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Tracking and disturbance rejection in a nonlinear control system with time delay

by

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Abstract: We consider the problem of designing a feedback control law in order to reject the unknown bounded disturbance and achieve tracking of reference inputs in control systems described by a class of nonlinear time-delay differential-algebraic equations. Based on the input-output feedback linearization technique and Lyapunov method for nonlinear state feedback synthesis, a robust globally asymptotical output tracking controller design methodology for nonlinear time-delay control systems with delays on the states and the input is developed. The underlying theoretical approaches are the differential geometry approach and the composite Lyapunov approach. For the view of practical application, the proposed control methodology has been successfully applied to the famous nonlinear automobile idle-speed control system problem.

Keywords: disturbance rejection, automobile idle-speed control system, differential geometry approach, composite Lyapunov approach.

1. Introduction

Recently, robust stabilization of systems with time delays has been treated as a challenging and interesting problem. As we know, in general, existence of time delays degrades control performance and sometimes makes the closed-loop stabilization difficult, especially for nonlinear systems. Appropriate mathematical descriptions incorporating time delays are the differential-difference equations, i.e., differential equations with deviating arguments. Several related reports have shown that differential-difference equations have been widely applied in theory of automatic control, the theory of self-oscillating systems, the study of problems connected with combustion in rocket motion, the problem of long-range planning in economics, a series of biological problems and in many other areas of science and technology (Driver, 1977; Górecki et al., 1989). In the past, there has been a number of interesting developments in stability criteria and controller designs for time-delay control systems, but were mostly restricted to linear cases; see, e.g., Gosiewski and Olbrot (1980), Kwon and Pearson (1980), Lewis and Anderson (1980), Mori et al. (1981), Mori et al. (1982), Thowsen (1982), Mori (1985), Wang et al. (1991), Wang (1992), Yanushevsky (1992), Trinh and Aldeen (1996) and Phoojaruenchanachai et al. (1998). In general, the global stability test of control systems with time delays is not as easy, even in the linear case, as without time delays. It involves some disgusting tasks as solving nonlinear matrix equations (Kwon and Pearson, 1980). It is clear that the investigation of nonlinear time-delay systems is worthwhile. In this paper, the globally asymptotical tracking problem of a general class of nonlinear time-delay control systems is investigated.

A typical approach for the analysis of nonlinear time-delay system is the local linearization approach. First an approximated linearization model on the operating point is obtained and then a linear control is constructed for the linearization model. In particular, some stability criteria and stabilization approaches have been developed for this linear time-delay differential equations (Brierley et al., 1982; Mori, 1985; Cheres et al., 1989; Mahmoud and Muthairi, 1994; and Cao and Sun, 1998). It is known that each local approximated model is valid only for a certain range of operating points and so these results can only guarantee local stability of the original nonlinear time-delay system.

In the past few years differential geometry approach (Banks, 1988; Nijmeijer and Van Der Schaft, 1990; Isidori, 1989) has proved to be an effective means of analysis and design of nonlinear control systems as it was in the past for the Laplace transform, complex variable theory and linear algebra in relation to linear control systems.

The problem of designing a feedback controller, which results in a system that rejects the unknown bounded disturbance and asymptotically tracks a desired reference output is a significant subject for the design of an efficient control system. In this paper, we present a systematic analysis and a simple design scheme that guarantees the uniform ultimate bounded stability of a feedback-controlled time-delay system and achieves globally asymptotical output tracking and disturbance-rejection performance for a class of nonlinear time-delay control systems. Finally, the developed control methodology is successfully applied to an automobile idle-speed control system. Throughout the paper, notation $\|\cdot\|$

denotes the usual Euclidean norm or the corresponding induced matrix norm.

2. Problem formulation and main result

In this paper, we consider the following single-input single-output (SISO) nonlinear time-delay control system with unknown bounded disturbance:

$$\begin{aligned}\dot{x}(t) &= f(x(t)) + g(x(t))u(t) + f_d(x(t-\theta)) + g_d(x(t))u(t-\theta) + \sum\nolimits_n \\ &:= f(x(t)) + g(x(t))u(t) + \sum (x(t), x(t-\theta), u(t-\theta)) + \sum\nolimits_n \qquad \text{(1a)}\end{aligned}$$

$$\sum (x(t), x(t-\theta), u(t-\theta)) := f_d(x(t-\theta)) + g_d(x(t))u(t-\theta) \qquad \text{(1b)}$$

$$y(t) = h(x(t)) \qquad \text{(1c)}$$

where $x(t) := [x_1(t)\ x_2(t)\ \cdots\ x_n(t)]^T \in \Re^n$ are the state variables, $u \in \Re^1$ is the input, $y \in \Re^1$ is the output, $\sum_n$ is the unknown bounded disturbance (noise) and $f_d := [f_{d1}\ f_{d2}\ \cdots\ f_{dn}] \in \Re^n$, $g_d := [g_{d1}\ g_{d2}\ \cdots\ g_{dn}] \in \Re^n$ are the time-delay terms. $f(x(t)) := [f_1\ f_2\ \ldots\ f_n]^T$ and $g(x(t)) := [g_1\ g_2\ \cdots\ g_n]^T$ are smooth vector fields on $\Re^n$, and $h(x(t)) := h(x_1(t), x_2(t), .., x_n(t)) \in \Re^1$ is a smooth function. The nominal system is then defined as follows:

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t-\theta) \qquad \text{(2a)}$$

$$y(t) = h(x(t)). \qquad \text{(2b)}$$

The nominal system possesses relative degree r (Henson and Seborg, 1991), i.e., there exists a positive integer $1 \leqslant r < \infty$ such that

$$L_g L_f^k h(x(t)) = 0, \qquad k < r-1, \qquad \text{(3)}$$

$$L_g L_f^{r-1} h(x(t)) \neq 0 \qquad \text{(4)}$$

for all $X \in \Re^n$ and $t \in [0, \infty)$, where the operator L is the Lie derivative (Isidori, 1989). The desired output trajectory $y_d(t)$ and the unknown disturbance $\sum_n$ are bounded and

$$\left\| \left[y_d(t) y_d^{(1)}(t) \cdots y_d^{(r)}(t) \right] \right\| \leqslant B_d \qquad \text{(5a)}$$

$$\left\| \sum\nolimits_n \right\| \leqslant B_n \qquad \text{(5b)}$$

where B_d and B_n are some positive constants.

Definition 1 *Consider the following dynamical system*

$$\dot{z}(t) = f(t, z(t)), \quad z \in \Re^p, \quad z(t_0) := z_0$$

where $z \in \Re^p$ is the state and $f(\cdot)$ is a smooth function. We use $z(t; t_0, z_0)$ to denote the solution of system with $z(t_0; t_0, z_0) = z_0$. A closed set S is called *a global final attractor for the trajectories* $z(\cdot):[t,\infty)\to\Re^p$, $z(t_0)=z_0$, *of the system, if for any initial state* z_0, *there exists a finite constant* $T(z_0,S)\in[0,\infty)$ *such that*

$$z(t;t_0,z_0)\in S,\ \forall t\geqslant t_0+T(z_0,S).$$

It has been shown (Isidori, 1989) that the mapping

$$\phi:\Re^n\to\Re^n \tag{6}$$

is described as

$$\phi_i(x(t)):=L_f^{i-1}h(x(t)),\ i=1,2,\cdots,r \tag{7a}$$

and

$$L_g\phi_k(x(t))=0,\ k=r+1,r+2,\cdots,n\,. \tag{7b}$$

For the sake of convenience, define

$$\phi_i(x(t)):=\xi_i(t),\ i=1,2,\cdots,r \tag{8}$$

$$\phi_k(x(t)):=\eta_k(t),\ k=r+1,r+2,\cdots,n \tag{9}$$

is a diffeomorphism onto image.

Define the trajectory error to be

$$e_i(t):=\xi_i(t)-y_d^{(i-1)}(t),\ i=1,2,\cdots,r \tag{10}$$

$$e:=[e_1(t)\,e_2(t)\,\cdots\,e_r(t)]^T\in\Re^r \tag{11}$$

and

$$\overline{e}_i(t):=\varepsilon^{i-1}e_i(t),\ i=1,2,\cdots,r \tag{12}$$

$$\overline{e}:=[\overline{e}_1(t)\,\overline{e}_2(t)\,\cdots\,\overline{e}_r(t)]^T\in\Re^r \tag{13}$$

where $\varepsilon\geq 1$ is some adjustable constant, the uncertainties

$$\Delta\tilde{A}:=\begin{bmatrix}\dfrac{\partial h}{\partial x}\sum(x(t),x(t-\theta),u(t-\theta))\\ \varepsilon\dfrac{\partial}{\partial x}L_f^1h\sum(x(t),x(t-\theta),u(t-\theta)\\ \vdots\\ \varepsilon^{r-1}\dfrac{\partial}{\partial x}L_f^{r-1}h\sum(x(t),x(t-\theta),u(t-\theta))\end{bmatrix} \tag{14a}$$

$$\Delta\psi:=\begin{bmatrix}\dfrac{\partial\phi_{r+1}}{\partial x}\sum(x(t),x(t-\theta),u(t-\theta))\\ \dfrac{\partial\phi_{r+2}}{\partial x}\sum(x(t),x(t-\theta),u(t-\theta))\\ \vdots\\ \dfrac{\partial\phi_n}{\partial x}\sum(x(t),x(t-\theta),u(t-\theta))\end{bmatrix} \tag{14b}$$

$$\Phi_\xi := \begin{bmatrix} \dfrac{\partial h}{\partial x} \\ \varepsilon \dfrac{\partial}{\partial x} L_f^1 h \\ \vdots \\ \varepsilon^{r-1} \dfrac{\partial}{\partial x} L_f^{r-1} h \end{bmatrix} \tag{14c}$$

$$\Phi_\eta := \begin{bmatrix} \dfrac{\partial \phi_{r+1}}{\partial x} \\ \dfrac{\partial \phi_{r+2}}{\partial x} \\ \vdots \\ \dfrac{\partial \phi_n}{\partial x} \end{bmatrix} \tag{14d}$$

and

$$\xi(t) := [\xi_1(t)\ \xi_2(t)\ \cdots\ \xi_r(t)]^T \in \Re^r \tag{15}$$

$$\eta(t) := [\eta_{r+1}(t)\ \eta_{r+2}(t)\ \cdots\ \eta_n(t)]^T \in \Re^{n-r} \tag{16}$$

$$q(\xi(t), \eta(t)) := [L_f \phi_{r+1}(t)\ L_f \phi_{r+1}(t)\ \cdots\ L_f \phi_n(t)]^T\ . \tag{17}$$

Define a phase-variable canonical matrix A_c to be

$$A_c := \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 \cdots & 0 \\ & \vdots & & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -\alpha_1 & -\alpha_2 & -\alpha_3 & \cdots & -\alpha_r \end{bmatrix} \tag{18}$$

where $\alpha_1, \alpha_2, \cdots, \alpha_r$ are any chosen parameters such that A_c is a Hurwitz matrix and let P be a symmetric positive definite matrix satisfying the Lyapunov equation

$$A_c^{-1} P + P A_c = -I\,. \tag{19}$$

Assumption 1 *For all $t \geqslant 0$, $\eta \in \Re^{n-r}$ and $\xi \in \Re^r$, there exists a positive constant L such that the following inequality holds:*

$$\| q(\xi, \eta) - q(0, \eta) \| \leqslant L \|\xi\|\ . \tag{20}$$

Assumption 2 *For the delayed term, there exist positive constants γ_1, γ_2, γ_1^*, γ_2^*, γ_3^*, γ_{11}, γ_{12}, l_1, l_2, l_1^*, l_2^*, l_3^*, l_{11}, l_{12} and l_3 such that*

$$\left\| 2P\Delta\tilde{A} \right\| \leqslant \gamma_1 \|\xi(t)\| + \gamma_2 \|\eta(t)\| + \gamma_{11} \|\xi(t-\theta)\| + \gamma_{22} \|\eta(t-\theta)\| \tag{21a}$$

$$\|\Phi_\xi\| \leqslant \gamma_1^* \|\xi(t)\| + \gamma_2^* \|\eta(t)\| + \gamma_3^* \tag{21b}$$

and

$$\|\Delta\psi\| \leqslant l_1 \|\xi(t)\| + l_2 \|\eta(t)\| + l_3 + l_{11} \|\xi(t-\theta)\| + l_{22} \|\eta(t-\theta)\| \quad \text{(22a)}$$
$$\|\Phi_\eta\| \leqslant l_1^* \|\xi(t)\| + l_2^* \|\eta(t)\| + l_3^* \,. \quad \text{(22b)}$$

Now we present our main result.

THEOREM 1 *Suppose that there exists a continuously differentiable function $V_0 : \Re^{n-r} \to \Re^+$ such that the following three inequalities hold for all $\eta \in \Re^{n-r}$:*

(a) $$k_1 \|\eta\|^2 \leqslant V_0(\eta) \leqslant k_2 \|\eta\|^2 , \quad k_1, k_2 > 0 \quad \text{(23a)}$$

(b) $$(\nabla_\eta V_0)^T q(0,\eta) \leqslant -k_3 \|\eta\|^2 , \quad k_3 > 0 \quad \text{(23b)}$$

(c) $$\|\nabla_\eta V_0\| \leqslant k_4 \|\eta\| , \quad k_4 > 0, \quad \text{(23c)}$$

then there exist constants ε, μ, t_{d1}, t_{d2} such that the output tracking error of system (1) satisfying Assumption (1)-(2), and subject to the tracking controller u defined by

$$u = \left[L_g L_f^{r-1} h(x(t))\right]^{-1} \Big\{ -L_f^r h(x) + y_d^{(r)} - \varepsilon^{-r} \alpha_1 \left[L_f^0 h(x) - y_d\right] - \varepsilon^{1-r} \alpha_2 \left[L_f^1 h(x) - y_d^{(1)}\right] - \cdots - \varepsilon^{-1} \alpha_r \left[L_f^{r-1} h(x) - y_d^{(r-1)}\right] \Big\} \quad \text{(24)}$$

is finally attracted in every closed sphere $B_{\overline{r}} := \left\{ \begin{bmatrix} \overline{e} \\ \eta \end{bmatrix} : \|\overline{e}\|^2 + \|\eta\|^2 \leqslant \overline{r}^2, \overline{r} > \underline{r} \right\}$, *where*

$$\underline{r} = \sqrt{\frac{N_1}{N_2}} \quad \text{(25a)}$$

$$N_1 := (\varepsilon\gamma_1 B_d + \varepsilon\gamma_{11} B_d + 2\|P\| B_n \gamma_1^* B_d \varepsilon + 2\|P\| B_n \gamma_3^* \varepsilon)^2 + \left[\frac{\mu k_4 l_3 + \mu k_4 L B_d + \mu k_4 l_1 B_d + \mu k_4 l_{11} B_d + k_4 B_n l_1^* B_d + k_4 B_n l_3^*}{\sqrt{\mu k_3}}\right]^2 > 0 \quad \text{(25b)}$$

$$N_{21} := \frac{1}{2} - \varepsilon\gamma_1 - t_{d1} - \varepsilon^2\gamma_{11}^2 - \varepsilon^2\gamma_{22}^2 - 2\|P\| B_n \gamma_1^* \varepsilon > 0, \quad t_{d1} > 0 \quad \text{(25c)}$$

$$N_{22} := \frac{3}{4}\mu k_3 - \mu k_4 l_2 - \mu t_{d2} - k_4 B_n l_2^* - (\mu k_4 L + \varepsilon\gamma_2 + \mu k_4 l_1 + 2\|P\| B_n \gamma_2^* + k_4 B_n l_1^*)^2 - \frac{1}{2} > 0, \quad t_{d2} > 0 \quad \text{(25d)}$$

$$N_{23} := t_{d1} - \frac{1}{4} - (\mu k_4 l_{11})^2 > 0 \quad \text{(25e)}$$

$$N_{24} := \mu t_{d2} - \frac{1}{4} - (\mu k_4 l_{22})^2 > 0 \quad \text{(25f)}$$

$$N_2 = \min\{N_{21}, N_{22}, N_{23}, N_{24} > 0\} \quad \text{(25g)}$$

whose radius $\overline{r}$ can be selected appropriately small by a suitable choice of relative parameters, i.e., the sphere $B_{\overline{r}}$ is a global final attractor for the output tracking error of system (1).

Proof. Applying the co-ordinate transformation (7) yields

$$\begin{aligned}\dot{\xi}_1(t) &= \frac{\partial \phi_1}{\partial x}\frac{dx}{dt} = \frac{\partial h(x(t))}{\partial x}\Big[f(x(t)) + g(x(t))\cdot u(t-\theta) + \textstyle\sum + \sum_n\Big]\\ &= L_f^1 h(x(t)) + L_g L_f^0 h(x(t))u(t) + \frac{\partial h(x)}{\partial x}\textstyle\sum + \frac{\partial h(x)}{\partial x}\sum_n\\ &= \xi_2(t) + \frac{\partial h(x)}{\partial x}\textstyle\sum + \frac{\partial h(x)}{\partial x}\sum_n \end{aligned} \tag{26}$$

$$\begin{aligned}\dot{\xi}_2(t) &= \frac{\partial \phi_2}{\partial x}\frac{dx}{dt} = \frac{\partial L_f^1 h(x(t))}{\partial x}\Big[f(x(t)) + g(x(t))\cdot u(t) + \textstyle\sum + \sum_n\Big]\\ &= L_f^2 h(x(t)) + L_g L_f^1 h(x(t))u(t) + \frac{\partial L_f^1 h(x(t))}{\partial x}\textstyle\sum + \frac{\partial L_f^1 h(x(t))}{\partial x}\sum_n\\ &= \xi_3(t) + \frac{\partial L_f^1 h(x(t))}{\partial x}\textstyle\sum + \frac{\partial L_f^1 h(x(t))}{\partial x}\sum_n \end{aligned} \tag{27}$$

$$\vdots$$

$$\begin{aligned}\dot{\xi}_{r-1}(t) &= \frac{\partial \phi_{r-1}}{\partial x}\frac{dx}{dt}\\ &= \frac{\partial L_f^{r-2} h(x(t))}{\partial x}\Big[f(x(t)) + g(x(t))\cdot u(t-\theta) + \textstyle\sum + \sum_n\Big]\\ &= L_f^{r-1} h(x(t)) + L_g L_f^{r-2} h(x(t))u(t) + \frac{\partial L_f^{r-2} h(x(t))}{\partial x}\textstyle\sum\\ &\quad + \frac{\partial L_f^{r-2} h(x(t))}{\partial x}\textstyle\sum_n\\ &= \xi_r(t) + \frac{\partial L_f^{r-2} h(x(t))}{\partial x}\textstyle\sum + \frac{\partial L_f^{r-2} h(x(t))}{\partial x}\sum_n \end{aligned} \tag{28}$$

$$\begin{aligned}\dot{\xi}_r(t) &= \frac{\partial \phi_r}{\partial x}\frac{dx}{dt} = \frac{\partial L_f^{r-1} h(x(t))}{\partial x}\Big[f(x(t)) + g(x(t))\cdot u(t) + \textstyle\sum + \sum_n\Big]\\ &= L_f^r h(x(t)) + L_g L_f^{r-1} h(x(t))u(t) + \frac{\partial L_f^{r-1} h(x(t))}{\partial x}\textstyle\sum\\ &\quad + \frac{\partial L_f^{r-1} h(x(t))}{\partial x}\textstyle\sum_n \end{aligned} \tag{29}$$

$$\begin{aligned}\dot{\eta}_k(t) &= \frac{\partial \phi_k(x)}{\partial x}\frac{dx}{dt} = \frac{\partial \phi_k(x)}{\partial x}\Big[f(x(t)) + g(x(t))\cdot u(t) + \textstyle\sum + \sum_n\Big]\\ &= \frac{\partial \phi_k(x)}{\partial x}f + \frac{\partial \phi_k(x)}{\partial x}gu(t) + \frac{\partial \phi_k(x)}{\partial x}\textstyle\sum + \frac{\partial \phi_k(x)}{\partial x}\sum_n\\ &= \frac{\partial \phi_k(x)}{\partial x}f + \frac{\partial \phi_k(x)}{\partial x}\textstyle\sum + \frac{\partial \phi_k(x)}{\partial x}\sum_n\\ &= L_f\phi_k(x) + \frac{\partial \phi_k(x)}{\partial x}\textstyle\sum + \frac{\partial \phi_k(x)}{\partial x}\sum_n\\ &= q_k + \frac{\partial \phi_k(x)}{\partial x}\textstyle\sum + \frac{\partial \phi_k(x)}{\partial x}\sum_n,\\ &\quad k = r+1, r+2, \cdots, n\,. \end{aligned} \tag{30}$$

Define

$$c(\xi(t),\eta(t)) := L_f^r h(x(t)) \tag{31}$$

$$d(\xi(t),\eta(t)) := L_g L_f^{r-1} h(x(t)) \tag{32}$$

$$q_k(\xi(t),\eta(t)) = L_f \phi_k(x), \quad k = r+1, r+2, \cdots, n\,. \tag{33}$$

Thus, the dynamic equations of system (1) in the new co-ordinates are as follows:

$$\dot{\xi}_i(t) = \xi_{i+1}(t) + \frac{\partial}{\partial x} L_f^{i-1} h(x(t)) \textstyle\sum + \frac{\partial}{\partial x} L_f^{i-1} h(x(t)) \textstyle\sum_n, \quad i = 1, 2, \cdots, r-1 \tag{34}$$

$$\dot{\xi}_r(t) = c(\xi(t),\eta(t)) + d(\xi(t),\eta(t))u(t) + \frac{\partial}{\partial x} L_f^{r-1} h(x(t)) \textstyle\sum + \frac{\partial}{\partial x} L_f^{r-1} h(x(t)) \textstyle\sum_n \tag{35}$$

$$\dot{\eta}_k(t) = q_k(\xi(t),\eta(t)) + \frac{\partial}{\partial x}\phi_k \textstyle\sum + \frac{\partial}{\partial x}\phi_k \textstyle\sum_n, \quad k = r+1, \cdots, n\,. \tag{36}$$

$$y(t) = \xi_1(t) \tag{37}$$

Define

$$v := y_d^{(r)} - \varepsilon^{-r}\alpha_1 \left[L_f^0 h(x) - y_d\right] - \varepsilon^{1-r}\alpha_2 \left[L_f^1 h(x) - y_d^{(1)}\right] - \cdots - \varepsilon^{-1}\alpha_r \left[L_f^{r-1} h(x) - y_d^{(r-1)}\right]\,. \tag{38}$$

According to equations (6), (10), (31) and (32), the tracking controller can be rewritten as

$$u = d^{-1}\left[-c + v\right]\,. \tag{39}$$

From equations (35) and (39), the dynamic equations of system (1) can be derived as follows:

$$\begin{bmatrix} \dot{\xi}_1(t) \\ \dot{\xi}_2(t) \\ \vdots \\ \dot{\xi}_{r-1}(t) \\ \dot{\xi}_r(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0\cdots & 0 \\ & \vdots & & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \\ \vdots \\ \xi_{r-1}(t) \\ \xi_r(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} v + \begin{bmatrix} \frac{\partial}{\partial x} h \sum \\ \vdots \\ \vdots \\ \frac{\partial}{\partial x} L_f^{r-2} h \sum \\ \frac{\partial}{\partial x} L_f^{r-1} h \sum \end{bmatrix} + \begin{bmatrix} \frac{\partial}{\partial x} h \\ \vdots \\ \vdots \\ \frac{\partial}{\partial x} L_f^{r-2} h \\ \frac{\partial}{\partial x} L_f^{r-1} h \end{bmatrix} \textstyle\sum_n \tag{40}$$

$$\begin{bmatrix} \dot{\eta}_{r+1}(t) \\ \dot{\eta}_{r+2}(t) \\ \vdots \\ \dot{\eta}_{n-1}(t) \\ \dot{\eta}_{n}(t) \end{bmatrix} = \begin{bmatrix} q_{r+1}(t) \\ q_{r+2}(t) \\ \vdots \\ q_{n-1}(t) \\ q_{n}(t) \end{bmatrix} + \begin{bmatrix} \frac{\partial}{\partial x}\phi_{r+1}\sum \\ \vdots \\ \vdots \\ \frac{\partial}{\partial x}\phi_{n-1}\sum \\ \frac{\partial}{\partial x}\phi_{n}\sum \end{bmatrix} + \begin{bmatrix} \frac{\partial}{\partial x}\phi_{r+1} \\ \vdots \\ \vdots \\ \frac{\partial}{\partial x}\phi_{n-1} \\ \frac{\partial}{\partial x}\phi_{n} \end{bmatrix} \sum\nolimits_n \tag{41}$$

$$y = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \end{bmatrix}_{1\times r} \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \\ \vdots \\ \xi_{r-1}(t) \\ \xi_r(t) \end{bmatrix}_{r\times 1} = \xi_1(t)\,. \tag{42}$$

Combining equations (10), (12), (19) and (38), it can be easily verified that equations (2)-(42) can be transformed into the following form with uncertainties:

$$\dot{\eta}(t) = q(\xi(t),\eta(t)) + \Delta\Psi + \Phi_\eta \sum\nolimits_n = q_1(t,\eta(t),\overline{e}(t)) + \Delta\Psi + \Phi_\eta \sum\nolimits_n \tag{43a}$$

$$\varepsilon\,\dot{\overline{e}}(t) = A_c\overline{e} + \varepsilon\Delta\tilde{A} \tag{43b}$$

$$y(t) = \xi_1(t)\,. \tag{44}$$

Let

$$V_1\left(\overline{e}\right) = \varepsilon\overline{e}^T P\overline{e} + t_{d1}\int_{t-\theta}^{t} \overline{e}^T(\tau)\overline{e}(\tau)d\tau \tag{45a}$$

$$V_2\left(\overline{e}\right) = V_0(\eta) + t_{d2}\int_{t-\theta}^{t} \eta^T(\tau)\eta(\tau)d\tau \tag{45b}$$

where t_{d1}, t_{d2} are positive constants and P is the positive definite solution of the following Riccati equation:

$$A_c^T P + PA_c = -I\,. \tag{46}$$

With these Lyapunov functions $V_1(\overline{e})$ and $V_2(\eta)$ at hand, we consider $V(\overline{e},\eta)$ defined by a weighted sum of $V_1(\overline{e})$ and $V_2(\eta)$,

$$V(\overline{e},\eta) := V_1(\overline{e}) + \mu V_0(\eta), \tag{47}$$

as a composite Lyapunov function of the system (43) (Khorasani and Kokotovic, 1986; Marino and Kokotovic, 1988), where μ is a strictly positive constant to be adjusted. In view of (24)-(25), (45) and (46), the derivative of $V(\overline{e},\eta)$ along the trajectories of (43) is given by

$$
\begin{aligned}
\dot{V} &= \dot{V}_1 + \mu\dot{V}_2 = \varepsilon\left(\dot{\overline{e}}^T P\overline{e} + \overline{e}^T P\dot{\overline{e}}\right) + t_{d1}\overline{e}^T\overline{e} - t_{d1}\overline{e}^T(t-\theta)\overline{e}(t-\theta) \\
&\quad + \mu\left[\dot{V}_0 + t_{d2}\eta^T\eta - t_{d2}\eta^T(t-\theta)\eta(t-\theta)\right] \\
&= \overline{e}^{-T}(A_c^T P + PA_c)\overline{e} + \varepsilon\Delta\tilde{A}^T P\overline{e} + \varepsilon\overline{e}^T P\Delta\tilde{A} + t_{d1}\|\overline{e}\|^2 - t_{d1}\|\overline{e}(t-\theta)\| \\
&\quad + \varepsilon\left(\Phi_\xi \textstyle\sum_n\right)^T P\overline{e} + \varepsilon\overline{e}^T P\left(\Phi_\xi \textstyle\sum_n\right) \\
&\quad + \mu\left\{\frac{\partial V_0}{\partial\eta}\left[q(\xi,\eta) + \Delta\Psi + \Phi_\eta \textstyle\sum_n\right] + t_{d2}\|\eta\|^2 - t_{d2}\|\eta(t-\theta)\|^2\right\} \\
&= (-1 + t_{d1})\|\overline{e}\|^2 + 2\varepsilon\overline{e}^T P\Delta\tilde{A} - t_{d1}\|\overline{e}(t-\theta)\|^2 + 2\varepsilon\,\overline{e}^T P\left(\Phi_\xi \textstyle\sum_n\right) \\
&\quad + \mu\left\{\begin{array}{l}\dfrac{\partial V_0}{\partial\eta}\left[q(\xi,\eta) - q(0,\eta)\right] + \dfrac{\partial V_0}{\partial\eta}q(0,\eta) + \dfrac{\partial V_0}{\partial\eta}\Delta\Psi + \dfrac{\partial V_0}{\partial\eta}\left(\Phi_\eta \textstyle\sum_n\right) \\ + t_{d2}\|\eta\|^2 - t_{d2}\|\eta(t-\theta)\|^2\end{array}\right\} \\
&\leqslant (-1 - t_{d1})\|\overline{e}\|^2 + \varepsilon\|\overline{e}\|\left(\gamma_1\|\xi\| + \gamma_2\|\eta\| + \gamma_{11}\|\xi(t-\theta)\|\right. \\
&\quad \left. + \gamma_2\|\eta(t-\theta)\|\right) - t_{d1}\|\overline{e}(t-\theta)\|^2 + 2\varepsilon\|\overline{e}\|\|P\|\|\Phi_\xi\|\|\textstyle\sum_n\| \\
&\quad + \mu\left\{\begin{array}{l} Lk_4\|\eta\|\|\xi\| - k_3\|\eta\|^2 + k_4\|\eta\|\left(l_1\|\xi\| + l_2\|\eta\| + l_3\right. \\ \left. + l_{11}\|\xi(t-\theta)\| + l_{22}\|\eta(t-\theta)\|\right) \\ + t_{d2}\|\eta\|^2 - t_{d2}\|\eta(t-\theta)\|^2 + k_4\|\eta\|\|\Phi_\eta\|\|\textstyle\sum_n\| \end{array}\right\} \\
&\leqslant (-1 + t_{d1} + \varepsilon\gamma_1 + 2\|P\|B_n\gamma_1^*\varepsilon)\|\overline{e}\|^2 + \|\overline{e}\|\left(\varepsilon\gamma_1 B_d + \varepsilon\gamma_{11}B_d\right. \\
&\quad \left. + 2\|P\|B_n\gamma_1^*)B_d\varepsilon + 2\|P\|B_n\gamma_3^*\varepsilon)\right) \\
&\quad + \|\eta\|^2\left(\mu k_4 l_2 - \mu k_3 + \mu t_{d2} + k_4 B_n l_2^*\right) \\
&\quad + \|\overline{e}\|\|\eta\|\left(\mu k_4 L + \varepsilon\gamma_2 + \mu k_4 l_1 + 2\|P\|B_n\gamma_2^* + k_4 B_n l_1^*\right) \\
&\quad + \|\eta\|\left(\mu k_4 l_3 + \mu k_4 L B_d + \mu k_4 l_1 B_d + \mu k_4 l_{11} B_d + k_4 B_n l_1^* B_d + k_4 B_n l_3^*\right) \\
&\quad - t_{d1}\|\overline{e}(t-\theta)\|^2 - \mu\, t_{d2}\|\eta(t-\theta)\|^2 + \varepsilon\gamma_{11}\|\overline{e}\|\|\overline{e}(t-\theta)\| \\
&\quad + \varepsilon\gamma_{22}\|\overline{e}\|\|\eta(t-\theta)\| + \mu k_4 l_{22}\|\eta\|\|\eta(t-\theta)\| + \mu k_4 l_{11}\|\eta\|\|\overline{e}(t-\theta)\| \\
&\leqslant (-1 + t_{d1} + \varepsilon\gamma_1 + 2\|P\|B_n\gamma_1^*\varepsilon)\|\overline{e}\|^2 + \frac{1}{4}\|\overline{e}\|^2 \\
&\quad + (\varepsilon\gamma_1 B_d + \varepsilon\gamma_{11}B_d + 2\|P\|B_n\gamma_1^* B_d\varepsilon + 2\|P\|B_n\gamma_3^*\varepsilon)^2 \\
&\quad + \|\eta\|^2(\mu k_4 l_2 - \mu k_3 + \mu t_{d2} + k_4 B_n l_2^*) \\
&\quad + \|\eta\|^2(\mu k_4 L + \varepsilon\gamma_2 + \mu k_4 l_1 + 2\|P\|B_n\gamma_2^* + k_4 B_n l_1^*)^2 \\
&\quad + \frac{1}{4}\|\overline{e}\|^2 + \frac{1}{4}\|\eta\|^2\mu k_3 \\
&\quad + \left\{\frac{\mu k_4 l_3 + \mu k_4 L B_d + \mu k_4 l_1 B_d + \mu k_4 l_{11} B_d + k_4 B_n l_1^* B_d + k_4 B_n l_3^*}{\sqrt{\mu k_3}}\right\}^2
\end{aligned}
$$

$$
\begin{aligned}
&- t_{d1} \left\|\overline{e}(t-\theta)\right\|^2 - \mu t_{d2} \left\|\eta(t-\theta)\right\|^2 + \frac{1}{4}\left\|\overline{e}(t-\theta)\right\|^2 + \left\|\overline{e}\right\|^2 (\varepsilon\gamma_{11})^2 \\
&- t_{d1} \left\|\overline{e}(t-\theta)\right\|^2 - \mu t_{d2} \left\|\eta(t-\theta)\right\|^2 + \frac{1}{4}\left\|\overline{e}(t-\theta)\right\|^2 + \left\|\overline{e}\right\|^2 (\varepsilon\gamma_{11})^2 \\
&+ \frac{1}{4}\left\|\eta(t-\theta)\right\|^2 + \left\|\overline{e}\right\|^2 (\varepsilon\gamma_{22})^2 + \frac{1}{4}\left\|\eta\right\|^2 \\
&+ \left\|\eta(t-\theta)\right\|^2 (\mu k_4 l_{22})^2 + \frac{1}{4}\left\|\eta\right\|^2 + \left\|\overline{e}(t-\theta)\right\|^2 (\mu k_4 l_{11})^2 .
\end{aligned}
$$

Therefore, we get

$$
\begin{aligned}
\dot{V} \leqslant &-\Big(\frac{1}{2} - \varepsilon\gamma_1 - t_{d1} - \varepsilon^2\gamma_{11}^2 - \varepsilon^2\gamma_{22}^2 - 2\left\|P\right\| B_n\gamma_1^*\varepsilon\Big)\left\|\overline{e}\right\|^2 \\
&- \Big[\frac{3}{4}\mu k_3 - \mu k_4 l_2 - \mu t_{d2} - k_4 B_n l_2^* - \big(\mu k_4 L + \varepsilon\gamma_2 + \mu k_4 l_1 \\
&+ 2\left\|P\right\| B_n\gamma_2^* + k_4 B_n l_1^*\big)^2 - \frac{1}{2}\Big]\left\|\eta\right\|^2 \\
&- \Big[t_{d1} - \frac{1}{4} - \big(\mu k_4 l_{11}\big)^2\Big]\left\|\overline{e}(t-\theta)\right\|^2 \\
&- \Big[\mu t_{d2} - \frac{1}{4} - (\mu k_4 l_{22})^2\Big]\left\|\eta(t-\theta)\right\|^2 \\
&+ \left\{ \begin{array}{l} (\varepsilon\gamma_1 B_d + \varepsilon\gamma_{11} B_d + 2\left\|P\right\| B_n\gamma_1^* B_d\varepsilon + 2\left\|P\right\| B_n\gamma_3^*\varepsilon)^2 \\ + \left[\dfrac{\mu k_4 l_3 + \mu k_4 L B_d + \mu k_4 l_1 B_d + \mu k_4 l_{11} B_d + k_4 B_n l_1^* B_d + k_4 B_n l_3^*}{\sqrt{\mu k_3}}\right]^2 \end{array} \right\} \\
&\leqslant -N_2\big(\left\|\overline{e}\right\|^2 + \left\|\eta\right\|^2 + \left\|\eta(t-\theta)\right\|^2\big) + N_1 := -N_2\big(\left\|y_1\right\|^2 + \left\|y_1(t-\theta)\right\|^2\big) + N_1
\end{aligned}
$$

where $\left\|y_1\right\|^2 := \left\|\overline{e}\right\|^2 + \left\|\eta\right\|^2$.

Define

$$\underline{r} := \sqrt{\frac{N_1}{N_2}}.$$

For $\left\|y\right\| > \underline{r}$, we have $\dot{V} < 0$. Hence any sphere defined by

$$B_{\overline{r}} := \left\{ \begin{bmatrix} \overline{e} \\ \eta \end{bmatrix} : \left\|\overline{e}\right\|^2 + \left\|\eta\right\|^2 \leqslant \overline{r}^2, \overline{r} > \underline{r} \right\}$$

is a global final attractor for the tracking error system of the nonlinear control systems (1). ■

According to the previous theorem and discussions, an efficient algorithm for deriving the tracking and disturbance rejecting control is proposed as follows:

(Step 1): Calculate the relative degree r of the given control system.

(Step 2): Choose the diffeomorphism ϕ such that the Assumptions 1 and 2 are satisfied.

(Step 3): Adjust some parameters $\alpha_1, \alpha_2, \cdots, \alpha_r$ such that the matrix A_c is

Hurwitz and calculate the positive definite matrices P of the Lyapunov equations (19) by some software package, such as Matlab.

(Step 4): Based on the famous Lyapunov approach, design a Lyapunov function to solve the conditions (23a)-(23c).

(Step 5): Appropriately tune the parameters ε, μ, t_{d1}, t_{d2} such that (25a)-(25g) are satisfied and go to the next step. Otherwise, go to Step 3 and repeat the overall designing procedures.

(Step 6): According to the equation (24), the desired controller u can be constructed such that the tracking and disturbance rejecting performances are guaranteed. That is, the system dynamics enters a neighborhood of zero state and remains within it thereafter.

3. Practical application–nonlinear automobile idle-speed control system with time-delay

Consider the following automobile engine for the idle-speed control system (ISCS) with the disturbance torque shown in Fig. 1. The input of the ISCS is the throttle position $\alpha(t)$ that controls the rate of airflow into the manifold.

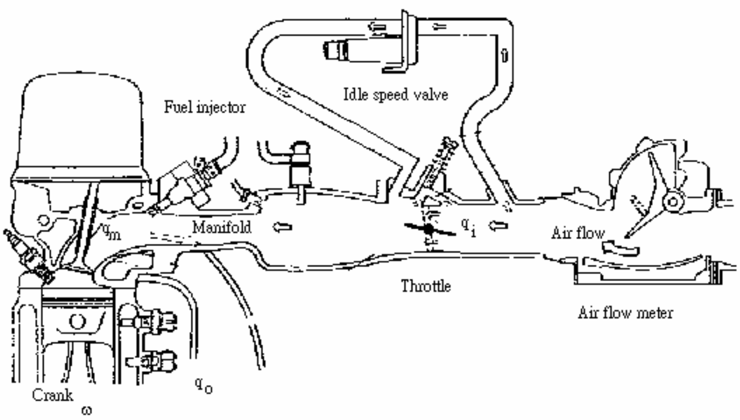

Figure 1. An automobile engine representation for the idle-speed control system.

The relative variables are described as follows:

T_d : disturbance torque due to application of auto accessories = constant

$q_i(t)$: amount of airflow across throttle into manifold

$q_o(t)$: amount of air leaving intake manifold through intake valves

$q_m(t)$: average air mass in manifold

$\tau_D(t)$: time delay in engine

J : inertia of engine

$\omega(t)$: engine speed

$T(t)$: engine torque

B : viscous-friction coefficient of engine.

Some assumptions and dynamic relations between engine parameters are stated as follows:

(A1) $\dfrac{dq_i(t)}{dt} = K_1\alpha(t)$

(A2) $\dfrac{dq_o(t)}{dt} = K_2 q_m(t) + K_3\omega(t)$

(A3) $T(t) = K_4 q_m(t - \tau_D)$

(A4) $\dfrac{dq_m(t)}{dt} = \dfrac{dq_i(t)}{dt} - \dfrac{dq_o(t)}{dt}$

(A5) $T(t) = J\dfrac{d\omega(t)}{dt} + B\omega(t) + T_d$

where K_1, K_2, K_3 and K_4 are constants. It is an easy routine to arrive at the following dynamic equation:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} \begin{bmatrix} -\dfrac{B}{J}x_1 \\ -K_3x_1 - K_2x_2 \end{bmatrix} + \begin{bmatrix} \dfrac{K_4}{J}x_2(t-\tau_D) \\ 0 \end{bmatrix} + \begin{bmatrix} -\dfrac{T_d}{J} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ K_1 \end{bmatrix} u$$

$$y(t) = x_2(t) := h(x(t))$$

where $\omega(t) := x_1(t)$, $q_m(t) := x_2(t)$ and $u(t) := \alpha(t)$. The objective of this problem is to reject the unknown bounded disturbance and make the rate of airflow across throttle into manifold equal to the rate of air leaving intake manifold through intake valves. For the sake of simplicity, the following dynamic equation is adopted for simulation:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -x_1 - 1 \\ -x_1 - x_2 \end{bmatrix} + \begin{bmatrix} x_2(t)(t-0.1) \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \tag{48a}$$

$$y(t) = h(x(t)) := x_2(t)\,. \tag{48b}$$

For the automobile idle-speed control system with the disturbance torque shown in Fig. 1, our goal is to find a tracking controller u that will reject the unknown bounded disturbance and steer the output tracking error $\overline{e} = y - y_d = x_2 - 0$ of the closed-loop system, starting from any initial value, to be asymptotically attracted in a sphere $B_{\overline{r}}$ whose radius can be selected appropriately small. Let us arbitrarily choose $\alpha_1 = 1$ such that $A_c = -1$ is a Hurwitz matrix and $P = 0.5$. The original system (48) is a system of relative degree one. From (24), we obtain the desired tracking controller

$$u = x_1 + x_2 - \varepsilon^{-1}x_2\,. \tag{49}$$

It can be verified that with the choice of $\varepsilon = 1$, $\mu = 1$ and $V_0(\eta) = \eta_2^2$, the relative conditions of Theorem 1 are satisfied with $B_d = 0$, $L = 0$, $k_1 = k_2 = 1$,

$k_3 = k_4 = 2$, $\gamma_1 = \gamma_2 = 0$, $\gamma_{11} = \gamma_{22} = 0$, $l_1 = l_2 = l_3 = 0$, $l_{11} = 0.01$, $l_{22} = 0$, $\gamma_1^* = \gamma_2^* = 0$, $\gamma_3^* = 1$, $l_1^* = l_2^* = 0$, $l_3^* = 1$, $t_{d1} = 0.35$, $t_{d2} = 1$, $N_{21} = 0.15$, $N_{22} = 0.5$, $N_{23} = 0.1$, $N_{24} = 1.75$ and $N_2 = 0.0875$. Hence the tracking controller (49) will steer the output tracking error $\overline{e} = y - y_d = x_2 - 0$ of the closed-loop system, starting from any initial value, to be asymptotically attracted in a sphere $B_{\overline{r}}$ in view of Theorem 1. Some output trajectories of the uncontrolled and feedback-controlled system are depicted in Fig. 2 and Fig. 3, respectively.

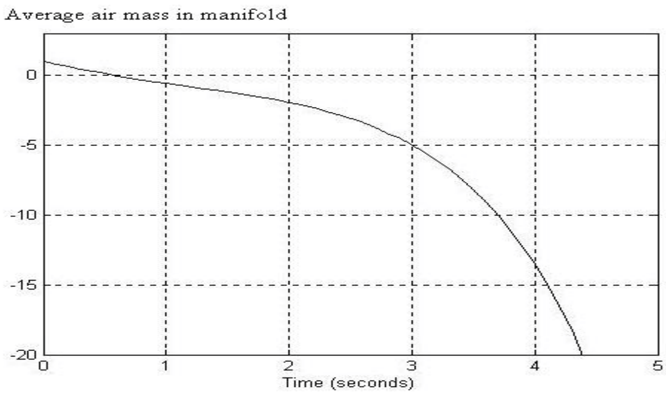

Figure 2. Output trajectory of the uncontrolled time-delay system.

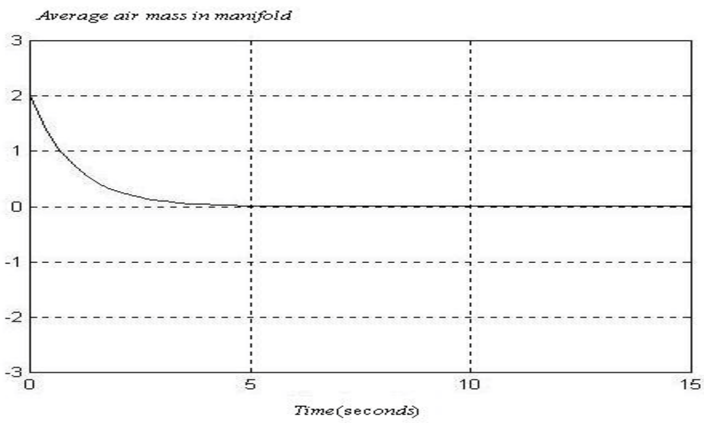

Figure 3. Output trajectory of the feedback-controlled time-delay system.

4. Conclusion

A nonlinear state feedback control law is designed which ensures globally that the tracking error in the closed-loop system lies within any adjustable bound if the reference inputs are bounded. The discussion and practical application of input-output feedback linearization of nonlinear time-delay control systems by parameterized co-ordinate transformation have been presented. A practical example of an automobile idle-speed control system with time delay demonstrated the applicability of the proposed differential geometry approach and the composite Lyapunov approach. Simulation result exploited the fact that the proposed methodology is successfully applied to input-output linearization problem and achieves an interesting disturbance-rejection performance of the controlled system for globally asymptotical output tracking.

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