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Completion and clarification to the paper *Tests for relation type – equivalence or tolerance – in finite set of elements*

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Some parts of my paper *TESTS FOR RELATION TYPE...* need completion and clarification. The formulas (33) and (38), which express – respectively – the significance level and the probability of the second type error in the tests, include the factor $1-2\delta$ (notation from the cited paper). This factor is used for the purpose of taking into account the fact that the estimation result is errorless - with some probability.

From the formal point of view the factor $1-2\delta$ ought to be replaced with the probability $P(\hat{\chi}_1^R,\ldots,\hat{\chi}_{\hat{n}_R}^R \equiv \chi_1^{*R},\ldots,\chi_{n_R}^{*R})$ or $P(\hat{\chi}_1^T,\ldots,\hat{\chi}_{\hat{n}_T}^T \equiv \chi_1^{*T},\ldots,\chi_{n_T}^{*T})$. The values of both probabilities are difficult to obtain in the analytic way. They can be evaluated with the use of simulation approach or approximated. The experience of the author – based on simulation approach and some formal considerations – leads to the conclusion that the factor $1-2\delta$ can be used as a rough approximation. This presumption results from the following premises:

1. The evaluations of probabilities of the first type and second type errors in the tests (before correction with the factor $P(\hat{\chi}_1^R,\ldots,\hat{\chi}_{\hat{n}_R}^R \equiv \chi_1^{*R},\ldots,\chi_{n_R}^{*R})$ or $P(\hat{\chi}_1^T,\ldots,\hat{\chi}_{\hat{n}_T}^T \equiv \chi_1^{*T},\ldots,\chi_{n_T}^{*T})$), based on the right-hand side of the Chebyshev inequality (the expression $1/k^2$), are higher than the actual values of the probabilities. This results from the following facts: the evaluations are based on two-sided Chebyshev inequality and the upper bound (rough evaluation) of the variance $\mathrm{Var}(S)$. In fact, the tests exploit "one side" of the inequality (in other words: one tail of probability distribution) and the actual variance (evaluated with the use of maximal probability δ of error in comparisons) is typically significantly lower than the evaluation used.
2. The result of the test can be valid also in the case, when the estimation result $\hat{\chi}_1^R,\ldots,\hat{\chi}_{\hat{n}_R}^R$ or $\hat{\chi}_1^T,\ldots,\hat{\chi}_{\hat{n}_T}^T$ is not exactly equal to – respectively – $\chi_1^{*R},\ldots,\chi_{n_R}^{*R}$ or $\chi_1^{*T},\ldots,\chi_{n_T}^{*T}$, but the value of the statistic S correctly indicates the type of the relation in the set $\mathbf{X}$. It is not possible to determine the probability of such event, under assumptions made, but it increases the probability of correct decision in the test.

3. The result of the test can be also not incorrect, when the sign of the test statistics S is incorrect (e.g. $S < 0$, while the tolerance relation exists in the set $\mathbf{X}$). This happens when the value of the random variable S is close to zero and the probability δ is close to 0.5; typically the test does not indicate any decision in this case (the value of the test statistic is contained in the non-decision region).

The facts mentioned in the points 1 – 3 above and some simulation experience suggests that the formulas (33) and (38) for both probabilities of errors in the tests seem reliable. Another approach to the problem is to evaluate the probability of errorless estimation result with the use of bootstrap techniques, i.e. to perform the simulation experiment for the estimated form of both relations; such an approach needs additional computations. The author of the paper will present the results of simulation experiments in forthcoming papers. Let us note that similar conclusions result from a formal analysis of simple cases of the relations, i.e. m equal 3 or 4 and independent comparisons of each pair, with known probabilities.

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