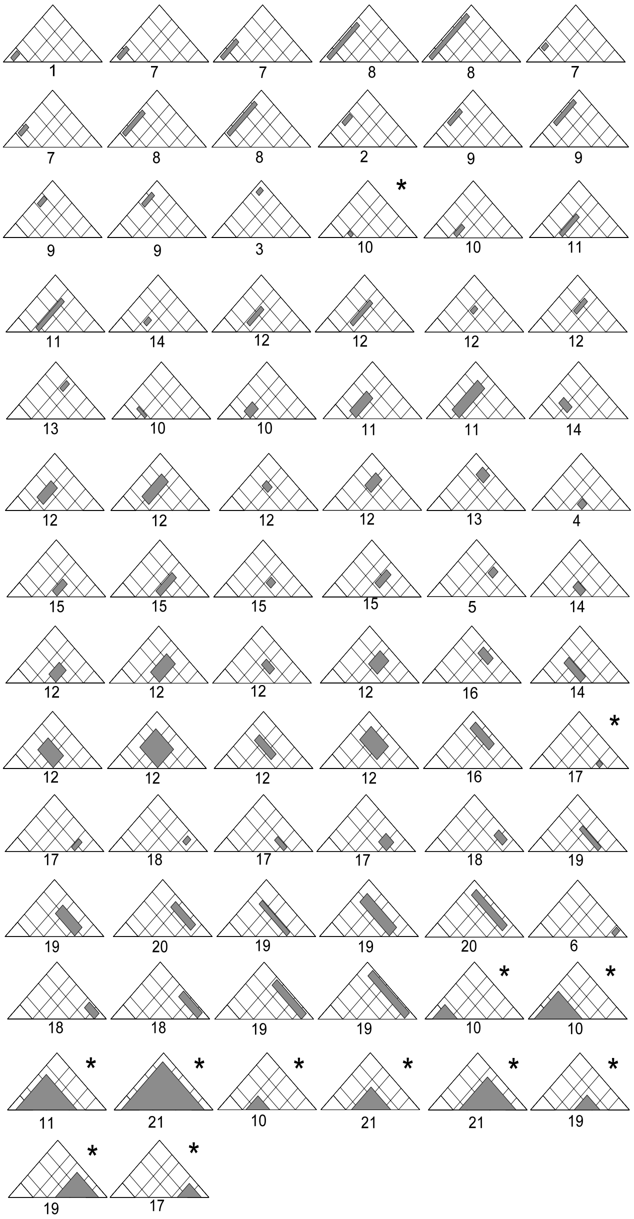

Figure 15. All possible topological relations between I_1^R and RRZs of I_2^R ($B_2^- \neq \emptyset$, $B_2^+ \neq \emptyset$ and $\underline{I_2} \neq \emptyset$). The number below each situation corresponds to the number in Table 3, indicating the set of possible relations between I_1 and I_2 (* denotes situations that are not included in Bassiri's relations, Bassiri et al., 2009).

approximations $\underline{R(I)}$, while combinations of the black zone and the grey zone are upper approximations $\overline{R(I)}$. More exactly, CTIs in the black zone ($\underline{R(I)}$) are definitely in the relation R to I, while CTIs in the white zones ($\neg\overline{R(I)}$) are definitely not in the relation R to I. In the grey region ($\overline{R(I)} - \underline{R(I)}$), intervals that may or may not in the relation R to I. Note that for Type 1 relations, both $\overline{R(I)}$ and $\underline{R(I)}$ are non-empty. Whereas for Type 2 relations, $\underline{R(I)}$ is empty.

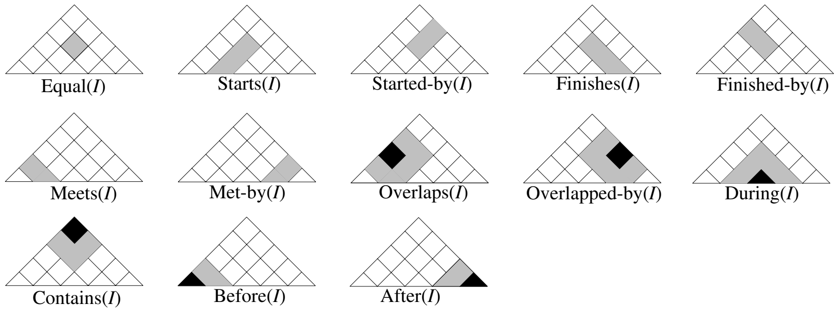

Figure 16. Rough sets of intervals in Allen's relations to I

4.5. Queries about relations between RTIs and CTIs

As discussed in Sections 3.2, the set of CTIs in a certain relation to another CTI I are modelled as zones in TM, i.e. $R(I)$. These zones can be used to model constraints of temporal queries of I. By operations on these zones, CTIs that satisfy these constraints can be obtained. Conjunctive queries can be answered by the intersection operation, e.g. $R(I_1) \cap R(I_2) \cap R(I_3)$. For example, CTIs during I_v and overlaping I_u, i.e. $\{I | overlaps(I, I_u) \wedge during(I, I_v)\}$, can be obtained by intersecting zones of $overlaps(I_u)$ and $during(I_v)$ in TM. As discussed in Section 4.4, if I_u and I_v are described by RTIs, $overlaps(I_u)$, $during(I_v)$, will also be rough sets. Fig. 17 illustrates the process of obtaining the set of CTIs that overlap I_u and during I_v, when I_u and I_v are described by RTIs. $\overline{overlaps(I_u)}$ and $\overline{during(I_v)}$, $\underline{overlaps(I_u)}$ and $\underline{during(I_v)}$ are intersected respectively and then both intersections are combined together to form another rough set $< \overline{overlaps\,(I_u) \bigcap during\,(I_v)}, \underline{overlaps(I_u) \bigcap during(I_v)} >$. Similarly, disjunctive queries can be answered by the union operation, e.g. $R(I_1) \cup R(I_2) \cup R(I_3)$. Intervals containing I_w and after I_x are obtained by the union of $contains(I_w)$ and $after\ (I_x)$. If I_w and I_x have upper and lower approximations, the resulting set $\{I | contains(I, I_w) \vee after(I, I_x)\}$ will also be a rough set (Fig. 18).

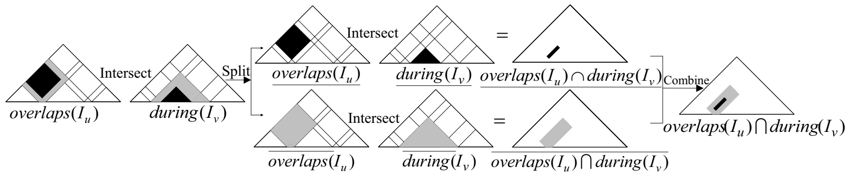

Figure 17. Querying for the set $\{I|overlaps(I, I_u) \wedge during(I, I_v)\}$ in TM

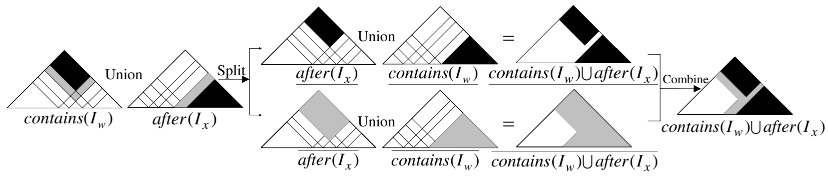

Figure 18. Querying for the set $\{I|contains(I, I_w) \wedge after(I, I_x)\}$ in TM

5. Handling fuzzy time intervals (FTI) in TM

According to the assumptions of Section 2.2, an FTI consist of a non-empty core and support, both of which occupy a convex CTI. If the fuzzy beginning and the fuzzy end are linear and monotonic, the intervals of the core and the support totally decide the pattern of the FTI. De Tré et al. (2006) modelled such trapezoidal FTIs in TM as linear segments between the interval point of the core and the interval point of the support (Fig. 19). The position, inclination and length of the linear segment totally determine the configuration of an FTI. This approach shows potential in displaying simple FTIs in historical databases (Fig. 20). However, it is restricted to cases in which people are solely interested in the configuration of the core and support, but neglect the functions of the fuzzy beginning and end, for example, FTIs with fuzzy beginnings and ends defined by the same function. For FTIs defined by diverse membership functions, then the approach cannot fully describe their differences. At this point, FTIs with arbitrary functions cannot be adequately modelled within TM. Further extensions and modifications to TM are still to be elaborated for complex FTIs and the relations between them, which is left for future work. In this section, we will solely investigate the relations between FTIs and CTIs. In contrary to relations between RTIs and CTIs which are represented as discrete geometries, relations between FTIs and CTIs are modelled as continuous fields in the 2D space. In this paper, 'fields' refer to 2D areas that are described in a continuous manner. In a field, every possible position is described by a variable or a set of variables (Longley et al., 2001). With operations on these continuous fields, queries about CTI-FTI relations can be answered in a more intuitive way.

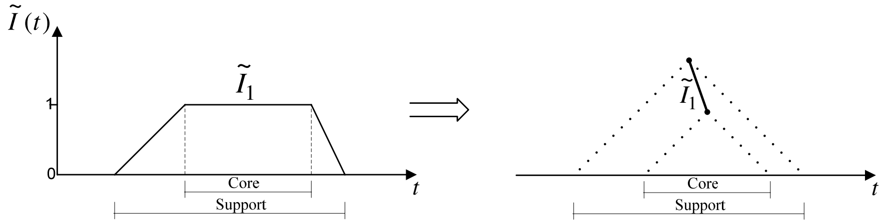

Figure 19. Representing a simple FTI as a linear segment in TM (De Tré et al., 2006)

Number	Country	Name	*support*([a, b])	*core*([a, b])
1	Belgium	Leopold II	[1865, 1908]	[1865, 1908]
2	Belgium	Albert I	[1908, 1934]	[1908, 1934]
3	France	Loubet	[1898, 1907]	[1899, 1906]
4	France	Fallières	[1906, 1913]	[1907, 1913]
5	Italy	Vittorio Emanuele III	[1899, 1944]	[1900, 1945]
6	Spain	Maria Cristina	[1885, 1902]	[1885, 1902]
7	Spain	Alfonso XIII	[1902, 1931]	[1903, 1930]
8	United Kingdom	Victoria	[1837, 1901]	[1837, 1901]
9	United Kingdom	Edward VII	[1901, 1910]	[1902, 1909]
10	United Kingdom	George V	[1910, 1935]	[1909, 1936]

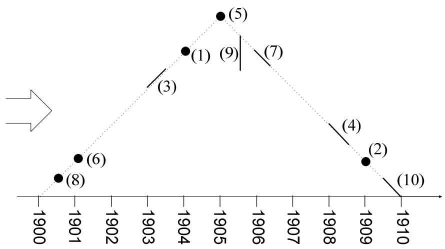

Figure 20. Representing trapezoidal FTIs in a historical database by use of TM (De Tré et al., 2006), restricted to the study period between 1900 and 1910

5.1. Relations between CTIs and FTIs

Relations between two CTIs are determined by binary operators ($<$, $=$ and $>$) between their beginnings and ends. However, these operators do not exist for fuzzy time intervals. Relations between an FTI and another interval (either CTI or FTI) are expressed by a real number between 0 and 1, which quantifies the truth of whether the two intervals are in this relation. Different functions have been proposed to obtain truth values of FTI relations (Nagypál and Motik, 2003; Ohlbach, 2004; Schockaert and De Cock, 2008; Schockaert et al., 2008). Every function takes special care of a certain aspect of expressivity and reasoning ability. All these functions can be represented in TM, yielding continuous fields with different patterns. In this paper, we adopt Schockaert's functions (Schockaert et al., 2008) of FTI relations for its advantages in the reasoning aspect (e.g. reflectivity, symmetry and transitivity) which offers a better basis for further reasoning research. In TM, every point representing a CTI, is allotted a value between 0 and 1, expressing the graded truth of whether this CTI is in a certain relation to an FTI. Let $R(I_1, \widetilde{I_2})$ denote the truth value of whether a CTI I_1 is in the relation R to a FTI $\widetilde{I_2}$. Let $R(\widetilde{I_2})$ denote the fuzzy set of CTIs that is in relation R to $\widetilde{I_2}$. In Fig. 21 left, we illustrate some situations for the *during* relation between a CTI I_1 and an FTI $\widetilde{I_2}$. We refer to Schockaert's work (Schockaert and De Cock, 2008; Schockaert et al., 2008) for a detailed explanation of functions. If the whole part of I_1 is within the core of $\widetilde{I_2}$, then

$during(I_1, \widetilde{I_2}) = 1$. If there is a part of I_1 that is totally out of the support of $\widetilde{I_2}$, then $during(I_1, \widetilde{I_2}) = 0$. In between these two situations, I_1 is partially during I_2, then $0 < during(I_1, \widetilde{I_2}) < 1$. In this way, every I in $I\mathbb{R}$ may have a value of $during(I, \widetilde{I_2})$. The set of CTIs $during(\widetilde{I_2})$ forms a fuzzy set, denoted as $during(\widetilde{I_2})$, and is modelled as a continuous field in TM (Fig. 21 right). Analogously, fuzzy sets of other Allen relations with respect to $\widetilde{I_2}$ can also be represented by continuous fields (Fig. 22). This approach is not restricted to trapezoidal FTIs as we illustrated. It can also apply to FTIs described by other functions.

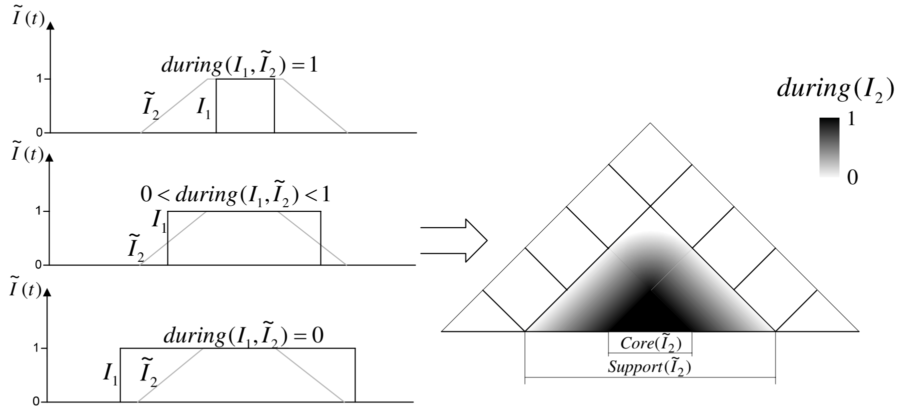

Figure 21. Representing the fuzzy set $during(\tilde{I}_2)$ in TM

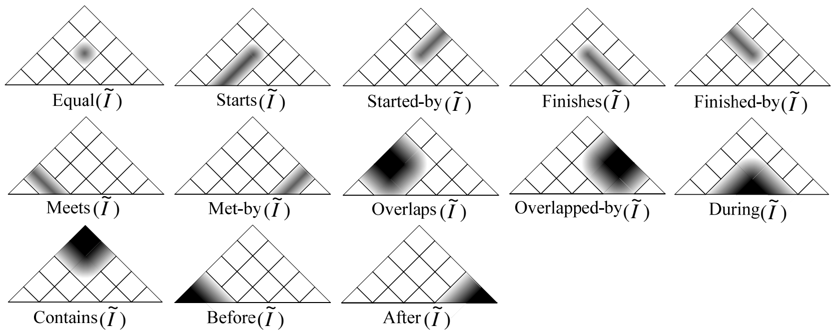

Figure 22. Fuzzy sets of 13 Allen's relations to $\tilde{I}_2$ of Fig. 21

5.2. Queries about relations between FTIs and CTIs

By use of TM, queries about relations between CTIs and FTIs can be answered. As discussed in the previous subsection, constraints of relations between CTIs and an FTI can be modelled by continuous fields in TM. By operations on these continuous fields, one may obtain the set of CTIs that satisfy these constraints. The obtained set of CTIs is also a fuzzy set and modelled as a continuous field in TM. For example, CTIs that contain $\tilde{I}_1$ constitute a fuzzy set, i.e. $contains(\tilde{I}_1)$, while CTIs that contain $\tilde{I}_2$ form another fuzzy set, i.e. $contains(\tilde{I}_2)$. Both $contains(\tilde{I}_1)$ and $contains(\tilde{I}_2)$ are modelled as continuous fields in TM. The intersection of $contains(\tilde{I}_1)$ and $contains(\tilde{I}_2)$ yields the fuzzy set of CTIs that both contain $\tilde{I}_1$ and $\tilde{I}_2$, i.e. $contains(\tilde{I}_1) \bigcap contains(\tilde{I}_2)$ (Fig. 23). In TM, the intersection of fuzzy sets can be obtained by applying fuzzy intersection to every point in the two-dimensional $I\mathbb{R}$. In Fig. 23, the fuzzy intersection uses the minimum t-norm (Dubois and Prade, 2000). Of course, this approach is also compatible to other t-norms. Similarly, unions of fuzzy sets can also be obtained by operations on the continuous fields. For example, in Fig. 24, the union of $contains(\tilde{I}_1)$ and $contains(\tilde{I}_2)$ is obtained by the maximum t-conorm operation (Dubois and Prade, 2000) applied to values in the two continuous fields of $contains(\tilde{I}_1)$ and $contains(\tilde{I}_2)$.

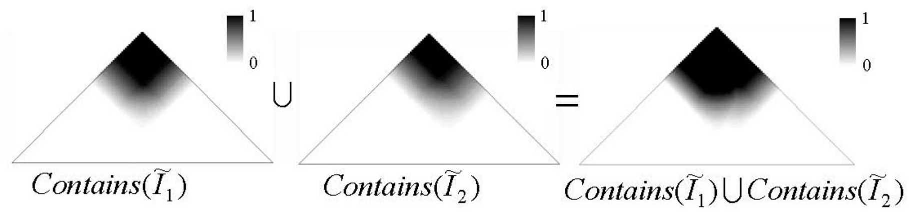

Figure 23. Querying for CTIs that both contain $\tilde{I}_1$ and $\tilde{I}_2$, using the minimum t-norm

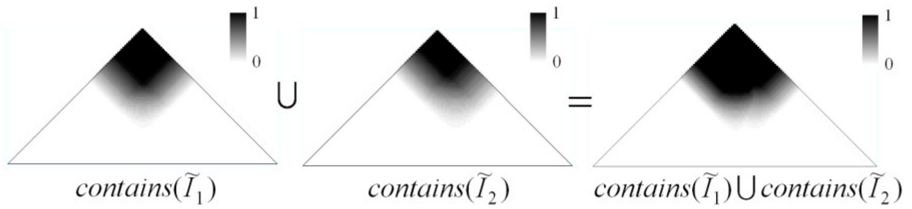

Figure 24. Querying for CTIs that either contain $\tilde{I}_1$ or $\tilde{I}_2$, using the maximum t-conorm

6. Solving practical problems

Because of the remaining problems and difficulties with FTI modeling in TM, as described in the previous section, in this section we solely deal with RTIs to illustrate the usefulness and applicability of TM for the handling of imperfect time intervals in a two-dimensional space. During World War One, aerial photos covering the Belgian-German front line in West-Flanders (Belgium) were taken at discrete time stamps. From these aerial photos, we can observe whether a military feature (e.g. a fire trench, gun position or barrack) was not yet present, present, or destroyed. Although the state of a feature is uncertain in between two time stamps, we assume that it does not change in between two snapshots which show similar states, such that the uncertainty only remains in between two snapshots showing different states. Certainly, this assumption relies on our knowledge that snapshots are dense enough to capture most of features' changes. When more volatile entities are considered, an appropriate temporal resolution will be required. In this context, RTIs are excellently suited to handle time modelling. Indeed, as no information is available about the state of a feature in between two time stamps, the construction of adequate membership functions for FTIs would induce an extra overhead and difficulties in the time modelling and handling. Together with the unsolved representation difficulties for FTIs in TM this justifies the choice to focus the case study under consideration on the use of RTIs.

Indeed, we might consider a period of snapshots showing similar states as a lower approximation for this state, its neighbouring uncertain intervals as boundary region, and all of them form the upper approximation (Fig. 25). Thus, a feature's lifetime can be meaningfully represented by an RTI. Basically, at least four photos are required to determine the lifetime of a feature (Fig. 25): (1) the last photo in which the feature is not yet present, (2) the first photo in which the feature is present, (3) the last photo in which the feature is present, and (4) the first photo in which the feature is destroyed or abandoned. The interval between the dates of photo (2) and (3) is the lower approximation of the feature's lifetime, while interval between the dates of photo (1) and photo (4) is the upper approximation. Intervals between the dates of photo (1) and (2), and intervals

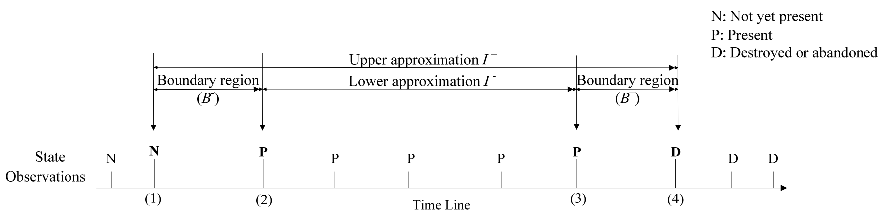

Figure 25. The RTI of a military feature

between the dates of photo (3) and (4) are boundary regions, which indicate respectively the range of the feature's construction and destroy dates. There are a few exceptions, where a feature was not yet present in one photo and already destroyed in the following photo. Since photo (2) and (3) are missing, the RTI for these features has an empty lower approximation. As described in Section 4.2, such RTIs are represented as triangles on the horizontal axis. The dates of the photos have been obtained from the database of the military features in which they were originally stored. From the database, people cannot easily capture distributions of features' RTIs. However, TM can display these RTIs in a more visible way.

Fig. 26 displays RTIs of a type of fire trenches (i.e. FT1) in TM. We assign darker colours to areas that have more overlapped polygons. From this figure we can see that most of FT1s are present in the beginning of the war, because there is a dark zone distributed at the left corner of the study area. Some other FT1s were randomly distributed through the war. However, in the very right corner of the study area, we can see a small group of FT1s that were specifically present at the end of the war. Compared with the data stored in a database, TM provides a more direct visualisation of the distribution of RTIs.

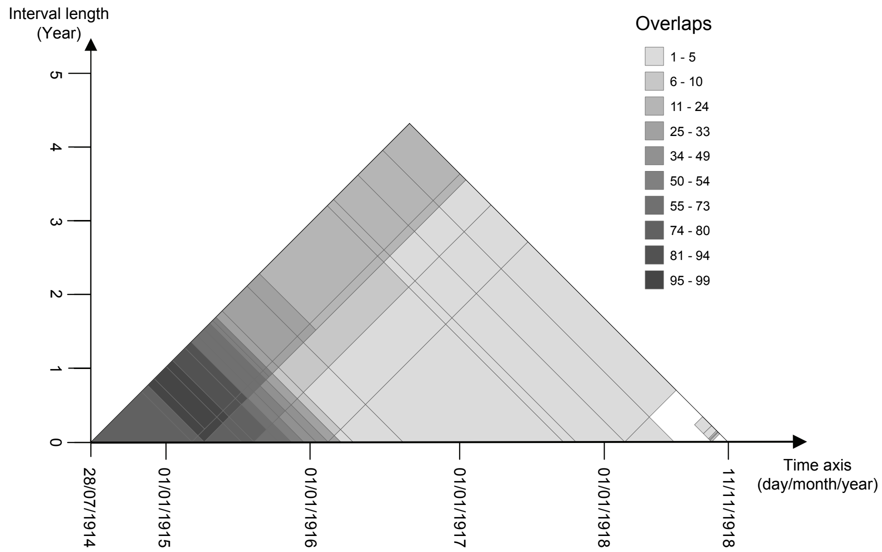

Figure 26. RTIs of FT1 features in TM

Moreover, TM may be combined with traditional geographical maps in order to support exploratory spatiotemporal analysis. In Fig. 27, RTIs of barracks' lifetimes are displayed in TM where we can see barracks are temporally distributed in two clusters. The construction dates of barracks in these two clusters overlap in most of the cases, whereas their destroy dates are clearly

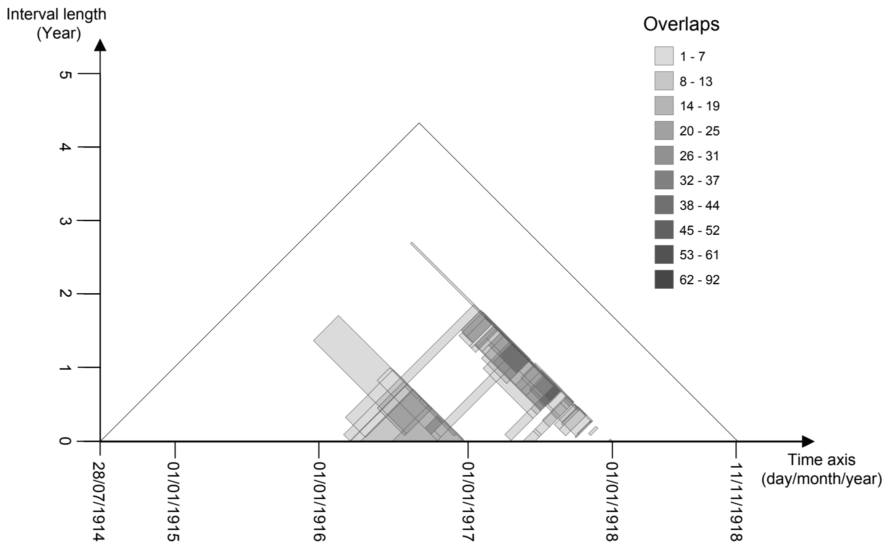

Figure 27. RTIs of barracks in TM

distributed in two distinctive periods. Barracks in the first cluster are mostly destroyed in 1916, while barracks in the second cluster are mostly destroyed in the second half of 1917. When checking the geographical distribution of these barracks (Fig. 28), one may observe that most barracks of the second cluster are further away from the front line than barracks of the first cluster. From this observation, people may infer that barracks near the front line were destroyed earlier than barracks further away from the front line. According to records of the war, fighting along the front line was getting increasingly intensive during the period of the two clusters. This fact can probably explains our finding in TM: barracks near the front line were destroyed or abandoned due to intensive fighting; while barracks further away from the front line survived for a longer time. In general, TM offers people a preliminary perception of the distribution of RTIs, which can not be easily done by traditional representations. This perception can guide people to set up specific hypothesis to be tested by selected methods.

7. Conclusions and future work

In this paper, we have discussed two types of imperfect time intervals (rough time intervals and fuzzy time intervals), and investigated them in the Triangular Model. In TM, RTIs can be represented by discrete geometries in a two-dimensional space. According to shapes and locations of these geometries, one can easily read information of an RTI. Compared with the linear represen-

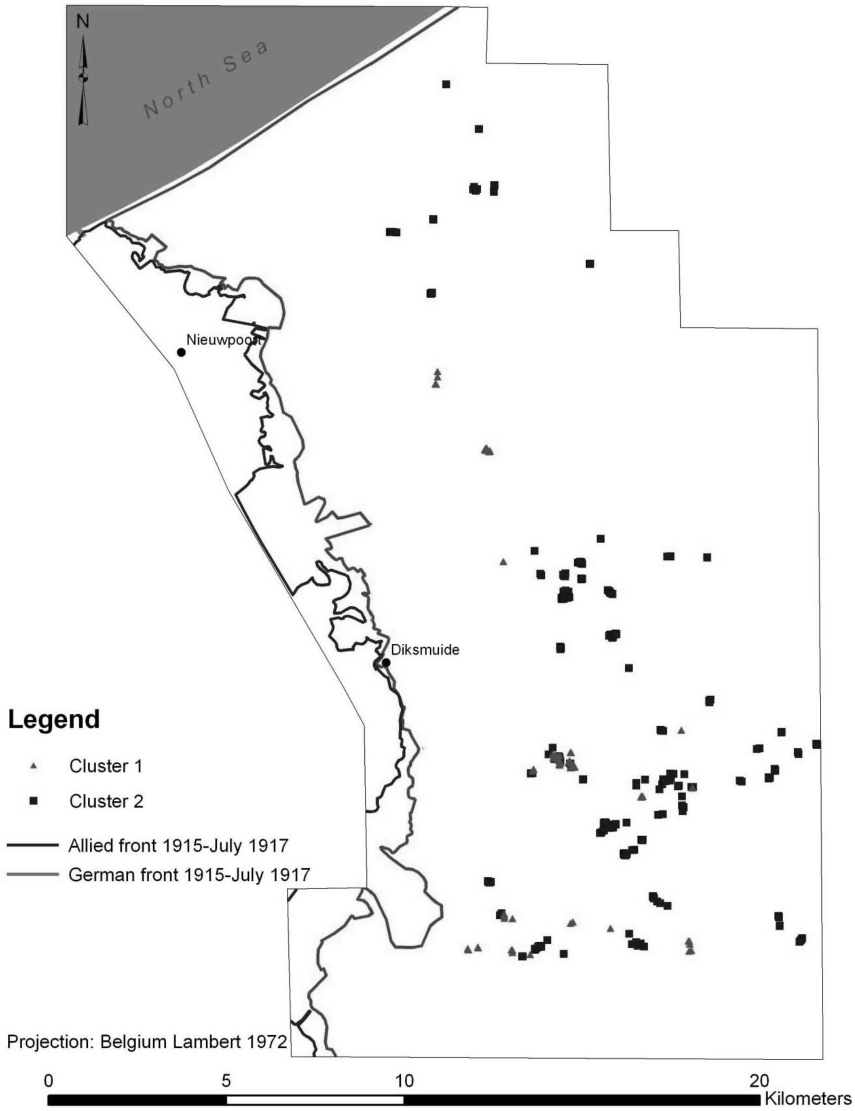

Figure 28. Geographical locations of the two barrack clusters

tation, TM is particularly advantageous in displaying large amounts of RTIs which broadly exist in temporal databases and data warehouses. Furthermore, TM can be treated as a visualisation tool that displays temporal information of geographical objects (see Section 6). By exploring geographical maps in combination with TM visualisations, people may perceive patterns or trends in spatiotemporal data sets, and then set hypotheses to be tested by additional analytical techniques, which is the major task of exploratory data analysis (Andrienko and Andrienko, 2006). On the other hand, temporal relations of RTIs can be investigated in TM. The set of CTIs that are in a specific relation to an RTI is represented by a 2D rough set in TM. Operations on such rough sets can be carried out by operations on corresponding geometries (Section 4.5). Moreover, by topological relations between an RTI and RRZs of the other RTI, one may easily deduce the possible relations between the two RTIs.

In TM, trapezoidal FTIs are represented as linear segments between interval points of the core and support (De Tré et al., 2006). This approach expresses temporal locations of the core and support and may offer an initial display of simple FTIs in temporal database. However, we still have to work out how to model more complex FTIs and relations between complex FTIs in TM. Possibly, more dimensions and diagrammatic elements have to be added to TM. We leave this for future work and in this paper limit our discussion on the handling of FTIs to relations between CTIs and FTIs. In TM, CTIs that are in a specific relation to FTI form a fuzzy set which is modelled as a continuous field in TM. By operations on these continuous fields, queries concerning CTIs and FTIs can be answered in an intuitive way.

TM provides an alternative representation of temporal information, where people can investigate imperfect time intervals or relations of imperfect time intervals from a different perspective. Compared to traditional linear model, the TM offers compact visualisations of imperfect time intervals. The representation of every time interval is fixed and unique, offering the possibility of displaying the distribution of large numbers of intervals. Moreover, sets (crisp, rough or fuzzy) of intervals can be graphically represented in TM, which is more perceptible compared to mathematical expressions. Objectively speaking, TM does not create a new nor extend an existing temporal calculus or logic, however, it takes special care of intuitive and visualisation aspects. As humans are better at reading and processing graphic representations than numerical representations, TM can be considered a valuable assistant tool for analysing and reasoning about imperfect time intervals.

In future research will take account of both theoretical and application aspects. The former issues include the continued work on the representation of complex FITs in TM (or extensions thereof). As well, the representation of FTI-FTI and FTI-RTI relations in TM will be addressed. Furthermore, we will investigate whether TM can express temporal semantics based on possibility theory and probability theory. Regarding the application issues, we plan to elaborate more use cases in order to evaluate TM within different contexts.

Also, we will implement TM as a software application with an interactive interface for people to handle temporal information in general and imperfect time intervals in particular. This interactive graphical tool may be a useful add-on for temporal database systems or information systems. The tool may assist users in analysing distributions of time intervals, or in defining and answering flexible queries by manipulating the 2D features within the TM space. For example, TM can be incorporated with a geographical information system in order to display temporal information of geographical objects. By interactively linking the TM view and the map view, people can flexibly analyze geographical objects from both spatial and temporal aspects.

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References

Allen, J.F. (1983) Maintaining Knowledge About Temporal Intervals. *Communications of the Acm* **26** (11), 832-843.

Allen, J.F. and Hayes, P.J. (1985) A Common-Sense Theory of Time. *Proceedings of the 9th international joint conference on artificial intelligence - Volume 1*, Los Angeles, California.

Andrienko, N. and Andrienko, G. (2006) *Exploratory Analysis of Spatial and Temporal Data*. Heidelberg, Springer-Verlag Berlin.

Bassiri, A., Malek, M., Alesheikh, A. and Amirian, P. (2009) Temporal Relationships between Rough Time Intervals. *Computational Science and Its Applications – ICCSA 2009*, 543-552.

Bittner, T. (2002) Approximate Qualitative Temporal Reasoning. *Annals of Mathematics and Artificial Intelligence* **36** (1), 39-80.

De Caluwe, R., Van Der Cruyssen, B., De Tré, G., Devos, F. and Maesfranckx, P. (1999) The Semantics and Modelling of Flexible Time Indications. *Computing with Words in Information/Intelligent Systems 1 (Foundations)*, Springer/Physica-Verlag, 229-256.

De Caluwe, R., Van Der Cruyssen, B., De Tré, G., Devos, F. and Maesfranckx, P. (1997) Fuzzy Time Indications in Natural Language Interfaces. *Flexible Query Answering Systems*, Kluwer Academic Publishers, 163-185.

De Tré, G., Van de Weghe, N., De Caluwe, R. and De Maeyer, P. (2006) Towards a Flexible Visualization Tool for Dealing with Temporal Data. *Flexible Query Answering Systems*, 109-120.

Dubois, D. and Prade, H. (2000) *Fundamentals of Fuzzy Sets*. Boston, USA, Kluwer Academic Publisher.

Dubois, D., Hadjali, A. and Prade, H. (2003) Fuzziness and Uncertainty in Temporal Reasoning. *Journal of Universal Computer Science* **9** (9), 1168-1194.

Dubois, D., Lang, J. and Prade, H. (1991) Timed Possibilistic Logic. *Fundamenta Informaticae* **15** (3-4), 211-234.

Freksa, C. (1992) Temporal Reasoning Based on Semi-Intervals. *Artificial Intelligence* **54** (1-2), 199-227.

Galton, A. (1990) A Critical Examination of Allen's Theory of Action and Time. *Artif. Intell.* **42** (2-3), 159-188.

Garrido, C., Marín, N. and Pons, O. (2009) Fuzzy Intervals of Represent Fuzzy Valid Time in a Temporal Relational Database. *International J. of Uncertainty, Fuzziness and Knowledge-Based Systems* **17** (1), 173-192.

Knight, B. and Ma, J. (1993) Time Representation: A Taxonomy of Temporal Models. *Artificial Intelligence Review* **7** (6), 401-419.

Kulpa, Z. (1997a) Diagrammatic Representation for a Space of Intervals. *Machine Graphics & Vision* **6**, 5-24.

Kulpa, Z. (1997b) Diagrammatic Representation of Interval Space in Proving Theorems About Interval Relations. *Reliable Computing* **3** (3), 209-217.

Kulpa, Z. (2006) A Diagrammatic Approach to Investigate Interval Relations. *Journal of Visual Languages and Computing* **17** (5), 466-502.

Ladkin, P.B. (1987) The Logic of Time Representation. Ph.D. Dissertation. University of California, Berkeley. AAI8813947.

Lin, Y. (1991) Two Theories of Time. *Journal of Applied Non-Classical Logics* **1** (1), 37-63.

Longley, P.A., Goodchild, M.F., Maguire, D.J. and Rhind, D.W. (2001) *Geographic Information Systems and Science*. Chichester, England, John Wiley & Sons, Ltd.

Nagypál, G. and Motik, B. (2003) A Fuzzy Model for Representing Uncertain, Subjective, and Vague Temporal Knowledge in Ontologies. *On the Move to Meaningful Internet Systems 2003: Coopis, Doa, and Odbase*, 906-923.

Ohlbach, H.J. (2004) Relations between Fuzzy Time Intervals. *Proceedings of the 11th International Symposium on Temporal Representation and Reasoning (Time 2004)*, Montreal, Canada. IEEE Computer Society.

Pawlak, Z. (1982) Rough Sets. *International Journal of Information and Computer Science* **11** (5), 341-356.

Pawlak, Z. (1991) *Rough Sets: Theoretical Aspects of Reasoning About Data*. Kluwer Academic Publishers.

Schockaert, S. and De Cock, M. (2008) Temporal Reasoning About Fuzzy Intervals. *Artificial Intelligence* **172** (8-9), 1158-1193.

Schockaert, S., De Cock, M. and Kerre, E.E. (2008) Fuzzifying Allen's Temporal Interval Relations. *IEEE Transactions on Fuzzy Systems* **16** (2), 517-533.

Shoham, Y. and Goyal, N. (1988) Temporal Reasoning in Artificial Intelligence. *Exploring Artificial Intelligence*, Morgan Kaufmann Publishers Inc., 419-438.

Van de Weghe, N., Docter, R., De Maeyer, P., Bechtold, B. and Ryckbosch, K. (2007) The Triangular Model as an Instrument for Visualising and Analysing Residuality. *Journal of Archaeological Science* **34** (4), 649-655.

Vila, L. (1994) A Survey on Temporal Reasoning in Artificial Intelligence. *AI Commun.* **7** (1), 4-28.

Zadeh, L.A. (1996a) Fuzzy Logic and Approximate Reasoning. *Fuzzy Sets, Fuzzy Logic, and Fuzzy Systems: Selected Papers by Lotfi A. Zadeh*, World Scientific Publishing Co., Inc., 238-259.

Zadeh, L.A. (1996b) Fuzzy Sets. *Fuzzy Sets, Fuzzy Logic, and Fuzzy Systems: Selected Papers by Lotfi A. Zadeh*, World Scientific Publishing Co., Inc., 19-34.