Control and Cybernetics

vol. **36** (2007) No. 4

Book review:

LINEAR PARTIAL DIFFERENTIAL EQUATIONS FOR SCIENTISTS AND ENGINEERS

by

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Partial differential equations (PDEs) originated from the study of geometrical surfaces and a wide variety of problems in mechanics. Since till today they are essential in modeling of natural phenomena, and have a wide range of applications in different branches of applied mathematics, physics, engineering, biology and chemistry, the PDEs form an essential part of the core mathematical syllabus for scientists and engineers.

The goal of the book here commented upon is to provide the readers with fundamental concepts, the underlying principles, wide range of applications and various methods of solution of linear PDEs, rather than an elegant exposition of general theory of PDEs in functional spaces.

The book is divided into fifteen chapters. Historical background of the development of PDEs theory and applications, as well as basic concepts and ideas of the field are sketched in the introductory chapter. Next, basic types of the first and second order linear PDEs are provided. Laplace, Helmholtz, heat, wave, elasticity equations as well as linear Schrödinger and linearized Korteweg – de Vries equations are recalled and employed to formulate mathematical models of physical phenomena. Models of vibrating strings and membranes, heat conduction in solids, as well as waves in elastic medium are presented. Moreover, spherical and also cylindrical wave equations are discussed.

The main part of the book is devoted to presentation of basic principles and analytical methods for solving linear PDEs. Fundamental principles, such as the superposition principle, conservation laws, the maximum and the minimum principles for boundary value problems are recalled. Results concerning the existence, uniqueness and well-posedness of solutions to linear boundary value problems are provided. The presentation of basic methods for solving linear PDEs include: the method of characteristics for solving first order linear and quasi-linear PDEs, the method of separation of variables, the Sturm – Liouville approach for selfadjoint equations consisting in solving the associated eigenvalue problems, the method of eigenvalue expansions, and the method of Green's functions. Separate chapter is devoted to Fourier, Laplace, Hankel, Mellin integral

transform methods and their applications in solving linear PDEs, including, especially, the fractional PDEs. For the sake of self contained exposition the theory of Fourier series is also recalled.

Since physical phenomena are usually nonlinear, so to illustrate the associated class of PDEs, one chapter deals with a few nonlinear equations including shock waves, nonlinear dispersive waves, Burgers', solitary waves equations. Two final chapters present briefly approximation and numerical methods for solving PDEs as well as tables of integral transforms, respectively. Special functions as well as their properties are assembled in the appendix. The list of bibliography consisting of 186 research papers and standard books is added. This allows the readers to have further insight into the subject matter. Each chapter is accompanied by exercises truly complementing the text. Answers to most exercises are provided at the end of the book.

This well written book is an excellent introduction to the basic theory and applications of linear PDEs. The authors provide a broad survey of many important topics and self contained exposition of linear PDEs. The development within each part is being rigorous and complete. Throughout the book the reader is acquainted with various approaches and techniques to find solutions to initial and boundary value problems. The book's wide scope and clear exposition make it a suitable text for undergraduate course in PDEs or for anyone who wants to start study the field of PDEs. The reader is assumed only to have completed a good basic first–year ancillary mathematics course. In particular, the book may serve as a textbook for basic courses in PDEs or in a course on advanced engineering mathematics. Moreover, graduate students, researchers as well as professionals in modern applied mathematics, mathematical physics or engineering will find this book as an excellent reference allowing for starting the interdisciplinary investigation in a variety of fields.

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Tyn Myint-U, Lokenath Debnath, *Linear Partial Differential Equations for Scientists and Engineers*, 4^{th} ed., Birkhäuser Verlag AG, Basel-Boston-Berlin, 2007, 778 pages, ISBN 978-0-8176-4393-5. Price (hardcover): 78.00 EUR