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Multi-criteria approach to comparison of inspection allocation for multi-product manufacturing systems in make-to-order sector*

by

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Abstract: This paper studies multi-product production manufacturing systems with in-line quality control. Quality control is carried at inspection stations located within a production line. Quality control has an impact on performance of the system: throughput decreases and resource utilization increases. Production lines are modeled as a multi-product Open Jackson Network, where stochastic character of routing is a result of quality control operations. Quality inspection can result in feedback to a work station of the manufacturing system. The approach developed allows for comparing different inspection allocations using three principal indices: overall cost, lead-time and equipment utilization.

Keywords: production line, queuing network, quality control, make-to-order.

1. Introduction

Sustaining stable quality of a final product is a central problem to many manufacturing systems. The spread of new quality management approaches, like lean manufacturing and six sigma, facilitates organizing a manufacturing system in a manner helping to reach perfection with zero defects. In order to achieve this, side by side with creation of a good attitude of workforce to quality problems, inspection stations are necessary at definite points of manufacturing process.

The make-to-order (MTO) manufacturing sector is growing. It plays an increasingly important role as requirements on product customization increase. Orders for the products tend to be on a make-to-order, make-to-print or engineer-to-order basis, often being specific to a particular customer with intermittent or no repetition of demands for the same product (Haskoe, Kingsman and Worthington, 2004).

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A typical company working in the MTO sector has to supply wide variety of products, usually in small quantities, with different types of requirements and customization. As each customer acts independently and asks for a different or least customized product, the arrival process over time has a strong stochastic nature. Each customer's order involves varying technological itinerary and processing times vary, as well. As a result, the MTO is a multi-product manufacturing system, i.e. parts for different kinds of final products are processed at the same time within a single manufacturing system.

In such a manufacturing system each customer's order needs to be interpreted as a job with assigned technological itinerary, according to its particular features. A job enters the MTO system and goes to the first work station on its itinerary. It typically joins a queue of other jobs waiting for their turn for the processing work to be carried out. Once the work on a job at the work station is completed, the job is transported to the next work station on its itinerary, where it again joins a queue of jobs awaiting processing. The lead-time is thus the sum of the set-up and processing times at each of the work stations on the technological itinerary plus the time spent waiting in queues in front of the work stations. It is reported that manufacturing lead times are often long and hard to predict almost entirely due to the large proportion of time spent in the queues (Papadopoulos and Heavey, 1996).

For make-to-order companies, sustaining stable high quality of final products is an important and demanding problem due to the variety of products being manufactured and due to individual quality requirements for almost every job. In order to ensure satisfying quality, inspection stations are necessary at some stages of the manufacturing system.

A quality check can be performed after completion of an operation of a manufacturing process. In general, the more inspections there are, the higher the expected quality of the final product. On the other hand, the more inspections, the more time is necessary and the system is more expensive to establish and to maintain. Efficient inspection strategy depends on many factors, including defect probability at each technological operation, inspection cost, time of inspection and budget.

Here arises an optimization problem of minimizing total inspection cost, while assuring a minimum acceptable outgoing quality level (AOQL). In order to solve this problem certain decision variables have to be taken into consideration: number of inspection stations, number of inspections executed, cost of establishing inspection station, cost of inspection at particular station, inspection reliability, inspection methodology (no inspection, sampling, full inspection) etc.

The main aim of this paper is to develop a model for a make-to-order manufacturing system, which considers quality assurance influence on performance. It enables examination and comparison of configurations of inspection stations allocations and assessment of the impact on the system. In order to compare different MTO system configurations it is proposed to apply the AHP methodology (Saaty, 2005).

This paper is organized as follows. We begin by literature review. Then, the problem is described with introduction of basic notions and computational complexity. A mathematical model based on an Open Jackson Network theory is developed in the subsequent section. Finally, an example is presented.

2. Literature review

There are numerous publications on inspection allocation. Here, some past studies, devoted to the issues similar to the above described problems, are reviewed. The issues dealt with in this context are as follows. Production system can be serial, assembly or non-serial (Buzacott and Shanthikumar, 1992). In a serial system, the input material passes through all workstations sequentially form the first to the last one, in a non-serial system the input material takes one of several technological itineraries through a production system, i.e., certain workstations may be involved in joining the outputs of multiple previous workstations, while in an assembly production system at least one work station is involved in building a semi-product or a final product from several different semi-products arriving from some previous workstations. This paper is focused on non-serial production systems.

The check carried at inspection stations can be full, fractional, repeated or dynamic. Inspection can be perfect, type I error, type II error, or both error types.

Constraints can be constituted by AOQL (acceptable outgoing quality level), inspection time, number of inspections, budget limit, number of repeated measurements or throughput (Mandroli, Shrivastava and Ding, 2006). Methods for solving the inspection allocation problem can be divided into two categories: exact and heuristic. Among exact methods dynamic programming, non-linear programming and integer programming techniques can be mentioned. Using these techniques is very computationally expensive. The second category consists of approximation techniques, which yield a nearly optimal solution at a considerably lower computational effort. This category includes genetic algorithms, simulated annealing, tabu search, ant algorithms (Rau and Cho, 2009).

Rebello, Agnetis and Mirchandani (1995) discussed exact and/or heuristic methods of solution for following models: when inspection/rework stations are to be located; when inspection stations are already located but their operating modes (rework or scrap) are to be determined; when both locations and operating modes are to be determined. The criteria included cost minimization, yield maximization, and minimization of undetected faulty units. Jewkes (1995) modeled a single stage manufacturing system processing, possibility of inspection, and repair, if necessary. First, the optimal inspection effort is determined for the situation, when defective items are repaired immediately. Next, results from literature on multi-armed bandits were used to optimally schedule the processing and repair tasks jointly with determining the optimal inspection effort. She observed dependence of results upon different arrival rates (Poisson process). The minimized criterion function was sum of costs: cost of inspection, repair per unit time, cost per unit of allowing a defective item leave the system undetected and holding cost per unit time (cost associated with the expected queuing and service times per unit).

Gurnani, Drezner and Akella (1996) considered a two-stage production line evaluated in two cases: when there are inspection sites after each production stage and when inspection is carried out after the final stage. As a function of inspection site, the capacity required to test and repair the parts would vary. The production-inspection model integrates the issues of inspection location, inspection capacity, and production capacity. For an n stage line they propose to calculate the total cost of holding safety stock at various stations and the cost of test/repair capacities and to choose the combination with the lowest total cost.

Bai and Yun (1996) present an inspection effort allocation model for a serial multi stage production system for situations where automatic inspections machines are limited in number and locations of these machines are obtained by minimizing the cost function, which includes three cost components: rework, inspection and penalty cost. For situations, where the numbers of stages and inspections machines are large, a heuristic algorithm using dynamic programming is proposed. Van Volsem, Dullaert and Van Landeghem (2005) considered a serial multi stage production system, in which products travel sequentially from stage 1 to stage n. After each of the processing stations, one of three inspection options can be chosen: no inspection, full inspection, or sampling inspection. A fusion between a discrete event simulation to model the multi-stage process subject to inspection and to calculate the resulting inspection costs, and an Evolutionary Algorithm to optimize the inspection strategies, is suggested. The criterion function is the sum of: total rework, penalty and inspection costs.

Y.-R. Shiau (2003) considered a limited number of inspection stations of each class of inspection stations for solving the inspection allocation problem in a multiple quality characteristics advanced manufacturing system. The costs, dealt with in this work include the costs of manufacturing, inspection, reworking, and discarding. Shiau (2003) proposed that each kind of cost should be analyzed for each quality characteristic of a product. Since determining the optimal inspection allocation plan seems to be impractical, as the problem size becomes large, two decision criteria (i.e. sequential order of work stations and tolerance interval) are employed separately to develop two different heuristic solution methods in this work. In a later paper (Shiau, Lin and Chuang, 2007) it is, however, proposed that better performance of an advanced manufacturing system can be achieved if process and inspection planning can be performed concurrently to manage the limited manufacturing resources. The unit cost model is constructed to represent the overall performance of an advanced manufacturing system by considering both internal and external costs. Process and inspection planning can then be concurrently solved by practically reflecting the customer requirements. As determining the optimal manufacturing resource allocation plan seems to be impractical due to problem size, in this research, genetic algorithm is successfully applied with the realistic unit cost embedded. The performance of each method in these papers is measured in comparison with the enumeration method that generates the optimal solution.

Several papers demonstrate methods of measuring quality costs in multi-stage manufacturing systems. The mathematical model by Chang, Hyun and Park (1996) quantifies the production cycle time and quality costs. Four types of costs are considered: prevention cost (process monitoring, production system repair and maintenance cost), appraisal cost (inspection, production system repair and maintenance cost), internal failure cost (scrap, false positive and false negative faults) and external failure cost (customer dissatisfaction). Oppermann, Sauer and Wohlrabe (2003) compare costs of three strategies: full inspection, no inspection and SPC. Quality costs constitute the measurement system for comparing different inspection strategies. The costs are calculated by the use of mathematical models—the quality cost models. To analyze and optimize the quality processes of a complete production line dynamic programming is used.

3. Problem statement

Let us consider a multi-stage multi-product manufacturing system with k work stations, within which t products follow technological itineraries. Each itinerary consist of p work stations, $p \leq k$. Quality control is performed by h inspection stations ($h \leq k$). Work station consists of a set of identical machines (one or more), working in parallel, and a waiting buffer. Inspection stations can follow any work station. Let's assume that in all inspection stations a full inspection strategy is applied.

In general the described above a multi-stage multi-product production system (Fig. 1) can be represented as a digraph $G(S, E)$. Vertices of the digraph, $S = \{s_1, s_2, s_3, ..., s_k\}$, are work stations and arcs of the digraph, $E = \{e_1, e_2, e_3, ..., e_l\}$, where $e_l = (s_i, s_j)$, show possible semi product transfer between work stations. Subsets of E form technological itineraries for each final product.

In Fig. 1 the following notation is used: b_i – buffer at i-th stage, s_i – one or several machines working in parallel at i-th stage, I_i – inspection station after i-th work station.

The inspection station is modeled as a part of a work station without an input buffer and with the same capacity as the preceding work station. The result of an inspection operation can be: acceptance (the job is forwarded to the next work station on its itinerary), rework (the job is transferred to a certain work station on its itinerary that it has already passed through), scrap (the detected fault discriminates the job from further processing and its manufacturing process starts from the beginning). The result of the control operation is modeled using probabilities. The sum of acceptance, rework and scrap probabilities is equal to 1. The decision about qualifying a job into one of the three categories is

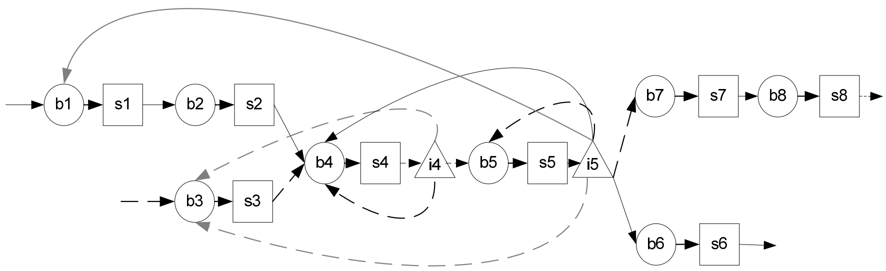

Figure 1. Multi-stage multi-product manufacturing system with eight work stations, two products and two inspection stations after work stations 4 and 5 (decision variant)

almost always subject to error. In general, we distinguish two types of error: type I (false positive) and type II (false negative). Thus, among accepted jobs some are in reality bad (should be classified as rework or scrap) and vice versa, the rework and scrap jobs may in reality be good.

Let us introduce some basic notions employed later on. The manufacturing system without inspection stations allocation will be called a *class of systems*. The class of systems (Fig. 2) will describe only technological itineraries of all t products. Within a class several families (Fig. 3) of systems can be discerned. A *family of systems* is a class with introduced inspection stations allocation, but without determined back loops for rework jobs. Finally, a manufacturing system with allocated inspection stations and determined back loops will be called a *decision variant* (Fig. 1).

The number of decision variants for a class of systems depends on the number of final products and the lengths of their technological itineraries, i.e. the number of work stations, through which a job is carried:

$$z = \prod_{i=1}^{t} z_i = \prod_{i=1}^{t} (h_i + 1)! \tag{1}$$

Here, z is the number of decision variants for a class of manufacturing systems; t is the number of final products; h_i is the length of technological itinerary of product i.

For the manufacturing systems, defined above, a decision maker has to choose the best configuration of inspection stations regarding quality of the final product and the manufacturing system performance. In general, the decision maker can analyze all possible allocations of different numbers of inspection stations and different backward loops. In reality, the choice is limited by technological constraints, so that the search region is reduced to a smaller number of decision variants.

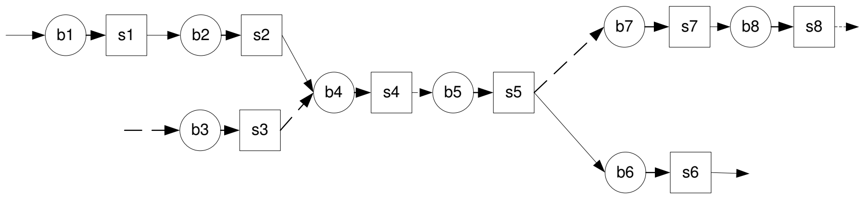

Figure 2. A class of systems with eight work stations and two products

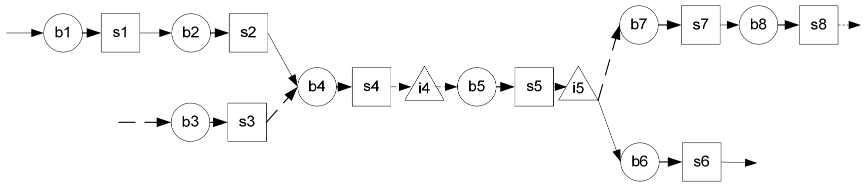

Figure 3. A family of systems with eight work stations, two products and two inspection stations after work stations 4 and 5

The problem described is hard to analyze when we take into account such stochastic factors as random times of order arrivals, unknown type and quantity of final products. One approach that can be used to evaluate performance of the multi-stage multi-product production system with quality inspection is the queuing networks theory. We propose to model such a system as an Open Jackson Network with multiple customer classes (tOJN − t classes of customers Open Jackson Network) (Jackson, 1957; Gross, Harris, 1998). In this model, a work station consists of one or more servers, processing jobs in parallel, independently, a waiting buffer and an inspection station.

Queuing network modeling of manufacturing systems has been addressed by many researchers. A comprehensive, but a little dated review can be found in Papadopoulos and Heavey (1996) and a more recent, but no so ample, in Baldwin et al. (2003). In general, so far, queuing networks were used to model manufacturing systems under the following restrictions: no parts are scrapped, only a single part type is modeled (Papadopoulos and Heavey, 1996). For some years Gershwin and his team have been working on quality/quantity modeling and analysis of production lines subject to uncertainty (Schick, Gershwin and Kim, 2005). Their approach is based on Markov chain analysis. Gershwin developed a model of a linear production system with inspection stations, which is decomposed into a set of two virtual machine lines for which exact result were found.

The tOJN approach makes it possible to: 1) take into account the stochastic nature of incoming customer orders (arrival time, type of demand, order size), 2) find the intensities of all streams within the system, 3) decompose an MTO manufacturing system into separate queuing systems and calculate performance measures of each of them.

4. Model of a make-to-order manufacturing system

In our methodology the following assumptions were made:

The manufacturing system can produce t different products at the same time. A technological itinerary is assigned to each product.

Customer demands enter the manufacturing system according to the Poisson process.

Each work station consists of an input buffer with FIFO queuing discipline, one or several machines (servers) working in parallel, and can be extended with an inspection station. According to Kendall's (1953) notation a work station is an M/M/n queuing system.

Processing times of technological operations carried out in work stations are distributed exponentially and are the same for all different products.

Processing time at each work station is independent of preceding processing times.

All machines at each work station work without breaking down.

All input buffers are infinite.

Transfer time between work stations is omitted.

The queuing network is in a steady state.

No quality upgrading can take place at the processing stages.

The mean arrival rate at a work station i is denoted γ_i. When jobs complete service at work station i, they go to work station j with probability r_{ij} (regardless of the state of the system), $i = 1, 2, 3, \ldots, k$. There is a probability r_{i0} that a customer order leaves the system at the work station i upon completion of service. To the manufacturing system a virtual exit work station $(k+1)$ is added to all technological itineraries in order to be able to acquire information about the last work station in the itinerary.

Let $\lambda_i^{(n)}$ be the total mean flow rate of $n = 1, 2, ..., t$ product to work station $i = 1, 2, ...k$, the combined flow from other work stations and from the outside. The traffic equation allows for finding the total flow rate for each node:

$$\lambda_i^{(n)} = \gamma_i^{(n)} + \sum_{j=1}^{k} \lambda_i^{(n)} r_{ij}^{(n)}, \tag{2}$$

or, in a vector-matrix form,

$$\lambda^{(n)} = \gamma^{(n)} + \lambda^{(n)} R^{(n)} \tag{3}$$

where $R^{(n)} = \{r_{ij}^{(n)}\}$ is a routing matrix reflecting passage probabilities with dimension $(k+1) \times (k+1)$. Here, the routing matrix is enlarged by one column and row in order to incorporate an additional virtual exit work station.

Using (2) we can obtain the total incoming flow into all work stations, $\lambda^{(n)}$:

$$\lambda^{(n)} = \gamma^{(n)}(I - R^{(n)})^{-1}. \tag{4}$$

The invertibility of $I-R$ is assured as long as there is at least one node releasing its output to the outside and no node is totally absorbing.

The total incoming stream of jobs into each work station is a sum of each class of job streams:

$$\lambda_i = \sum_{n}^{t} \lambda_i^{(n)}. \tag{5}$$

Using this model we can obtain the following mean value performance characteristics: 1) utilization at each node ρ_i, 2) number of jobs in each node, L_i, and in the entire network, L, 3) number of waiting jobs at each node and in the entire network, 4) processing time at each node, T_i, and in the entire network, T, 5) waiting time at each node, W_i, and in the entire network, W.

The result of a quality control operation is stochastic. Its result is independent, previous quality check do not influence the result of the next check. In this case, it is possible to describe inspection results for each category of decision as probabilities. For work station i we can have probability p_{ij} – a job fulfills quality requirements and moves to a downstream node, p_{ii} – a job is returned for rework in the same working centre, p_{il} – a job is returned to one of upstream work station for rework and then continues itinerary as usual, p_{i0} – a job is scrapped (this stream of jobs is redirected to a first work station of an appropriate technological itinerary).

5. Optimization model

5.1. Input parameters

1. Production system structure may be represented as a digraph $G(S,E)$. Vertices of the digraph, $S = \{s_1, s_2, s_3, ..., s_k\}$, are the work stations and arcs of the digraph, $E = \{e_1, e_2, e_3, ..., e_l\}$, where $e_l = (r_i, r_j)$, $l = \overline{1, L}$, are the links connecting them.

2. Customers' orders stream for products indexed n is given as a vector $\Gamma^{(n)} = [\ \gamma_1^{(n)} \quad \gamma_2^{(n)} \quad \ldots \quad \gamma_k^{(n)}\ ]^T$. Those orders concern production of $F = \{f_1, f_2, ..., f_t\}$ final products. Total input stream of jobs is a sum of product input streams $\Gamma = \sum_{n=1}^{t} \Gamma^{(n)}$, due to the superposition property of Poisson distribution.

3. Each work station r_i consists of a set of one-type production posts $N = \begin{bmatrix} n_1 & n_2 & \dots & n_k \end{bmatrix}^T$, performing some kind of technological operation. The work stations are characterized by: $\lambda^{(n)} = [\ \lambda_1^{(n)} \quad \lambda_2^{(n)} \quad \dots \quad \lambda_k^{(n)}\ ]^T$ – intensity of the incoming stream of product n, the total input stream intensity being a sum of product input streams $\lambda_i = \sum_{n=1}^{t} \lambda_i^{(n)}$, $\mathrm{M} = \begin{bmatrix} \mu_1 & \mu_2 & \dots & \mu_k \end{bmatrix}^T$ – productivity of a work station, $\Omega = \begin{bmatrix} \omega_1 & \omega_2 & \dots & \omega_k \end{bmatrix}^T$ – probability that the result of technological operations performed within the vertex is a success. With a work station the following cost parameters are associated: $Co = \begin{bmatrix} co_1 & co_2 & \dots & co_k \end{bmatrix}^T$- the cost of technological operation, $Cr = \begin{bmatrix} cr_1 & cr_2 & \dots & cr_k \end{bmatrix}^T$ – the cost of reworking one item at the work station.

4. Each production process $P = \begin{bmatrix} p^{(1)} & p^{(2)} & \dots & p^{(t)} \end{bmatrix}^T$ is carried through a route of work stations, representing a chain of technological operations, where $p^{(n)}$ is a matrix of dimensions $(k+1) \times (k+1)$. If a work station belongs to the technological itinerary of a product, $p_{ij}^{(n)} = 1$, and the next work station is j, otherwise $p_{ij}^{(n)} = 0$.

5. An inspection station is characterized by the following parameters: $Ci = \begin{bmatrix} ci_1 & ci_2 & \dots & ci_k \end{bmatrix}^T$– the cost of quality control of one item at the inspection station, $\mathrm{A} = \begin{bmatrix} \alpha_1 & \alpha_2 & \dots & \alpha_k \end{bmatrix}^T$– the type I error probability, i.e. the probability of false positive – accepting a defected item at the inspection station, $\mathrm{B} = \begin{bmatrix} \beta_1 & \beta_2 & \dots & \beta_k \end{bmatrix}^T$ – the type II error probability, i.e. the probability of false negative – rejecting a good item at the inspection station.

5.2. Control parameters

Inspection stations allocation is characterized by vector $I = \begin{bmatrix} i_1 & i_2 & \dots & i_k \end{bmatrix}^T$, $i_i \in \{0, 1\}$. If an inspection station is present at the i-th work station, then $i_i = 1$, otherwise $i_i = 0$. All inspection stations are also described by a work station where rework process is carried out, $Re = \begin{bmatrix} re_1 & re_2 & \dots & re_k \end{bmatrix}^T$, $re_i \in \{1, 2, \dots, k\}$.

5.3. Criterion functions

Cycle time is the time a job spends in a manufacturing system. Thus, it is the sum of the set-up and processing times at each of the work stations on the job itinerary plus all of the time spent waiting in queues in front of the work stations. In this case it can be calculated as a sum of processing times at each work station of a production process:

$$T = \sum_{n=1}^{t} \frac{\sum_{i=1}^{k} (\gamma_i^{(n)} \cdot T_i \cdot x_i^{(n)})}{\sum_{n=1}^{t}\sum_{i=1}^{k} \gamma_i^{(n)}}, \tag{6}$$

where T_i is a cycle time of i-th work station, calculated using formulas from the queuing theory, X is a $k \times n$ matrix. If a work station belongs to a product technological itinerary, $x_i^{(n)} = 1$, otherwise $x_i^{(n)} = 0$.

Utilization of available manufacturing resources should be maximized. In this case average utilization is used. Therefore, the second criterion function has the following form:

$$U = \sum_{i=1}^{k} \frac{(1-\rho_i)}{k}, \tag{7}$$

where $\rho = \begin{bmatrix} \rho_1 & \rho_2 & \dots & \rho_k \end{bmatrix}^T$ is a work station utilization index $\rho_i = \frac{\lambda_i}{n_i \cdot \mu_i}$.

Quality control cost is a composite function C and it includes four components:

1) *Testing cost* - cost of inspection execution for one job,

$$CT = \sum_{i=1}^{k} \lambda_i^T \cdot ci_i \cdot i_i. \tag{8}$$

2) *Rework cost* – cost of executing a special technological operation at each work station in order to repair a faulty job,

$$CR = \sum_{i=1}^{k} pr_i \cdot cr_i \cdot i_i, \tag{9}$$

where pr_i is the probability that jobs from the i-th work station will be directed to rework.

3) *Scrap cost* – manufacturing cost up to the inspection station,

$$CS = \sum_{i=1}^{k} \left(ps_i \cdot \sum_{j=1}^{k} co_i \right), \tag{10}$$

where ps_i is the probability that jobs from i-th work station will be identified as scrap.

4) *Inspection errors costs* – cost of making wrong decisions regarding jobs: α - type I (false positive) and β - type II (false negative),

$$CE = \sum_{n=1}^{t} \sum_{j=1}^{k} \left(\alpha_j \cdot \sum_{i=1}^{k} (co_i \cdot x_{in}) + \beta_j \sum_{i=1}^{n} (co_i \cdot x_{in}) \right). \tag{11}$$

5.4. Utility function

In order to find the optimal production system configuration, the weighted sum method will be used (Ehrgott and Wiecek, 2005). It is one of the most popular approaches. For the above-stated problem the global criterion function Ψ is:

$$\Psi = w_1 \cdot T + w_2 \cdot U + w_3 \cdot C = \min, \tag{12}$$

$$C = w_4 \cdot CT + w_5 \cdot CR + w_6 \cdot CS + w_7 \cdot CE, \tag{13}$$

where $w_1, w_2, w_3, w_4, w_5, w_6, w_7$ are weights, $w_1 + w_2 + w_3 = 1$ and $w_4 + w_5 + w_6 + w_7 = 1$.

The weights have to be fixed by the decision-maker, basing on her/his knowledge about the production system and its environment. This is usually a difficult assignment and for this purpose the Analytic Hierarchy Process (AHP) method is applied.

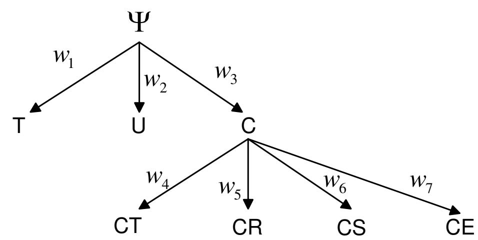

Figure 4. The structure of the global criterion function

The AHP is a technique of relative measurement. With this approach a scale of priorities is derived from the pair-wise comparisons expressed through numerical values taken from the AHP absolute fundamental scale of 1-9. The AHP method consists of three steps (Korpela, Lehmusvaara and Tuominen, 2001):

— decomposition of a complex multi-criterion problem into a hierarchy, where at each level there are some criteria that can be further decomposed at the next hierarchy level,
— use of relative reciprocal goal importance measuring methodology at every level of hierarchy,
— priority synthesis.

The main advantage of the AHP method is the possibility of applying it to problems expressed in different units, like in case of production capacity optimization where the criteria are in units of time and money.

5.5. Constraint on acceptable outgoing quality level

The goal of inspection allocation is to work out a manufacturing system configuration with a minimum overall quality assurance infrastructure cost. This can be achieved by removing all inspection stations or not introducing them. The effect, though, is unsatisfactory quality of manufactured products. Thus, a constraint on an Acceptable Outgoing Quality Level (AOQL) is introduced. AOQL is the probability that a final product will comply with all quality indicators. The outgoing quality level for an MTO system is calculated as follows:

$$OQL = \sum_{j=1}^{t} \left[\frac{\sum_i \gamma_{i,j}}{\sum_{n=1}^{t} \sum_{i=1}^{k} \gamma_{i,n}} \cdot \prod_i \xi_{i,j} \right], \tag{14}$$

$$\xi_{i,j} = \begin{cases} 1 & \text{if } X_{i,j} = 0 \\ \omega_i & \text{if } X_{i,j} = 1 \land i_i \neq 1 \\ 1 - \alpha_i & \text{otherwise} \end{cases} . \tag{15}$$

So, the constraint is:

$$OQL \geq AOQL \tag{16}$$

6. Illustrative example

Consider a class of MTO manufacturing systems, producing two different products (Fig. 2). For the first product, the technological itinerary goes through work stations: 1, 2, 4, 5, 6 and for the second product, through stations: 3, 4, 5, 7, 8. A decision-maker has to decide how many and where to assign inspection stations. The decision has to be taken considering future product quality, performance of the manufacturing system and different kinds of cost associated with it. For this MTO class 518,400 decision variants should be investigated without applying further constraints.

In this example seven decision variants, belonging to three different families of systems, are considered and their performance is assessed.

In decision variant 1 (Fig. 5), belonging to the first family, there are three inspection stations, first after work station 2, with jobs for rework sent to work station 1, second inspection station after work station 4 and jobs for rework of the first product are sent to work station 2 and of the second product to work station 3. The last, third inspection station is situated after work station 5 and rework goes to work stations 3 or 5.

Consider now decision variant 4 (Fig. 6), belonging to the second family, where also three inspection stations are present, first after work station 3 and

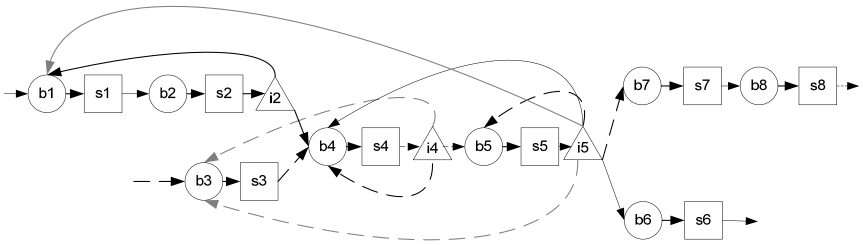

Figure 5. Decision variant 1

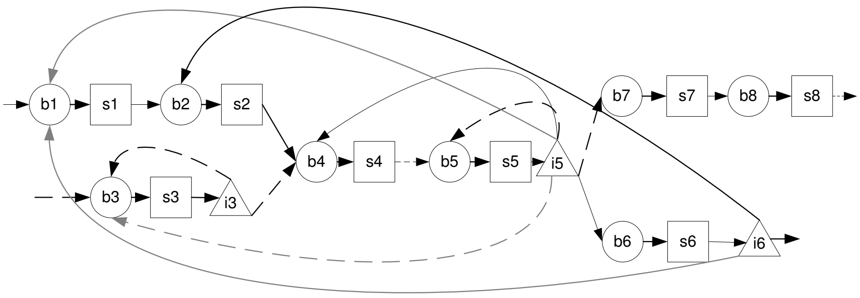

Figure 6. Decision variant 4

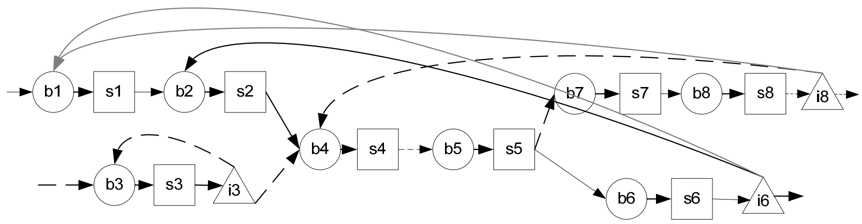

Figure 7. Decision variant 6

jobs for rework are sent to the same station, second inspection station is after work station 5 and jobs for rework of the first product are sent to work station 3 and of the second product to work station 5. The last, third inspection station is situated after work station 6 and rework goes to work station 2.

One more example is decision variant 6 (Fig. 7), belonging to the third family, with four inspection stations, first after work station 1, jobs for rework being sent to the same station, second inspection station is after work station 3 and jobs for rework are sent to work station 3. Third inspection station is situated after work station 6 and rework goes to work station 2. The last inspection station is after work station 8 and jobs for rework are sent to work station 4.

All seven decision variants are shown in Table 1.

Table 1. Decision variant configurations

Decision	Inspection station							
variant	pos	rw	pos	rw	pos	rw	pos	rw
1	2	1	4	2.3	5	2.5		
2	2	1	4	1.4	5	4.3		
3	2	2	4	1.3	5	1.3		
4	3	3	5	5.3	6	2		
5	3	3	5	4.4	6	1		
6	1	1	3	3	6	2	8	4
7	1	1	3	3	6	6	8	8

pos: position of inspection station; rw: address of the rework link

For all decision variants the following common conditions were applied. The input stream rate for product 1 is $\gamma^{(1)} = 10$ and for product 2: $\gamma^{(2)} = 10$. Moreover, assume that the manufacturing system is working with the 3 sigma quality level, i.e. 93.3% of technological operations are successful, 2.23% need rework and 4.47% are defective. This can be expressed through matrices for decision variant 1 (one matrix for each product):

$$R^{(1)} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.067 & 0 & 0 & 0.933 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0447 & 0.0223 & 0 & 0 & 0.933 & 0 & 0 & 0 & 0 \\ 0.0447 & 0.0223 & 0 & 0 & 0 & 0.933 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R^{(2)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0.0447 & 0 & 0.0223 & 0 & 0.933 & 0 & 0 & 0 & 0 \\ 0.0447 & 0 & 0 & 0 & 0.0223 & 0.933 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Parameters of inspection and work stations for both products are given in Table 2.

Table 2. Model parameters

s	μ	n	co	ci	ω	α	β
1	12	2	1	2	0.95	0.01	0.01
2	12	2	1	2	1	0.01	0.01
3	12	2	1	2	0.95	0.01	0.01
4	12	2	1	2	1	0.01	0.01
5	12	2	1	2	1	0.01	0.01
6	12	2	1	2	0.95	0.01	0.01
7	12	2	1	2	0.95	0.01	0.01
8	12	2	1	2	0.95	0.01	0.01

Now let us calculate weighting coefficients expressing decision-maker's view on importance of criteria. The Analytic Hierarchy Process was applied and all criteria were pair-wise compared. First, three main criteria: Cycle time (T), Utilization (U) and Quality control costs (C). The comparison was done with the use of Saaty's scale (from 1: equal importance of criteria, to 9: first criterion is absolutely more important than the second one, Saaty, 2005). The matrix shown in Table 3 is the result. The same was done for second tier criteria, being quality control costs, i.e., testing cost (CT), rework cost (CR), scrap cost (CS), inspection error cost (CE). The comparison matrix is shown as Table 4.

Table 3. Pair-wise comparisons of criteria at the first tier

	T	**U**	**C**
T	1	4.00	0.50
U	0.25	1	0.25
C	2.00	4.00	1

Table 4. Pair-wise comparisons of criteria at the second tier

	CT	**CR**	**CS**	**CE**
CT	1	0.50	0.33	2
CR	2.00	1	1.00	3.0000
CS	3.00	1.00	1	3
CE	0.50	0.33	0.33	1.00

The ultimate weighing coefficients, obtained with the AHP technique, were: w_1=0.346, w_2=0.11, w_4=0.092, w_5=0.186, w_6=0.207, w_7=0.059. Inconsistence ratios are 0.05 and 0.02, meaning that decision-maker's judgment was consistent. Finally, eq. (12) takes in this case the following form:

$$\Psi = 0.346 \cdot T + 0.11 \cdot U + 0.092 \cdot CT + 0.186 \cdot CR + 0.207 \cdot CS + 0.059 \cdot CE. \quad (17)$$

By employing Open Jackson Network model it is possible to calculate the values for criteria T, U and C. For the 1st decision variant we obtain T = 1.07928, U = 0.44969, CT = 5.45072, CR = 0.01115, CS = 0.894 and CE= 0.35. The value of the utility function is Ψ = 0.80862. The same calculations as for the 1st decision variant were made for the 4th decision variant and the following results were obtained T = 1.04711, U = 0.45464, CT = 4.24067, CR = 0.00892, CS = 0.8493, CE= 0.38 and the global utility function value Ψ = 0.73321. The results for all considered decision variants are gathered in Table 5.

Table 5. Results for seven decision variants

Criterion	Decision variant						
	1	2	3	4	5	6	7
T	1.07928	1.12562	1.12689	1.04711	1.0498	0.92372	0.85799
U	0.44969	0.44878	0.44633	0.45464	0.45439	0.45694	0.46408
CT	5.45072	5.44633	5.47194	4.24067	4.21945	4.28318	4.28318
CR	0.01115	0.01115	0.01115	0.00892	0.00892	0.00892	0.00892
CS	0.894	0.894	0.894	0.8493	0.8493	0.8046	0.8046
CE	0.35	0.35	0.35	0.38	0.38	0.5	0.5
C	0.70905	0.70865	0.711	0.58988	0.58793	0.59158	0.59158
Ψ	*0.80862*	*0.82433*	*0.82579*	*0.73321*	*0.73305*	*0.69169*	*0.66973*

The difference between the best (7) and the worst (3) decision variants is about 23% (Fig. 8). The average throughput time for variant 3 is 1.13 hours and for variant 7 – 0.86 hours (or roughly 31%).

In the illustrative example presented, for the sake of simplicity all costs, error probabilities and servicing rates were the same level for all work stations, thus the major impact on the results came from the configuration. It demonstrates the importance for the make-to-order manufacturing systems of allocation of inspection stations. The multi-criteria methodology enables incorporation of different points of view on the system in one model leaving, at the same time, room for a decision-maker to adjust for specific needs.

7. Conclusions

The approach to modeling of multi-product production systems, presented here, allows for solving the problem of introducing quality control for assuring acceptable outgoing quality level of final products, while accounting performance

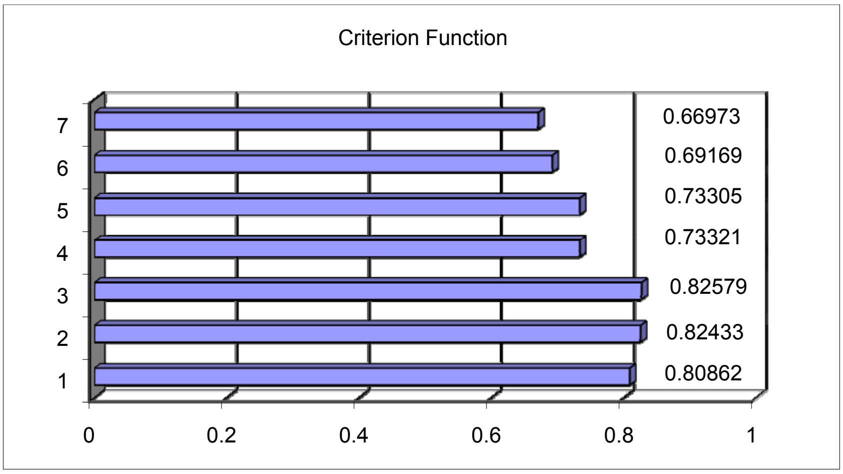

Figure 8. Values of criterion Ψ for all variants

measures like lead-time and utilization. The proposed approach is based on decomposition of the manufacturing system into separate streams of product types and independent work stations which were modeled as M/M/n queuing systems.

Using the methodology presented it is possible to compare any number of different configurations of quality control. The model proposed incorporates two problems which used to be treated separately: performance evaluation and quality control. The model is open, new criterion functions can be developed and the ones utilized in this article can be modified according to the decision-maker's needs.

This methodology could be extended by introducing an algorithm of search for an optimal decision variant. Now, with application of just AHP all possible decision variants have to be analyzed and the decision-maker does not have any hint about the direction where he should be looking for.

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