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Book review:

AN INVITATION TO VARIATIONAL METHODS IN DEFFERENTIAL EQUATIONS

by

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The book constitutes an elementary introduction to variational methods in differential equations. As the author states in the preface, the book is based on his textbook published in Portuguese in 1986, which was slightly revised after 18 years of teaching the subject to students, starting their postgraduate studies in differential equations and nonlinear analysis. Accordingly, the readers are assumed to posses only a basic knowledge in calculus, nonlinear analysis and differential equations.

In the first chapter, the author introduces the readers into the topic of the book, by presenting five instructive examples of two-points boundary value problems for ordinary differential equations, that illustrate problems of existence and uniqueness of solutions, as well as the use of the variational approach. The next chapters are the following:

2. Critical points via minimization,
3. The deformation theorem,
4. The mountain-pass theorem,
5. The saddle-point theorem,
6. Critical points under constraints,
7. A duality principle,
8. Critical points under symmetries,
9. Problems with S^1-symmetry,
10. Problems with lack of compactness,
11. Lack of compactness for bounded Ω.

In the appendix, Ekeland's variational principle is presented with a proof.

The above outline of the contents shows that the book covers quite a broad material, which is presented in an elementary and reader-friendly manner. The basic results are proved and simple examples of their applications are provided. Each chapter is supplied with a couple of exercises. The bibliography consists of 77 items, almost all of them published before the Portuguese edition of the book. The subject and authors index is provided.

The book provides a nicely written elementary introduction to variational methods in differential equations and can be recommended, as the first textbook, for students and researchers interested in the topic.

K. Malanowski

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