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An application of feedback linearization to the tracking and almost disturbance decoupling control of nonlinear system with uncertainties

by

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Abstract: This paper studies the tracking and almost disturbance decoupling problem of nonlinear systems with uncertainties, based on the feedback linearization approach. The main contribution of this study is to construct a controller, under appropriate conditions, such that the resulting closed-loop system is valid for any initial condition and bounded tracking signal with the following characteristics: input-to-state stability with respect to disturbance inputs and almost disturbance decoupling, i.e., the influence of disturbances on the L_2 norm of the output tracking error can be arbitrarily attenuated by changing some adjustable parameters. Two examples, which cannot be solved by the first paper on the almost disturbance decoupling problem, are proposed in this paper to exploit the fact that the tracking and the almost disturbance decoupling performances are easily achieved by the proposed approach. In order to demonstrate the practical applicability, the paper has investigated the AMIRA ball and beam system.

Keywords: almost disturbance decoupling, feedback linearizable, differential geometry approach, composite Lyapunov approach.

1. Introduction

Two well-known tasks of stabilization and tracking are important topics in the field of control. The tracking problem is generally more complicated than the stabilization problem for nonlinear control systems. Many approaches for nonlinear systems have been introduced, including feedback linearization, variable structure control (sliding mode control), backstepping, regulation control, nonlinear H^∞ control, internal model principle and H^∞ adaptive fuzzy control. Recently, variable structure control has been introduced to deal with nonlinear system (Khalil, 1996). However, chattering behavior that may create unmodeled high-frequency due to the discontinuous switching and imperfect implementation, and even drive system to instability is inevitable in the variable structure control scheme. Beckstepping has been a powerful tool for synthesizing controllers for a class of nonlinear systems. However, a disadvantage with the backstepping approach is the explosion of complexity, which is caused by the complicated repeated differentiations of some nonlinear functions (Yip and Hedrick, 1998; Swaroop et al., 2000). An output tracking approach utilizes the scheme of the output regulation control (Isidori and Byrnes, 1990) in which the outputs are assumed to be excited by an exosystem. However, the nonlinear regulation problem requires finding the difficult solution of partial-differential algebraic equation. Another problem of the output regulation control is that the exosystem states need to be switched to describe changes in the output and this will create transient tracking errors (Peroz et al., 2002). In general, the nonlinear H^∞ control involves solving the Hamilton-Jacobi equation, which is a difficult nonlinear partial-differential equation (Ball et al., 1993; Isidori and Kang; 1995; A.J. Van der Schaft, 1992). Only for some particular nonlinear systems we can derive a closed-form solution (Isidori, 1994). The control approach based on internal model principle converts the tracking problem to nonlinear output regulation problem. This approach depends on solving a first-order partial-differential equation of the center manifold (Isidori and Byrnes, 1990). For some special nonlinear systems and desired trajectories, the asymptotic solutions of this equation via ordinary differential equations have been developed (Huang and Rugh, 1990; Gopalswamy and Hedrick, 1993). Recently, H^∞ adaptive fuzzy control has been proposed to systematically deal with nonlinear systems (Chen et al., 1996). The drawback with H^∞ adaptive fuzzy control is that the complex parameter update law makes this approach impractical. During the past decade significant progress has been made in the research of control approaches for nonlinear systems based on the feedback linearization theory (Isidori, 1989; Nijmeijer and Van Der Schaft, 1990; Slotine and Li, 1991; Khalil, 1996). Moreover, feedback linearization approach has been applied successfully to address many real controls. These include the control of electromagnetic suspension system (Joo and Seo, 1997), pendulum system (Corless and Leitmann, 1981), spacecraft (Sheen and Bishop, 1998), electrohydraulic servosystem (Alleyne, 1998), car-pole system (Bedrossian, 1992) and bank-to-turn missile system (Lee et al., 1997).

For many practical control systems, it is difficult to obtain completely accurate mathematical models for them. Thus, there are inevitable uncertainties in their constructed models. Therefore, the design of a robust controller that deals with uncertainties of a control system is a significant subject for the design of an excellent control system. In this paper, we present a systematic analysis and a simple design scheme that guarantees the globally asymptotical stability of feedback-controlled uncertain system and achieves output tracking and almost disturbance decoupling performances for a class of nonlinear control systems with uncertainties.

The almost disturbance decoupling problem being the problem of design of a controller which attenuates the effect of the disturbance on the output terminal to an arbitrary degree of accuracy was originally considered for linear and nonlinear control systems by Willems (1982) and Marino, Respondek and Van Der Schaft (1989), respectively. Henceforward, the problem has attracted considerable attention and many significant results have been developed for both linear and nonlinear control systems (Weiland and Willems, 1989; Marino and Tomei, 1999; Qian and Lin, 2000). Marino, Respondek and Van Der Schaft (1989) show that for the following plants which cannot satisfy the main structure conditions, the almost disturbance decoupling problem may not be solvable:

$$\begin{aligned} &\dot{x}_1(t) = \tan^{-1} x_2 + \theta(t) \\ &\dot{x}_2(t) = u \\ &y = x_1 \end{aligned}$$

and

$$\begin{aligned} &\dot{x}_1(t) = x_2 + \theta_1(t) \\ &\dot{x}_2(t) = x_2^3\theta_2(t) + u \\ &y = x_1 \end{aligned}$$

where u, y denote the input and output respectively and θ, θ_1, θ_2 are the disturbances. On the contrary, these examples can be easily solved via the approach proposed in this paper. Moreover, in order to show the significant applicability possibilities, in this paper we also derive tracking controller with almost disturbance decoupling for the AMIRA ball and beam system. Throughout the paper, the notation $\|\cdot\|$ denotes the usual Euclidean norm or the corresponding induced matrix norm.

2. Tracking and almost disturbance decoupling controller design

In this paper, we consider the following nonlinear control system with uncertainties and disturbances:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} f_1(x_1, x_2, .., x_n) \\ f_2(x_1, x_2, .., x_n) \\ \vdots \\ f_n(x_1, x_2, .., x_n) \end{bmatrix} + \begin{bmatrix} g_1(x_1, x_2, .., x_n) \\ g_2(x_1, x_2, .., x_n) \\ \vdots \\ g_n(x_1, x_2, .., x_n) \end{bmatrix} u + \begin{bmatrix} \Delta f_1 \\ \Delta f_2 \\ \vdots \\ \Delta f_n \end{bmatrix} + \begin{bmatrix} \Delta g_1 \\ \Delta g_2 \\ \vdots \\ \Delta g_n \end{bmatrix} u + \sum_{i=1}^{p} q_i^\star \theta_i \tag{1a}$$

$$y(t) = h(x_1, x_2, .., x_n) \tag{1b}$$

i.e.,

$$\begin{aligned}\dot{X}(t) =& f(X(t)) + g(X(t))u \ + \Delta f + \Delta g \cdot u + \sum_{i=1}^{p} q_i^{\star}\theta_i \\ y(t) =& h(X(t))\end{aligned}$$

where $X(t) := \begin{bmatrix} x_1(t) \ x_2(t) \ \cdots \ x_n(t) \end{bmatrix}^T \in \Re^n$ is the state vector, $u \in \Re^1$ is the input, $y \in \Re^1$ is the output, $\theta := \begin{bmatrix} \theta_1(t) \ \theta_2(t) \ \cdots \ \theta_p(t) \end{bmatrix}^T$ is a bounded time-varying disturbances vector, $\Delta f := \begin{bmatrix} \Delta f_1 \ \Delta f_2 \ \cdots \ \Delta f_n \end{bmatrix} \in \Re^n$ and $\Delta g := \begin{bmatrix} \Delta g_1 \ \Delta g_2 \ \cdots \ \Delta g_n \end{bmatrix} \in \Re^n$ are the system uncertainties, f, g, $q_1^*, \cdots, q_p^*$ are smooth vector fields on $\Re^n$, and $h(X(t)) \in \Re^1$ is a smooth function. The nominal system is then defined as follows:

$$\dot{X}(t) = f(X(t)) + g(X(t))u \tag{2a}$$

$$y(t) = h(X(t)). \tag{2b}$$

The nominal system (2) is of relative degree r (Henson and Seborg, 1991), i.e., there exists a positive integer $1 \leqslant r < \infty$ such that

$$L_g L_f^k h(X(t)) = 0, \quad k < r-1, \tag{3}$$

$$L_g L_f^{r-1} h(X(t)) \neq 0 \tag{4}$$

for all $X \in \Re^n$ and $t \in [0, \infty)$, where the operator L is the Lie derivative (Isidori, 1989). The desired output trajectory $y_d(t)$ and its first r derivatives are all uniformly bounded and

$$\left\| \left[y_d(t), y_d^{(1)}(t), \cdots, y_d^{(r)}(t) \right] \right\| \leqslant B_d \tag{5}$$

where B_d is some positive constant. For the uncertainties, there exist smooth functions $\delta_1(\cdot), \delta_2(\cdot) : \Re^n \to \Re$ such that the uncertainties Δf and Δg in (1) are assumed to satisfy

$$\Delta f(X) = g(X)\delta_1(X)$$

and

$$\Delta g(X) = g(X)\delta_2(X)$$

where $g(X)\delta_1(X)$ and $g(X)\delta_2(X)$ are referred to as the matched uncertainties.

Under the assumption of the well-defined relative degree, it has been shown (Isidori, 1989) that the mapping

$$\phi : \Re^n \to \Re^n \tag{6}$$

defined as

$$\phi_i(X(t)) := \xi_i(t) = L_f^{i-1} h(X(t)),\ i = 1, 2, \cdots, r \tag{7}$$

$$\phi_k(X(t)) := \eta_k(t),\ k = r+1, r+2, \cdots, n \tag{8}$$

and satisfying

$$L_g \phi_k(X(t)) = 0,\ k = r+1, r+2, \cdots, n \tag{9}$$

is a diffeomorphism onto image. For the sake of convenience, define the trajectory error and the trajectory error multiplied with some adjustable positive constant ε, respectively, to be

$$e_i(t) := \xi_i(t) - y_d^{(i-1)}(t),\ i = 1, 2, \cdots, r \tag{10}$$

$$e := [e_1(t)\ e_2(t) \cdots e_r(t)]^T \in \Re^r \tag{11}$$

and

$$\overline{e_i}(t) := \varepsilon^{i-1} e_i(t),\ i = 1, 2, \cdots, r \tag{12}$$

$$\overline{e} := [\overline{e_1}(t)\ \overline{e_2}(t) \cdots \overline{e_r}(t)]^T \in \Re^r\,. \tag{13}$$

Let

$$\xi(t) := [\xi_1(t)\ \xi_2(t) \cdots \xi_r(t)]^T \in \Re^r \tag{14a}$$

$$\eta(t) := [\eta_{r+1}(t)\ \eta_{r+2}(t) \cdots \eta_n(t)]^T \in \Re^{n-r} \tag{14b}$$

$$q(\xi(t), \eta(t)) := [L_f\phi_{r+1}(t)\ L_f\phi_{r+1}(t)\ \cdots\ L_f\phi_n(t)]^T := [q_{r+1}\, q_{r+2}\ \cdots\ q_n]^T\,. \tag{14c}$$

Define a phase-variable canonical matrix A_c to be

$$A_c := \begin{bmatrix} 0 & 1 & 0 & & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ & \vdots & & & & \vdots \\ 0 & 0 & 0 & & \cdots & 1 \\ -\alpha_1 & -\alpha_2 & -\alpha_3 & & \cdots & -\alpha_r \end{bmatrix}_{r \times r} \tag{15}$$

where $\alpha_1, \alpha_2, \cdots, \alpha_r$ are any chosen parameters such that A_c is Hurwitz, and the vector B to be

$$B := \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \end{bmatrix}_{r \times 1}^T\,. \tag{16}$$

Let P be the positive definite solution of the following Lyapunov equation

$$A_c^T P + P A_c = -I \tag{17}$$

$$\lambda_{\max}(P) := \text{the maximum eigenvalue of } P \tag{18}$$

$$\lambda_{\min}(P) := \text{the minimum eigenvalue of } P. \tag{19}$$

Assumption 1 *For all* $t \geqslant 0$, $\eta \in \Re^{n-r}$ *and* $\xi \in \Re^{r}$, *there exists a positive constant* M *such that the following inequality holds:*

$$\|q_{22}(t,\eta,\bar{e}) - q_{22}(t,\eta,0)\| \leqslant M(\|\bar{e}\|) \tag{20}$$

where $q_{22}(t,\eta,\bar{e}) := q(\xi,\eta)$.

Assumption 2 *There exists known function* $\beta_2(\cdot) : \Re^{n} \to \Re^{+}$ *such that*

$$\left\|\delta_1 d - \delta_2 c + \delta_2 y_d^{(r)} - \delta_2 \varepsilon^{-r}\bar{\bar{e}} + (1+\delta_2)\frac{\partial h}{\partial X}\left(K_x^T X\right)\right\| \leqslant \beta_2 \|\bar{e}\| \tag{21a}$$

$$d := L_g L_f^{r-1} h(X(t)) \tag{21b}$$

$$c := L_f^{r} h(X(t)) \tag{21c}$$

where $K_x := [k_{x1}\ k_{x2}\ \cdots\ k_{xn}]$ *and* $k_{xi}, i = 1,2,...,n$ *are real constants. For the sake of stating precisely the investigated problem, define*

$$\bar{\bar{e}} := \alpha_1 \overline{e_1} + \alpha_2 \overline{e_2} + \cdots + \alpha_r \overline{e_r} \tag{22}$$

and recall some definitions of class K, *class* KL *and input-to-state stability.*

Definition 1 (Khalil, 1996) *A continuous function* $\alpha : [0,a) \to [0,\infty)$ *is said to belong to class* K *if it is strictly increasing and* $\alpha(0) = 0$.

Definition 2 (Khalil, 1996) *A continuous function* $\beta : [0,a) \times [0,\infty) \to [0,\infty)$ *is said to belong to class* KL *if, for each fixed* s, *the mapping* $\beta(r,s)$ *belongs to class* K *with respect to* r *and, for each fixed* r, *the mapping* $\beta(r,s)$*is decreasing with respect to* s *and* $\beta(r,s) \to 0$ *as* $s \to \infty$.

Definition 3 (Khalil, 1996) *Consider the system* $\dot{x} = f(t,x,\theta)$, *where* $f : [0,\infty) \times \Re^{n} \times \Re^{n} \to \Re^{n}$ *is piecewise continuous in* t *and locally Lipschitz in* x *and* θ. *This system is said to be input-to-state stable if there exists a class* KL *function* β, *a class* K *function* γ *and positive constants* k_1 *and* k_2 *such that for any initial state* $x(t_0)$ *with* $\|x(t_0)\| < k_1$ *and any bounded input* $\theta(t)$ *with* $\sup_{t \geqslant t_0} \|\theta(t)\| < k_2$, *the state exists and satisfies*

$$\|x(t)\| \leqslant \beta(\|x(t_0)\|, t - t_0) + \gamma(\sup_{t_0 \leqslant \tau \leqslant t} \|\theta(\tau)\|) \tag{23a}$$

for all $t \geqslant t_0 \geqslant 0$.

Now we formulate the tracking problem with almost disturbance decoupling as follows:

Definition 4 (Marino and Tomei, 1999) *The tracking problem with almost disturbance decoupling is said to be globally solvable by the state feedback controller* u *for the transformed-error system by a global diffeomorphism (6), if the controller* u *enjoys the following properties:*

(i) It is input-to-state stable with respect to disturbance inputs.
(ii) For any initial value $\bar{x}_{e0} := [\bar{e}(t_0)\eta(t_0)]^T$*, for any* $t \geqslant t_0$ *and for any* $t_0 \geqslant 0$

$$|y(t) - y_d(t)| \leqslant \beta_{11}(\|x(t_0)\|, t - t_0) + \frac{1}{\sqrt{\beta_{22}}}\beta_{33}\left(\sup_{t_0 \leqslant \tau \leqslant t} \|\theta(\tau)\|\right) \tag{23b}$$

and

$$\int_{t_0}^{t} [y(\tau) - y_d(\tau)]^2 \, d\tau \leqslant \frac{1}{\beta_{44}}\left[\beta_{55}(\|\bar{x}_{e0}\|) + \int_{t_0}^{t} \beta_{33}\left(\|\theta(\tau)\|^2\right) d\tau\right] \tag{23c}$$

where β_{22},β_{44} *are some positive constants,* β_{33}, β_{55} *are class K functions and* β_{11} *is a class KL function.*

Some motivation for this study will be described here prior to stating our main contribution. Willems (1982) proposed the necessary and sufficient conditions for solving the almost disturbance decoupling problem via the almost controlled invariant subspaces. Marino, Respondek and Van Der Schaft (1987) obtained the necessary and sufficient conditions given in Willems (1982) in L^∞ case via the singular perturbation approach. In solvable cases a high-gain feedback structure is given involving the one in Young (1982) for a restricted subclass of linear systems. The L^k case for any k is solved in Marino, Respondek and Van Der Schaft (1988). Marino, Respondek and Van Der Schaft (1989) extended the results of Marino et al. (1988) to solve the nonlinear case via the same approach. Moreover, Marino et al. (1989) showed that for those plants, which cannot satisfy the main structure conditions, the almost disturbance decoupling problem may not be solvable. In this study we address the almost disturbance decoupling problem using the Lyapunov theory and linear algebra without the structural constraints given in Marino et al. (1989). Let us state the main result now as follows:

Theorem 1 *Suppose that there exists a continuously differentiable function* $V : \Re^{n-r} \to \Re^+$ *such that the following three inequalities hold for all* $\eta \in \Re^{n-r}$*:*

$$\text{(a)} \quad \omega_1 \|\eta\|^2 \leqslant V(\eta) \leqslant \omega_2 \|\eta\|^2, \quad \omega_1, \omega_2 > 0 \tag{24a}$$

$$\text{(b)} \quad \nabla_t V + (\nabla_\eta V)^T q_{22}(t, \eta, 0) \leqslant -2\alpha_x \|\eta\|^2, \quad \alpha_x > 0 \tag{24b}$$

$$\text{(c)} \quad \|\nabla_\eta V\| \leqslant \omega_3 \|\eta\|, \quad \omega_3 > 0, \tag{24c}$$

then the tracking problem with almost disturbance decoupling is globally solvable by the controller defined by

$$\begin{aligned} u = \left[L_g L_f^{r-1} h(X(t))\right]^{-1} \Big\{ &- L_f^r h(X) + y_d^{(r)} - \varepsilon^{-r}\alpha_1 \left[L_f^0 h(X) - y_d\right] \\ &- \varepsilon^{1-r}\alpha_2 \left[L_f^1 h(X) - y_d^{(1)}\right] - \cdots \\ &- \varepsilon^{-1}\alpha_r \left[L_f^{r-1} h(X) - y_d^{(r-1)}\right] + \frac{\partial h}{\partial X}(K_x^T X)\Big\} \end{aligned} \tag{25}$$

Moreover, the influence of disturbances on the L_2 norm of the tracking error can be arbitrarily attenuated by increasing the following adjustable parameter NN_2:

$$H(\varepsilon) := \begin{bmatrix} H_{11} & H_{12} \\ H_{12} & H_{22} \end{bmatrix}$$

$$:= \begin{bmatrix} 2\alpha_x - \frac{\omega_3^2}{\omega_1}\left\|\phi_\eta\right\|^2 & -\frac{1}{\sqrt{k(\varepsilon)}}\left[\frac{w_3 M}{\sqrt{2w_1\lambda_{\min}(P)}}\right] \\ -\frac{1}{\sqrt{k(\varepsilon)}}\left[\frac{w_3 M}{\sqrt{2w_1\lambda_{\min}(P)}}\right] & \frac{1}{\varepsilon\lambda_{\max}(P)} - \frac{2\varepsilon^r\left\|B^T P\right\|\beta_2 + 2(\frac{k(\varepsilon)}{\varepsilon})\|\phi_\xi\|^2\|P\|^2}{\varepsilon\lambda_{\min}(P)} \end{bmatrix}, \tag{26a}$$

$$\alpha_s(\varepsilon) = \frac{H_{11} + H_{22} - [(H_{11} - H_{22})^2 + 4H_{12}^2]^{1/2}}{4}, \tag{26b}$$

$$N := 2\alpha_s(\varepsilon), \tag{26c}$$

$$N_2 := \min\left\{\omega_1, \frac{k(\varepsilon)}{2}\lambda_{\min}(P)\right\}, \tag{26d}$$

$$\phi_\xi(\varepsilon) := \begin{bmatrix} \varepsilon\frac{\partial}{\partial X}hq_1^\star & \dots & \varepsilon\frac{\partial}{\partial X}hq_p^\star \\ \vdots & \vdots & \\ \varepsilon^r\frac{\partial}{\partial X}L_f^{r-1}hq_1^\star & \dots & \varepsilon^r\frac{\partial}{\partial X}L_f^{r-1}hq_q^\star \end{bmatrix}, \tag{26e}$$

$$\phi_\eta(\varepsilon) := \begin{bmatrix} \frac{\partial}{\partial X}\phi_{r+1}q_1^\star & \cdots & \frac{\partial}{\partial X}\phi_{r+1}q_p^\star \\ \vdots & \vdots & \\ \frac{\partial}{\partial X}\phi_n q_1^\star & \cdots & \frac{\partial}{\partial X}\phi_n q_q^\star \end{bmatrix}, \tag{26f}$$

where H is positive definite matrix and $k(\varepsilon) : \Re^+ \to \Re^+$ is any continuous function satisfying

$$\lim_{\varepsilon\to 0} k(\varepsilon) = 0 \text{ and } \lim_{\varepsilon\to 0}\frac{\varepsilon}{k(\varepsilon)} = 0 \tag{26g}$$

Proof. Applying the co-ordinate transformation (6) yields

$$\dot{\xi}_1(t) = \xi_2(t) + \sum_{i=1}^{p}\frac{\partial h(X)}{\partial X}q_i^\star\theta_i \tag{27}$$

$$\vdots$$

$$\dot{\xi}_{r-1}(t) = L_f^{r-1}h + \frac{\partial L_f^{r-2}h}{\partial X}\sum_{i=1}^{p}q_i^\star\theta_i = \dot{\xi}_r(t) + \sum_{i=1}^{p}\frac{\partial L_f^{r-2}h}{\partial X}q_i^\star\theta_i \tag{28}$$

$$\dot{\xi}_r(t) = c + d[(1+\delta_2)u + \delta_1] + \sum_{i=1}^{p}\frac{\partial L_f^{r-1}h}{\partial X}q_i^\star\theta_i \tag{29}$$

$$\dot{\eta}_k(t) = L_f\phi_k + \sum_{i=1}^{p}\frac{\partial\phi_k}{\partial X}q_i^\star\theta_i,\ k = r+1, r+2, \cdots, n. \tag{30}$$

Since

$$c(\xi(t),\eta(t)) := L_f^r h(X(t)) \tag{31}$$
$$d(\xi(t),\eta(t)) := L_g L_f^{r-1} h(X(t)) \tag{32}$$
$$q_k(\xi(t),\eta(t)) = L_f \phi_k(X), \quad k = r+1, r+2, \cdots, n \tag{33}$$

the dynamic equations of system (1) in the new co-ordinates are as follows:

$$\dot{\xi}_i(t) = \xi_{i+1}(t) + \sum_{i=1}^{p} \frac{\partial}{\partial X} L_f^{i-1} h q_i^\star \theta_i, \quad i = 1, 2, \cdots, r-1 \tag{34}$$
$$\dot{\xi}_r(t) = c(\xi(t),\eta(t)) + d(\xi(t),\eta(t))\left[(1+\delta_2)\,u + \delta_1\right] + \sum_{i=1}^{p} \frac{\partial}{\partial X} L_f^{r-1} h q_i^\star \theta_i \tag{35}$$
$$\dot{\eta}(t) = q_k(\xi(t),\eta(t)) + \sum_{i=1}^{p} \frac{\partial}{\partial X} \phi_k(X) q_i^\star \theta_i \quad k = r+1, \cdots, n \tag{36}$$
$$y(t) = \xi_1(t). \tag{37}$$

Define

$$\begin{aligned} v := {} & y_d^{(r)} - \varepsilon^{-r}\alpha_1\left[L_f^0 h(X) - y_d\right] - \varepsilon^{1-r}\alpha_2\left[L_f^1 h(X) - y_d^{(1)}\right] \\ & - \cdots - \varepsilon^{-1}\alpha_r\left[L_f^{r-1} h(X) - y_d^{(r-1)}\right] + \frac{\partial h}{\partial X}(K_x^T X) \end{aligned} \tag{38}$$

According to equations (7, 10, 31, 32) and (38), the tracking controller can be rewritten as

$$u = d^{-1}\left[-c + v\right]. \tag{39}$$

Substituting equation (39) into (35), the dynamic equations of system (1) can be shown as follows:

$$\begin{bmatrix} \dot{\xi}_1(t) \\ \dot{\xi}_2(t) \\ \vdots \\ \dot{\xi}_{r-1}(t) \\ \dot{\xi}_r(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0\cdots & 0 \\ & \vdots & & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \\ \vdots \\ \xi_{r-1}(t) \\ \xi_r(t) \end{bmatrix} +$$

$$+\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \{\delta_1 d - \delta_2 c + (1+\delta_2)\, v\} + \begin{bmatrix} \sum_{i=1}^{p} \frac{\partial}{\partial X} h q_i^\star \theta_i \\ \sum_{i=1}^{p} \frac{\partial}{\partial X} L_f^1 h q_i^\star \theta_i \\ \vdots \\ \sum_{i=1}^{p} \frac{\partial}{\partial X} L_f^{r-1} h q_i^\star \theta_i \end{bmatrix} \tag{40}$$

$$\begin{bmatrix} \dot{\eta}_{r+1}(t) \\ \dot{\eta}_{r+2}(t) \\ \vdots \\ \dot{\eta}_{n-1}(t) \\ \dot{\eta}_n(t) \end{bmatrix} = \begin{bmatrix} q_{r+1}(t) \\ q_{r+2}(t) \\ \vdots \\ q_{n-1}(t) \\ q_n(t) \end{bmatrix} + \begin{bmatrix} \sum_{i=1}^{p} \frac{\partial}{\partial X} \phi_{r+1} q_i^\star \theta_i \\ \sum_{i=1}^{p} \frac{\partial}{\partial X} \phi_{r+2} q_i^\star \theta_i \\ \vdots \\ \sum_{i=1}^{p} \frac{\partial}{\partial X} \phi_{n-1} q_i^\star \theta_i \\ \sum_{i=1}^{p} \frac{\partial}{\partial X} \phi_n q_i^\star \theta_i \end{bmatrix} \tag{41}$$

$$y = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \end{bmatrix}_{r\times 1} \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \\ \vdots \\ \xi_{r-1}(t) \\ \xi_r(t) \end{bmatrix}_{r\times 1} = \xi_1(t). \tag{42}$$

Combining equations (10, 12, 15) and (38), it can be easily verified that equations (40)-(42) can be transformed into the following form:

$$\dot{\eta}(t) = q(\xi(t), \eta(t)) + \phi_\eta \theta := q_{22}(t, \eta(t), \overline{e}(t)) + \phi_\eta \theta \tag{43a}$$

$$\varepsilon \dot{\overline{e}}(t) = A_c \overline{e} + B\varepsilon^r \left\{ \delta_1 d - \delta_2 c + \delta_2 y_d^{(r)} - \delta_2 \varepsilon^{-r} \overline{\overline{e}} + (1+\delta_2) \frac{\partial h}{\partial X} \left(K_x^T X \right) \right\} + \phi_\xi \theta \tag{43b}$$

$$y(t) = \xi_1(t). \tag{44}$$

We consider $L(\bar{e}, \eta)$ defined by a weighted sum of $V(\eta)$ and $W(\bar{e})$,

$$L(\bar{e}, \eta) := V(\eta) + k(\varepsilon) W(\bar{e}), \tag{45}$$

as a composite Lyapunov function of the subsystems (43a) and (43b) (Khorasani and Kokotovic, 1986; Marino and Kokotovic, 1988), where $W(\bar{e})$ satisfies

$$W(\tilde{e}) := \frac{1}{2} \bar{e}^T P \bar{e}. \tag{46}$$

In view of (17, 20, 21, 24) and (25), the derivative of L along the trajectories of (43a) and (43b) is given by

$$\begin{aligned}\dot{L} &= \left[\nabla_t V + (\nabla_\eta V)^T \dot{\eta}\right] + \frac{k}{2}\left[(\dot{\bar{e}})^T P\bar{e} + \bar{e}^T P(\dot{\bar{e}})\right] \\ &= \left[\nabla_t V + (\nabla_\eta V)^T \dot{\eta}\right] + \frac{k}{2\varepsilon}\left[\bar{e}^T\left(A_c^T P + PA_c\right)\bar{e}\right] \\ &+ \frac{k}{2}\varepsilon^{r-1}\left\{2B^T P\bar{e}\left(\delta_1 d - \delta_2 c + \delta_2 y_d^{(r)} - \delta_2\varepsilon^{-r}\bar{\bar{e}} + (1+\delta_2)\frac{\partial h}{\partial X}\left(K_x^T X\right)\right)\right\} \\ &+ k\left\{\frac{1}{\varepsilon}\theta^T\phi_\xi^T P\bar{e}\right\} \\ &\leq -\left(2\alpha_x - \frac{\omega_3^2}{\omega_1}||\phi_\eta||^2\right)V + 2\left(\frac{w_3 M}{\sqrt{2w_1\lambda_{\min}(P)}}\right)\sqrt{V}\sqrt{W} \\ &- \left(\frac{k}{\varepsilon\lambda_{\max}(P)} - \varepsilon^{r-1}\frac{k||B^T P||\beta_2}{\frac{1}{2}\lambda_{\min}(P)} - \frac{k^2||\phi_\xi||^2||P||^2}{\frac{\varepsilon^2}{2}\lambda_{\min}(P)}\right)W + \frac{1}{2}||\theta||^2 \\ &= -\begin{bmatrix}\sqrt{V} & \sqrt{kW}\end{bmatrix} H \begin{bmatrix}\sqrt{V} \\ \sqrt{kW}\end{bmatrix} + \frac{1}{2}\|\theta\|^2. \qquad (47)\end{aligned}$$

$$\text{i.e. } \dot{L} \leqslant -\lambda_{\min}(H)L + \frac{1}{2}\|\theta\|^2, \qquad (48)$$

where $\lambda_{\min}(H)$ denotes the minimum eigenvalue of the matrix H. Utilizing the fact that $\lambda_{\min}(H) = 2\alpha_s$, we obtain

$$\dot{L} \leqslant -2\alpha_s\left(\omega_1\|\eta\|^2 + \frac{k}{2}\lambda_{\min}(P)\|\bar{e}\|^2\right) + \frac{1}{2}\|\theta\|^2 \leqslant -NN_2\left(\|\eta\|^2 + \|\bar{e}\|^2\right) + \frac{1}{2}\|\theta\|^2. \qquad (49)$$

Define

$$\bar{e}_{1r} := \begin{bmatrix}\bar{e}_2 \\ \vdots \\ \bar{e}_r\end{bmatrix}. \qquad (50)$$

Hence

$$\dot{L} \leqslant -NN_2\left(\|\eta\|^2 + \|\bar{e}_1\|^2 + \|\bar{e}_{1r}\|^2\right) + \frac{1}{2}\|\theta\|^2. \qquad (51)$$

Application of (51) easily yields

$$\int_{t_0}^{t}\left(y(\tau) - y_d(\tau)\right)^2 d\tau \leqslant \frac{L(t_0)}{NN_2} + \frac{1}{2\,NN_2}\int_{t_0}^{t}\|\theta(\tau)\|^2 d\tau \qquad (52)$$

so that statement (23c) is satisfied. From (49), we get

$$\dot{L} \leqslant -NN_2\left(\|y_{total}\|^2\right) + \frac{1}{2}\|\theta\|^2 \tag{53a}$$

where

$$\|y_{total}\|^2 := \|\bar{e}\|^2 + \|\eta\|^2 . \tag{53b}$$

By virtue of Khalil (1996, Theorem 5.2), (53a) implies the input-to-state stability for the closed-loop system. Furthermore, it is easy to see that

$$\Delta_{\min}\left(\|\bar{e}\|^2 + \|\eta\|^2\right) \leq L \leq \Delta_{\max}\left(\|\bar{e}\|^2 + \|\eta\|^2\right) \tag{54}$$

i.e.

$$\Delta_{\min}\left(\|y_{total}\|^2\right) \leqslant L \leqslant \Delta_{\max}\left(\|y_{total}\|^2\right) \tag{55}$$

where $\Delta_{\min} := \min\left\{\omega_1, \frac{k}{2}\lambda_{\min}(P)\right\}$ and $\Delta_{\max} := \min\left\{\omega_2, \frac{k}{2}\lambda_{\max}(P)\right\}$. From (49) and (55), we get

$$\dot{L} \leqslant -\frac{NN_2}{\Delta_{\max}}L + \frac{1}{2}\left(\sup_{t_0\leqslant\tau\leqslant t}\|\theta(\tau)\|\right)^2 . \tag{56}$$

Hence,

$$L(t) \leqslant L(t_0)e^{-\frac{NN_2}{\Delta_{\max}}(t-t_0)} + \frac{\Delta_{\max}}{2\,NN_2}\left(\sup_{t_0\leqslant\tau\leqslant t}\|\theta(\tau)\|\right)^2, \quad t \geqslant t_0 \tag{57}$$

which implies

$$|e_1(t)| \leqslant \sqrt{\frac{2L(t_0)}{k\lambda_{\min}(P)}}e^{-\frac{NN_2}{2\Delta_{\max}}(t-t_0)} + \sqrt{\frac{\Delta_{\max}}{k\lambda_{\min}(P)\,NN_2}}\left(\sup_{t_0\leqslant\tau\leqslant t}\|\theta(\tau)\|\right). \tag{58}$$

Thus, statement (23b) is proved and then the tracking problem with almost disturbance decoupling is globally solved. ■

3. Illustrative example

Fig. 1 shows the hardware structure of the AMIRA ball and beam system. U-type aluminium profiles construct the platform and the organization of the ball and beam system which is covered at the side by four sheets of plexiglas. The steel ball rolls freely on the beam and its position is measured by a camera unit and the lighting module mounted below a small platform on top of the system. The beam is located in the center of the system and driven by a tooth-belt, a tooth wheel and a DC motor. The angle of the beam is measured by an incremental encoder mounted at the rear end of the beam shaft. Two limited switches are located below the beam to detect whether the beam reaches its maximum angle or not. The unmeasurable states, the speed of the ball and the angular speed of the beam, are estimated by a Luenberger reduced order observer. Due to the mounting of the beam, the maximum angle $\alpha_{\max}$ is approximated to 0.24 rad.

Figure 1. The AMIRA ball and beam experimental equipment

By balancing all the forces acting upon the system, it is easy to evaluate the kinetic energy, potential energy, dissipative forces, and generalized forces of the system. Inserting them into the Lagrange equation, we can obtain simultaneously the motion equations (Amira Gmbh, 1999)

$$\left(m + \frac{I_b}{r^r}\right)\ddot{x}' + \left(mr^2 + I_b\right)\frac{1}{r}\ddot{\alpha} - mx'\dot{\alpha}^2 = mg(\sin\alpha) \tag{59a}$$

$$\left[m\left(x'\right)^2 + I_b + I_w\right]\ddot{\alpha} + \left(2m\dot{x}'x' + bl^2\right)\dot{\alpha} + Kl^2\alpha$$
$$+ \left(mr^2 + I_b\right)\frac{1}{r}\ddot{x}' - mgx'\left(\cos\alpha\right) = ul\left(\cos\alpha\right) \tag{59b}$$

and the nonlinear state equation

$$\dot{x}_1 = x_2 \tag{60a}$$

$$\dot{x}_2 = \frac{a_2\left[\left(b_2x_1x_2 + b_3\right)x_4 + b_4x_3 - b_6x_1\cos\left(x_3\right)\right]}{a_1\left(mx_1^2 + b_1\right) - a_2b_5}$$
$$+ \frac{\left(mx_1^2 + b_1\right)\left(a_3\sin\left(x_3\right) + mx_1x_4^2\right) - a_2l\cos\left(x_3\right)u}{a_1\left(mx_1^2 + b_1\right) - a_2b_5} \tag{60b}$$

$$\dot{x}_3 = x_4 \tag{60c}$$

$$\dot{x}_4 = \frac{-\left(b_2 x_1 x_2 + b_3\right) x_4 - b_4 x_3 + b_6 x_1 \cos\left(x_3\right)}{m x_1^2 + b_1} - \frac{b_5 \left(a_3 \sin\left(x_3\right) + m x_1 x_4^2\right)}{a_1 \left(m x_1^2 + b_1\right) - a_2 b_5}$$
$$- \frac{a_2 b_5 \left[\left(b_2 x_1 x_2 + b_3\right) x_4 + b_4 x_3 - b_6 x_1 \cos\left(x_3\right)\right]}{\left(m x_1^2 + b_1\right)\left(a_1 \left(m x_1^2 + b_1\right) - a_2 b_5\right)}$$
$$+ \left[1 + \frac{a_2 b_5}{a_1 \left(m x_1^2 + b_1\right) - a_2 b_5}\right] \frac{l \cos\left(x_3\right) u}{m x_1^2 + b_1} \tag{60d}$$

where the notations have the following meanings: m = mass of the ball, r = roll radius of the ball, I_b = inertia moment of the ball, $a_2 = \left(mr^2 + I_b\right)\frac{1}{r}$, $b_2 = 2m$, b = friction coefficient of the drive mechanics, l = radius of force application, $b_3 = bl^2$, l_w = beam radius, K = stiffness of the drive mechanics, $b_4 = Kl^2$, g = gravity, $b_6 = mg$, I_w = inertia moment of the beam, $b_1 = I_b + I_w$, $a_3 = mg$, u = force of the drive mechanics, $a_1 = m + \frac{I_b}{r^2}$, $b_5 = \left(mr^2 + I_b\right)\frac{1}{r}$, $x_1 = x'$ = position of the ball, $x_2 = \dot{x}'$ = velocity of the ball, $x_3 = \alpha$ = angle of the beam to the horizontal, $\alpha_{\max}$ = maximum angle of the beam to the horizontal, $x_4 = \dot{\alpha}$ = angular velocity of the beam. Substituting all the physical values, i.e. $r = 0.02$ m, $l = 0.48$ m, $m = 0.0162$ kg, $M = 1.122$ kg, $b = 1$ Ns/m, $K = 0.001$ N/m, $l_w = 0.5$ m into (60), the state equation can be rewritten as follows:

$$\dot{x}_1 = x_2 \tag{61a}$$
$$\dot{x}_2 = \frac{a_{12} + a_{34}}{a_5} + \frac{b_{11}}{b_{22}} u + \theta_1 \tag{61b}$$
$$\dot{x}_3 = x_4 \tag{61c}$$
$$\dot{x}_4 = \frac{z_{12} + z_{34} + z_{56}}{z_7} + au \tag{61d}$$

where

$$\begin{aligned}
a_{12} =& 0.0000561038 x_1 x_2 x_4 + 0.0004157 x_4 + 0.000000415757 x_3 \\
& - 0.00027490882 x_1 \cos(x_3) \\
a_{34} =& 0.0025719 x_1^2 (\sin(x_3)) + 0.00026244 x_1^3 x_4^2 + 0.03711015 \sin(x_3) \\
& + 0.00378675 x_1 x_4^2 \\
a_5 =& 0.0015 x_1^2 + 0.021642252, b_{11} = -0.000848484 \cos(x_3), b_{22} \\
& = 0.0015 x_1^2 + 0.021642252 \\
z_{12} =& -0.0000486 x_1^3 x_2 x_4 - 0.00036018 x_1^2 x_4 - 0.00000036 x_1^2 x_3 \\
& + 0.000238 x_1^3 \cos(x_3) \\
z_{34} =& -0.0007013 x_1 x_2 x_4 - 0.005197 x_4 - 0.000005197 x_3 \\
& + 0.003436 x_1 \cos(x_3) \\
z_{56} =& -0.00000445 x_1^2 \sin(x_3) - 0.000000454 x_1^3 x_4^2 - 0.0000643 \sin(x_3) \\
& - 0.00000655 x_1 x_4^2
\end{aligned}$$

$$z_7 = 0.0000243x_1^4 + 0.00070126x_1^2 + 0.021645,$$
$$a = \frac{0.000735x_1^2\cos(x_3) + 0.0106\cos(x_3)}{0.0000243x_1^4 + 0.00070126x_1^2 + 0.021645}$$

and $\theta_1 = \sin(t-8)$ is assumed to be the disturbance form. Applying Theorem 1 to the AMIRA ball and beam system allows for finding a tracking controller u that will steer the angle of beam, starting from any initial value, to track the desired zero function. In order to achieve the goal, we choose $h(X) = x_1 + x_2 + x_3 + x_4$. Let us arbitrarily choose $\alpha_1 = 0.007$ such that $A_c = -0.007$ is Hurwitz and $P = 71.43$. The AMIRA ball and beam system is a system of relative degree one. It can be verified that, with the choice $V(t,\eta) = \eta_3^2$, Assumption 1 and conditions of Theorem 1 are satisfied with $\varepsilon = 0.0025$, $M = \sqrt{3}$, $\omega_1 = \omega_2 = 1$, $k = \sqrt{\varepsilon}$, $K_x = \begin{bmatrix} 0 & 0 & -0.7 & -0.7 \end{bmatrix}^T$, $\omega_3 = 2$, $\alpha_x = 1$, $N = 1.325$, $N_2 = 1$ and

$$H = \begin{bmatrix} 2 & -1.296 \\ -1.296 & 3.8 \end{bmatrix}.$$

Then, according to (25), the desired tracking controller can be expressed as

$$u = \left(a + \frac{b_{11}}{b_{22}}\right)^{-1}\Big\{ -2.8x_1 - 3.8x_2 - 3.5x_3 - 4.5x_4 - \frac{z_{12} + z_{34} + z_{56}}{z_7} - \frac{a_{12} + a_{34}}{a_5}\Big\}. \quad (62)$$

Consequently, it follows from Theorem 1 that the tracking controller will steer the angle of the beam to track the desired trajectory $y_d(t) = \sin t$. The tracking errors are shown in Fig. 2.

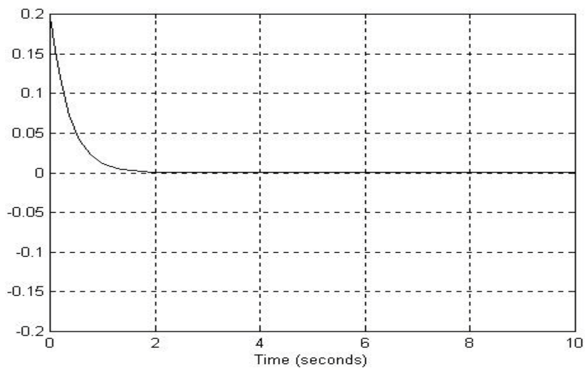

Figure 2. The tracking error for (61)

4. Examples comparative to existing approaches

Marino, Respondek and Van Der Schaft (1989) demonstrate that the almost disturbance decoupling problem cannot be solved for the following example:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} \tan^{-1}(x_2) \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \theta(t) \tag{63a}$$

$$y(t) = x_1(t) := h(X(t)) \tag{63b}$$

where u, y denote the input and output respectively, $\theta(t) := 0.5\sin t$ is the disturbance. On the contrary, this problem can be easily solved via the approach proposed in this paper. Following the same procedures as shown in the demonstrated example, the tracking problem with almost disturbance decoupling can be solved by the state feedback controller u defined as

$$u = \left(1 + x_2^2\right) \left[-\sin t - (0.03)^{-2} (x_1 - \sin t) - (0.03)^{-1} (\tan^{-1} x_2 - \cos t)\right] \tag{64}$$

The output trajectory of feedback-controlled system for (63) is depicted in Fig. 3 via MATLAB. It is also shown in Marino, Respondek and Van Der Schaft (1989) by the following example

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} x_2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} \theta_1(t) \\ x_2^3 \theta_2(t) \end{bmatrix}, \ \theta_1(t) = \theta_2(t) = 0.5\sin t \tag{65a}$$

$$y(t) = x_1(t) := h(X(t)) \tag{65b}$$

that the tracking and almost disturbance decoupling problem is not achieved. The feedback control algorithm proposed in this paper will solve it perfectly. Applying the same design procedures of Theorem 1 yields the desired tracking and almost disturbance decoupling controller as follows:

$$u = -\sin t - (0.03)^{-2} (x_1 - \sin t) + (0.03)^{-1} (x_2 - \cos t). \tag{66}$$

The output trajectory of the feedback-controlled system for (65) is depicted in Fig. 4 via MATLAB. From Figs. 3 and 4 it is obvious to see that the desired tracking and almost disturbance decoupling performance are achieved.

5. Conclusion

In this study we have constructed a feedback control algorithm which globally solves the tracking problem with almost disturbance decoupling for nonlinear systems with uncertainties. The discussion and practical application of feedback linearization of nonlinear control systems by parameterized co-ordinate transformation have been presented. Two comparative examples are proposed to show the significant contribution of this paper with respect to the existing

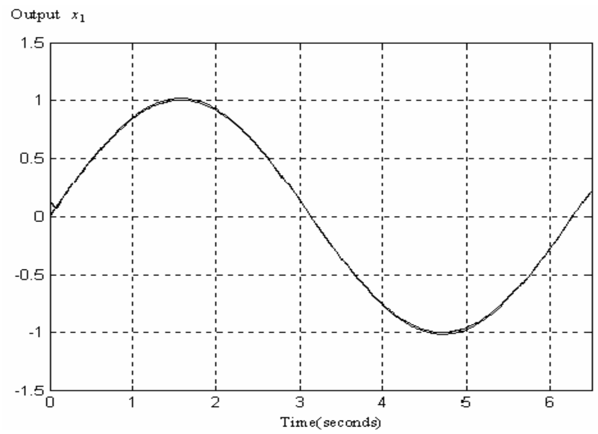

Figure 3. The output trajectory of feedback-controlled system for (63)

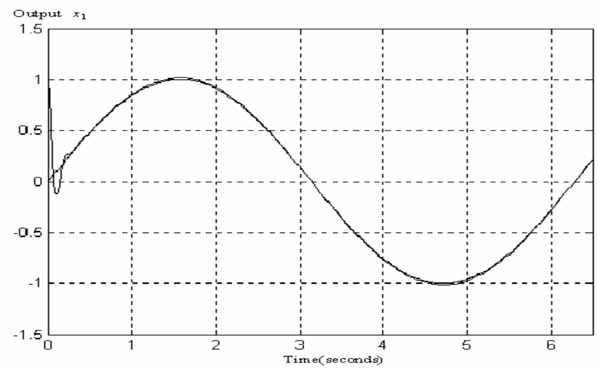

Figure 4. The output trajectory of feedback-controlled system for (65)

approaches. Moreover, a practical example of AMIRA ball and beam system demonstrates the applicability of the proposed differential geometry approach and the composite Lyapunov approach. Simulation results exploit the fact that the proposed methodology is successfully applied to feedback linearization problem and achieves the desired tracking and almost disturbance decoupling performances of the controlled system.

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