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Book review:

POINT PROCESS THEORY AND APPLICATIONS. MARKED POINT AND PIECEWISE DETERMINISTIC PROCESSES

by

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Theory and applications of point and piecewise deterministic processes are very important fields of stochastic analysis. Point processes, in this book, are interpreted as processes describing scattering of points, which are associated with occurrences of random events. The author presents the theory and application of simple point processes (SPPs) and marked point processes (MPPs). Main processes from the second class, i.e. piecewise deterministic processes (PDPs), have a finite number of jumps in finite time and they are deterministic between jumps. Both these classes of stochastic processes are very useful in finance, insurance, survival analysis and many other fields. The subclass of those PDPs, that are Markov processes, is especially important from the practical point of view.

The book is divided into three parts. The first is dedicated to theory, the second to applications and the third consists of two appendices. There are also Bibliographical Notes. The overview and definitions of probabilistic notions, i.e. conditional expectation, probability, and regular conditional distribution, are presented in Chapter 1 of Part I.

Chapters 2 and 3 contain definitions and constructions of basic processes used in the book. In Chapter 2 the simple and marked point processes are defined. Additionally, in the second section of this chapter, counting processes and random counting measures are introduced. There are also descriptions of spaces K, and K_E of sequences of timepoints of events and their marks, W, of counting process paths, and $\mathcal{M}$, of discrete counting measures. It is shown that SPPs and MPPs can be identified with counting processes and random counting measures, respectively. It is also emphasized that SPPs, MPPs, counting processes and random counting measures may be viewed as random variables with values in corresponding spaces, i.e. K, K_E, W and $\mathcal{M}$. Chapter 3 is dedicated to constructions of canonical point processes, counting processes and random counting measures. Its third section contains an outline of construction of PDPs from MPPs.

Chapter 4 is essential for understanding further chapters of the book. It is devoted to probability measures on the spaces W and $\mathcal{M}$. Furthermore, theory of compensators and compensating measures is presented. There are also elements of the martingale representation theory and some facts concerning stochastic integrals. It is shown that a piecewise continuous stochastic process, adapted to the filtration generated by a random counting measure, can be decomposed into a predictable process and a local martingale.

Chapter 5 contains theory of processes of Radon-Nikodym derivatives (likelihood processes) for probabilities on canonical space of counting process paths and the space of discrete counting measures, under the assumption that one of the probabilities is locally absolutely continuous with respect to the other. In particular, the structure of the likelihood process and change of measure by using local martingales are discussed.

Chapter 6 is dedicated to characterization of the independence between the marked point processes in terms of the structure of the compensating measures. There are also considerations about counting processes and random counting measures with independent increments. Moreover, some properties of Levy processes are presented. The author suggests treating this presentation only as information. However, in my opinion, it can be useful for many readers, who apply this class of stochastic processes.

Piecewise deterministic Markov processes are defined in Chapter 7. They are special cases of piecewise deterministic processes. Their detailed description is preceded by basic definitions and properties of Markov processes and chains. The author also presents renewal processes, processes derived from homogeneous Poisson measures and solutions of a class of stochastic differential equations as examples of piecewise deterministic Markov processes. For time-homogeneous processes, Ito's formula, the full infinitesimal generator, and stationarity are discussed. The form of the likelihood processes for piecewise deterministic Markov processes is the main subject of the final part of this chapter.

The sufficiently general approach to stochastic analysis in Part I enabled unification of the treatment of many classes of stochastic processes.

The theory of the marked point processes and piecewise deterministic Markov processes is a promising field of stochastic analysis. The evidence of this fact is a wide variety of their applications. Some examples of applications are presented in four chapters of Part II.

Chapter 8 is devoted to survival analysis. Problem of estimating an unknown survival distribution is considered. In the presented case a partially specified statistical model is used. The estimation is based on an independent, identically distributed sample subject to right-censoring, and martingale estimators are applied. Moreover, in Chapter 8, methods of estimation for the Cox regression model are discussed. The considered model describes the intensity of failure.

Chapter 9 consists of three different models. The first one is a piecewise deterministic Markov process, describing the evolution of a single-sex population, where each individual can give birth to a new individual and each individual may die with rates depending on age. The second model is dedicated to risk theory. Sum of a compound Poisson process and a linear drift is considered. The ruin probability and the Laplace transform of the time to ruin are calculated in two cases. In proofs of propositions the theory of piecewise deterministic time-homogeneous Markov process is used. The third model discussed in Chapter 9 is a model describing the development over time of a soccer game. For this model the author applies the likelihood process and change of measure techniques to obtain the dynamics of the game.

A very interesting example of application of a piecewise deterministic Markov process to mathematical finance is presented in Chapter 10. It is dedicated to pricing of risky assets. In place of Brownian motion, which is used in the basic financial models, price process of the underlying asset is described by using of a compound Poisson process. For this model, pricing of contingent claims is considered. Problems of self-financing trading strategies, arbitrage and pricing with application of martingale methods are discussed. The model with jumps, proposed by the author can approximate the basic diffusion model arbitrarily well.

Chapter 11 is dedicated to application of piecewise deterministic Markov processes to queueing theory. In the first part of this chapter GI/G/1 queue is considered. In this model arrivals occur according to a renewal process, random variables describing service times are independent and identically distributed, and there is no dependence between them and the arrival process. There is also a discussion about the stationarity of a simpler model, i.e. M/M/1 queue. The second part of Chapter 11 is devoted to applications of piecewise deterministic Markov processes to queueing networks.

The series of examples of applications presented in Part II is very valuable for readers. In my opinion, it would be better to present more of them in the form of theorems and propositions. However, the general impression from reading of this part of the book remains very positive.

Appendices in Part III concern differentiation of cadlag functions and some basic elements of stochastic analysis including filtrations and martingales. They are helpful for readers not familiar with the theory of stochastic processes. Bibliographical Notes in the closing part of book are very useful for readers who are interested in the theory, as well as in further development of models from Part II.

The theory presented in this book is designed for advanced readers. However, definitions and detailed proofs of the most important theorems and propositions make the exposition self-contained. Additionally, the author suggests which sections or proofs may be omitted at the first reading. Therefore, the book is useful for both students and researchers. Taking into account the variety of applications of marked point and piecewise deterministic processes, I recommend the book to readers interested in stochastic analysis.

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