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Application of a quantitative multiple criteria decision making (MCDM-1) approach to the analysis of investments in construction

by

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Abstract: The paper considers major principles of methods based on quantitative measurements. The problem of construction investments is of paramount importance because they determine the potential of construction and the structure of expenses. The calculation of investments is an expensive tool of enterprise management used in planning the investments. It can be perceived as the harmonization and evaluation of the models for investment decision-making. Methods based on a single criterion can hardly be used in solving the problems associated with sophisticated technological or marketing systems. It is a multiple attribute decision method, taking into consideration major efficiency criteria, that enables the effective methods of solving complicated problems. When the quantitative criteria are precisely defined, the developed quantitative multiple criteria decision making (MCDM-1) method can be successfully applied. The present paper is devoted to the analysis of the algorithm and methods of application of the suggested technique.

Keywords: decision-making, quantitative methods, weights of criteria, multiple criteria evaluation, estimation of efficiency of investments.

1. Introduction

The problem of construction investment planning is of paramount importance because investments determine the potential of construction and the structure of expenses. Successful performance of a company is also closely associated with the investment policy, since the latter embraces the expenses required as well as considerable financial resources. Efficient planning and management of investments have become not only an important but also a complicated problem in the dynamically changing environment.

The calculation of investments is an expensive tool of enterprise management in planning of investments. It can be perceived as the harmonization and evaluation of the models for investment decision-making. Therefore, in calculating investments, in addition to the methods and models based on the data obtained in the study of income and expenses, the multiple attribute decision methods assessing their profitability are applied.

Decision-making is associated with ranking problems aimed to obtain a particular preference order of solutions (Triantaphyllou, 2000; Wierzcbicki, 2001). Such problems arise when forming the portfolio of securities (Ghasemzadeh, Archer, 2000), assessing the performance of construction enterprises (Ginevičius, Podviezko, 2001) or purchasing goods (Jain Subhash, 1990), etc. Very often it is hardly possible to describe the essential features of an object using one criterion. To achieve this goal, a number of various attributes are needed. The problems of determining the most preferable object by the efficiency criteria are quite frequent. They include selection of production (Chetyrkin, 2001) and construction investment projects (Vasilyev, Panibratov, 1997; Larichev, Kochin, Ustinovichius, 2003; Ustinovichius, 2004) or consumer goods. Obviously, when the ranking for preference order is complete, the first object in a list is most preferable.

One of the possible approaches is based on classification made according to the type of information received from the decision maker (Larichev, 2002). It includes:

1. Methods based on quantitative measurements. This group consists of widely known methods from multicriteria utility theory (Keeney, Raiffa, 1976; Zavadskas et al., 1995) and some new methods (Hwang, Yoon 1981; Triantaphyllou 2000; Zavadskas et al., 2004; Brauers, 2004; Ustinovichius 2004).
2. Methods based on initial qualitative assessments, the results of which later take a quantitative form. This group consists of analytic hierarchy method (Saaty, 1994), as well as the methods based on fuzzy sets (Zadeh 1965; Peldschus and Zavadskas, 2005).
3. Methods based on quantitative measurements but using a few criteria to compare the alternatives (comparison preference method). This group consists of preference comparison methods (Roy, 1996; Slowiński, Greco, Matarazzo, 2002; Ustinovichius, Stasiulionis, 2001).
4. Methods based on qualitative data not using a transformation to quantitative variables. This group comprises verbal decision analysis (VDA) (Larichev et al., 1995; Larichev et al., 2003; Ustinovichius, Kochin, 2004).

An indirect assumption is made in the methods of the first group that a particular parameter is measured by the DM and the value obtained is the only basis for establishing his/her preference. However, recent psychological research and practical application have not proved it (Larichev et al., 1995). The determination of weights of quantitative attributes often causes errors (Borcherding, Schmeer, Weber, 1995). Slight differences in weights of attributes can dramatically change the order of the alternative preference (von Winterfeldt, Fischer, 1975). It is not easy to precisely evaluate possible quantitative measurement errors. For example, the equivalent quantitative assessment of qualitative attributes can yield a very wide spread in values. In addition, when the parameters are changing, low sensitivity will be observed in the case of weak alternative utility dependence on the changes of the parameters (von Winterfeldt, Fischer, 1975). However, it cannot be stated that such dependence is found in all cases.

Therefore, when assessing the second group of methods, the same conclusions can be drawn as in the case of approaches based on quantitative measurements (Larichev, 2002; Dyer, 1990):

1) these methods ensure the consistency of comparisons;
2) they are sensitive to measurement errors made by decision makers;
3) in the case of parametric variation, they can form cyclic relations in a set of alternatives.

All methods of the third group use pairwise comparisons based on specified indices of concordance or discordance (Roy, 1996). In fact, the hypothesis of the particular alternatives being more preferable than others is checked. A concordance index is found by analysing the coefficients of the attribute weights received from the decision maker. A discordance index is obtained by evaluating the criteria values of the alternatives. Concordance and discordance indices are not related, and are used separately. Each stage of analysis has its lowest and highest concordance indices, according to which the preference of one alternative with respect to another is determined. Some cycles may occur in a set of alternatives when using comparative preference (outranking) methods in the presence of various indices of concordance and discordance. There can be many alternatives in a cycle. However, the methods of the group mentioned above are not so sensitive to the DM errors of measurement. First, the preference of one alternative over another may be stated only if the assessment values change within a wide range. Second, the values can vary for the same concordance indices if the total sum of values remains unchanged when the concordance index is being calculated (Larichev, 2002).

To avoid the above difficulties, it is possible to use VDA methods. In the case of precisely defined quantitative attributes, the first version of Multiple Criteria Decision Making (MCDM-1) (Ustinovichius, 2004) can be suggested. The present paper is devoted to the analysis of the algorithm and methods of application of the suggested technique.

2. Application of a quantitative approach MCDM-1

Methods based on a single criterion can hardly be used in solving the problems associated with sophisticated technological or marketing systems. It is the multiple attribute decision method, taking into account major efficiency criteria, that enables the effective approach to solving complicated problems.

A description of numerous multiattribute decision methods can be found in the literature. However, application of one of these methods is not sufficient (Zavadskas, Ustinovichius, Podvezko, 2005). Various methods can be combined, grouped or applied in turns. A set of multiattribute decision methods (multicriteria approach) suggested is given in Fig. 1.

First, the applicable alternatives of technological and economic projects are developed. Then the criteria of efficiency (attributes) to be used in analysing the alternatives are established. Based on the judgements of experts, the weights of the criteria are determined, and the concordance of expert judgements is checked. If the judgements are in concordance, the efficiency values obtained can be relied upon. To avoid accidental errors, three methods of determining the utility function of an alternative should be used. They include methods of similarity to ideal solution (TOPSIS) (Hwang, Yoon, 1981), simple additive weighting (SAW) (MacCrimmon, 1968) and the multiple criteria complex proportional assesment of alternatives (COPRAS) (Zavadskas et al., 2004; Kaklauskas, Zavadskas, Raslanas, 2005).

Pairwise comparison for determining the weights of the efficiency criteria. Decision-making system may help to determine the weights of the attributes developed. This may be achieved by pairwise comparison expertise method AHP, suggested by T. Saaty (Saaty, 1977).

The method AHP can be used to assess the consistency (concordance) of estimates of individual experts by calculating the consistency and concordance index of comparisons. Pairwise evaluation may be considered sufficiently reliable only if the judgements of experts are in concordance. Therefore, when statistically processing the data obtained from the experts, the index of concordance should be determined and the causes of discordance identified. AHP does not provide for expert judgements concordance analysis, therefore, its application is justified only if a group of experts has reached a compromise solution and provided a joint matrix of pairwise comparison of alternatives. The use of the DELPHI method can contribute to harmonizing the estimates. Otherwise, the consistency of expert estimates may be determined by the concordance coefficient W suggested by W. Kendall and the respective values of the statistic χ^2 (Kendall, 1970). A matrix of the criterion ranks provided by each expert is used as a basis for calculating the concordance coefficient. Therefore, when a compromise group solution cannot be reached, the criteria weights can be determined by AHP and ranked. Then, the values of W and χ^2 should be calculated and the degree of consistency of group estimates determined (Podvezko, 2005; Zavadskas, Vilutiene, 2006). When individual and group estimates are consis-

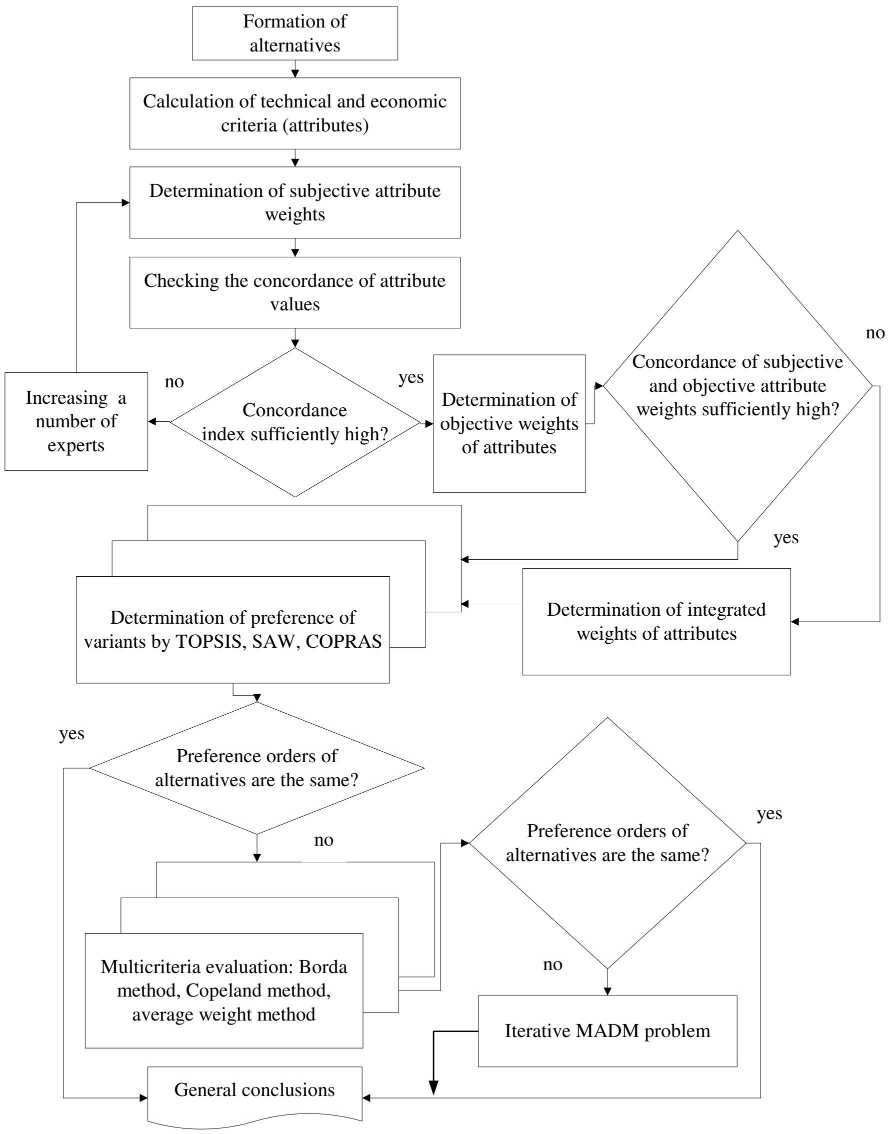

Figure 1. A family of multicriteria decision methods

tent, the criteria weights are calculated as arithmetical means of the weights for particular experts. The number of experts in the group is not limited.

A diagram of an extended version of the pairwise comparison approach is given in Fig. 2.

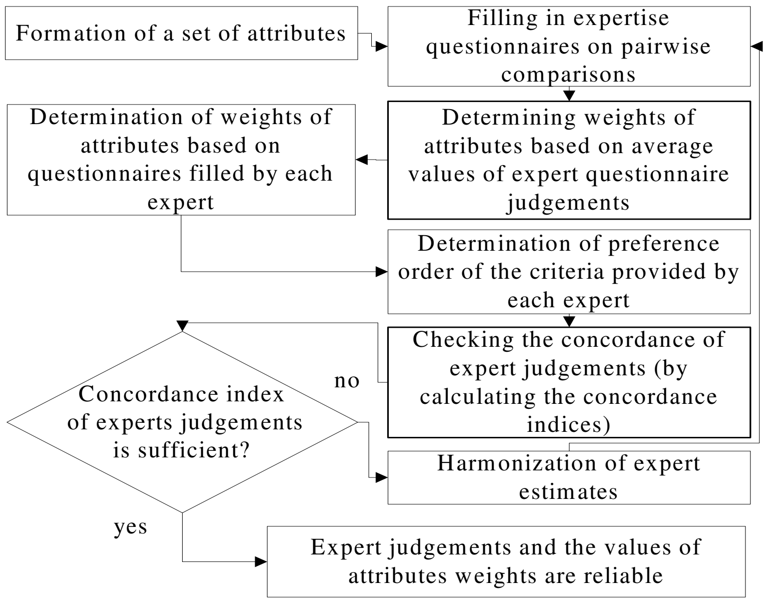

Figure 2. Determination of weights of efficiency criteria by pairwise comparison

Methods of determining objective and integrated weights of attributes. In various papers, the attributes are subdivided into objective and subjective. The attribute weight is obtained based on privileged data and vector technique (Saaty, 1977), least squares comparison (Chu, Kalaba, Spingam, 1979), Delphi (Hwang, Lin, 1987), LINMAP (Linear Programming Techniques for Multidimensional Analysis of Privileged) (Srinivasan, Shocker, 1973) and various computer-aided mathematical models (see, e.g., Pekelman, Sen, 1974). The latter technique uses mathematical programming for obtaining weights without privileged data, involving entropy approach (Hwang, Yoon, 1981) and multiattribute programming methods (Fan, 1994).

Subjective and objective approaches have a number of advantages and disadvantages. The weights obtained by a subjective approach reflect subjective judgements of a person resulting in ranking of the alternatives of the particular problem. Objective weights are obtained by mathematical methods based on the analysis of the initial data. As we can see, none of the two approaches is perfect. It may be suggested that an integrated approach could be most appropriate for determining the weights of the attributes. Currently, a number of papers aimed to combine subjective and objective approaches to solve MCDM problems have been published. However, the models considered present some difficulties for application. The authors themselves admit that integrated methods based on subjective and objective information are far from being perfect, and require further analysis.

Integrated, subjective and objective weights of the attributes. An objective weight of the criterion can be easily obtained by an entropy method. Assume that $B = \{B_1, B_2, \ldots, B_m\}$ is a discrete set of alternatives, $R = \{R_1, R_2, \ldots, R_n\}$ – a set of attributes, and $X = [x_{ij}]_{m\times n}$ – an alternative by attribute matrix, where x_{ij} is R_j attribute value of alternative B_i $(i = 1, 2, ..., m;$ $j = 1, 2, ..., n)$. For the sake of having the same scale of measurement, it is assumed that all the initial entry values in the matrix are in the range from 0 to 1. This is achieved by normalising the elements of the initial matrix.

The value of the objective weights of an attribute is determined by the attribute dominance level. Subjective weight values show the relevance of the attribute to the alternatives considered. In some cases, $\overline{q_j}$ (subjective weight) and q_j (objective weight) are essentially different, thereby considerably decreasing the accuracy of preference ordering of the alternatives. This may be accounted for the fact that minor factors can have a great influence on ranking the alternatives, and vice versa. Therefore, the first author (Ustinovichius, 2001a) suggests that the formula for determining the objective weight of the attribute by entropy approach be interpreted as follows:

$$\overline{q_j} = \frac{q_j^* q_j}{\sum\limits_{j=1}^{n} q_j^* q_j}, \quad (j = \overline{1, n}). \tag{1}$$

To obtain the integrated weight of attribute q_j^*, a system of equations given below should be solved:

$$\begin{cases} q_1^*(\overline{q_1}q_1 - q_1) + q_2^*\overline{q_1}q_2 + q_3^*\overline{q_1}q_3 + ... + q_n^*\overline{q_1}q_n + f = 0 \\ q_1^*\overline{q_2}q_1 + q_2^*(\overline{q_2}q_2 - q_2) + q_3^*\overline{q_2}q_3 + ... + q_n^*\overline{q_2}q_n + f = 0 \\ q_1^*\overline{q_3}q_1 + q_2^*\overline{q_3}q_2 + q_3^*(\overline{q_3}q_3 - q_3) + ... + q_n^*\overline{q_3}q_n + f = 0 \\ \cdots \\ q_1^*\overline{q_n}q_1 + q_2^*\overline{q_n}q_2 + q_3^*\overline{q_n}q_3 + ... + q_n^*(\overline{q_n}q_n - q_n) + f = 0 \\ q_1^* + q_2^* + q_3^* + ... + q_n^* = 1 \end{cases}, \tag{2}$$

here, f is the error coefficient of the system of equations.

Later on, the weights q_j^* will be used in multiattribute decision methods such as TOPSIS, COPRAS, SAW, etc.

Preference ranking of alternatives by the PROPORTIONAL (COPRAS) method (Zavadskas et al., 2004; Kaklauskas, Zavadskas, Raslanas, 2005). This method assumes direct and proportional dependence of significance and priority of investigated alternatives on a system of attributes. The system of attributes is determined and experts calculate their values and initial weights. The interested parties, taking into consideration their goals and the existing capabilities, can check and correct all this information.

The determination of significance and priority of alternatives is carried out in four stages.

Stage 1. The weighted normalized decision-making matrix D is constructed. The purpose of this stage is to receive dimensionless weighted values of the attributes. All attributes, originally having different dimensions, can be compared when their dimensionless values are known.

The following equation is used:

$$d_{ij} = \frac{x_{ij} q_j^*}{\sum_{i=1}^{m} x_{ij}}, \quad i = \overline{1,m}, \quad j = \overline{1,n}. \tag{3}$$

Stage 2. The sums of weighed normalized attributes describing the i-th alternative are calculated. The options are described by minimizing attributes S_{-i} and maximizing attributes S_{+i}. The sums are calculated as follows:

$$\begin{aligned} &S_{+i} = \sum_{j=1}^{n} d_{+ij}, \quad S_{-i} = \sum_{j=1}^{n} d_{-ij}, \\ &i = \overline{1,m}, \quad j = \overline{1,n}. \end{aligned} \tag{4}$$

In this case, the values S_{+i} (the greater this value, the more satisfied the interested parties) and S_{-i} (the lower this value, the better is goal attainment by the interested parties) express the degree of goal attainment by the interested parties in each alternative. In any case, the sums of 'pluses' S_{+i} and 'minuses' S_{-i} of all alternatives are always, respectively, equal to all sums of the weights of maximized and minimized attributes by

$$\begin{aligned} S_{+} &= \sum_{i=1}^{m} S_{+i} = \sum_{j=1}^{n} \sum_{i=1}^{m} d_{+ij}, \\ S_{-} &= \sum_{i=1}^{m} S_{-i} = \sum_{j=1}^{n} \sum_{i=1}^{m} d_{-ij}, \quad i = \overline{1,m}, \quad j = \overline{1,n}. \end{aligned} \tag{5}$$

Stage 3. The significance of the compared alternatives is determined by describing their positive (pluses) and negative (minuses) characteristics. Relative significance Q_i of each alternative B_i is calculated as follows:

$$Q_i = S_{+i} + \frac{\min\limits_i S_{-i} \sum_{i=1}^{m} S_{-i}}{S_{-i} \sum_{i=1}^{m} \frac{\min_i S_{-i}}{S_{-i}}}, \quad i = \overline{1,m}. \tag{6}$$

Stage 4. Determining the priority order of alternatives. The greater the Q_i, the higher the efficiency of an alternative.

The analysis of the method presented makes it possible to state that one can easily apply it to evaluating the alternatives and selecting the most efficient one, while being completely aware of the physical meaning of the process. Moreover, the method allows for the formulation of a reduced criterion Q_i that is directly proportional to the relative effect of the compared criteria values x_{ij} and weight q_j^* on the final result.

Determining preferences of the alternatives based on similarity to an ideal solution (Hwang, Yoon, 1981). Yoon and Hwang developed a technique based on the idea that the optimal alternative is most similar to an ideal solution, being at the longest distance from the "ideally" worst solution. This method is known as TOPSIS – Technique for Order Preference by Similarity to Ideal Solution.

A relative distance of any $i-$th alternative from an ideal one is obtained as:

$$K_i = \frac{L_i^-}{L_i^+ + L_i^-}, \quad i = \overline{1,m}, \quad \text{where } K_i \ [0,1], \tag{7}$$

where L_i^+ is a distance between the compared i–th alternative and the ideal one; L_i^- – a distance between the compared i–th alternative and the negatively ideal option.

The nearer to one is K_i value, the closer is the i–th alternative to a^+, i.e. the optimal alternative is the one which has the highest value of K_i.

Determining the efficiency of alternatives by Simple Additive Weighting (SAW) (MacCrimmon, 1968). SAW is a simple and widely used method. The method was summarized by MacCrimmon. Its major principles were also described in the paper of Churchman and Ackoff (1954) and Klee (1971).

In determining the efficiency of an alternative, the respective terms in a normalized matrix are multiplied by weights and summed up. The sum of the products of an optimal alternative will be maximum:

$$A = \left\{ A_i \left| \max_i \sum_{j=1}^{n} \overline{q}_j \overline{x}_{ij} \,/ \sum_{j=1}^{n} \overline{q}_j \right. \right\}, \tag{8}$$

here $\overline{x}_{ij}$ – normalized alternative vs. attribute matrix.

Multiple criteria evaluation by Borda, Copeland and average weight methods (Hwang, Yoon, 1981). The availability of several research methods raises the problem of the proper choice. The question "Which method is most suitable to solve the problem¿' is most important, but difficult to answer. Since any of multicriteria methods has its advantages and disadvantages, there is actually no answer to the above question.

In addition, the application of various methods of calculation may yield different results (preference order). A model for assessing multiple criteria techniques is based on Borda, Copeland and the average weight methods.

Multicriteria iterative decision problem. Each of the available quantitative methods of multicriteria evaluation has some peculiar features and inherent logic reflecting the specific characteristics of the alternatives compared. It is hardly possible to find any descriptions or recommendations concerning the particular method application. Using several multicriteria methods simultaneously allows us to identify some stable alternatives rated similarly by various techniques. However, numerous calculations have also shown different ranks of a certain number of alternatives, though the variations are slight. The need arises of considering different estimates of alternatives and try to achieve unambiguous ranking.

When Borda, Copeland and the average weight methods are applied to the analysis of alternatives, the uncertainty about the choice of the best option still remains. In this case, the author of the present investigation suggests the multicriteria iterative decision method (Fig. 3) to be used (Ustinovichius, 2001b).

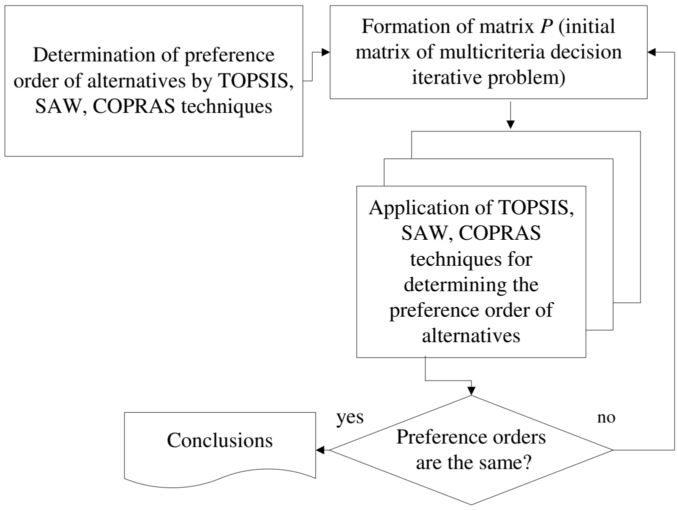

Figure 3. General structural scheme of iterative multicriteria decision analysis

In the beginning matrix P is generated:

$$P = [p_{i,j}], \quad i = \overline{1,m}; \quad j = \overline{1,n_{met}}, \tag{9}$$

where n_{met} is the number of decision support methods used, n_{met}=3 (in our case).

The matrix P is generated in the following way:

$$p_{i,1} = K_i, \quad p_{i,2} = A_i, \quad p_{i,3} = Q_i, \quad i = \overline{1,\ m}. \tag{10}$$

In other words, the first column of the matrix P is made up of the preference order obtained by TOPSIS, while the second column of matrix P is formed of the alternative preferences obtained by SAW and the third column consists of the alternative preferences yielded by COPRAS.

It should be noted that matrix P is the initial decision-making matrix. The number of columns represents the number of the applied multiobjective optimization methods, which can be extended. All the criteria in the matrix P are maximized because the methods TOPSIS, SAW and COPRAS rely on the highest values of K_i, A_i, Q_i in determining the most preferable option.

Matrix P is used as a basis for further multicriteria evaluation. The criteria of the methods used in previous evaluation of alternatives are used as new indicators in making the estimates more accurate. The weights of new indicators (methods) are determined by experts. The authors assume them to be equal in further calculations. When practically used, the suggested technique demonstrated quick convergence of the iterative process and unambiguous ranking of the compared alternatives.

3. Case study

To illustrate the technique developed, some alternatives of purchasing an office building for a company are considered. Suppose that a client (DM) needs to purchase office premises. There are four variants (B_1 – B_4) of office location. Four attributes are considered:

1) R_1 – price (10,000 \$),
2) R_2 – office area (m^2),
3) R_3 – distance from home to work (km),
4) R_4 – office location quality (in points).

The attributes R_2 and R_4 are maximized, while R_1 and R_3 are minimized. The data concerning office purchasing for a firm is presented in Table 1.

Table 1. Data on office purchasing

Alternatives	Attributes			
	R_1	R_2	R_3	R_4
B_1	3.0	100	10	7
B_2	2.5	80	8	5
B_3	1.8	50	20	11
B_4	2.2	70	12	9
	min	max	min	max

Following Table 1, the alternatives vs. attributes matrix takes the form of:

$$X = \begin{bmatrix} 3.0 & 100 & 10 & 7 \\ 2.5 & 80 & 8 & 5 \\ 1.8 & 50 & 20 & 11 \\ 2.2 & 70 & 12 & 9 \end{bmatrix}. \tag{11}$$

Suppose that the experts provided the following matrix B of pairwise evaluation of the attributes (Saaty, 1994):

$$C = \begin{bmatrix} 1 & 1/3 & 1/2 & 1/5 \\ 3 & 1 & 2 & 1/2 \\ 2 & 1/2 & 1 & 1/2 \\ 5 & 2 & 2 & 1 \end{bmatrix}. \tag{12}$$

The subjective weight of the attributes was determined by using expert pairwise evaluation as a subjective approach. The entropy method (Hwang and Yoon, 1981) was used as an objective approach to determine the objective weights of the criteria. The calculated weights are presented in Table 2.

Table 2. Weights of attributes

Weights of attributes	Attributes			
	R_1	R_2	R_3	R_4
Subjective weights	0.095	0.230	0.193	0.481
Objective weights	0.128	0.217	0.360	0.295

The values of an integrated weight are determined by solving a system of equations (2). Taking into account the data given in Table 2, a system of equations (2) may be written as:

$$\begin{cases} -0.1155q_1^* + 0.0207q_2^* + 3.43q_3^* + 0.0281q_4^* + f = 0 \\ 0.0294q_1^* - 0.1673q_2^* + 0.0829q_3^* + 0.0679q_4^* + f = 0 \\ 0.0246q_1^* + 0.0419q_2^* - 0.2904q_3^* + 0.0569q_4^* + f = 0 \\ 0.0614q_1^* + 0.1047q_2^* + 0.1733q_3^* - 0.1529q_4^* + f = 0 \\ q_1^* + q_2^* + q_3^* + q_4^* = 1 \end{cases}. \tag{13}$$

The values of integrated weights as well as the values of complex objective weights of the efficiency attributes (determined from formula (8)) are given in Table 3.

When the values of the objective weights of the attributes are applied, their effect on the efficiency of alternatives does not match that of subjective weights, which may adversely affect the accuracy of the results obtained. The use of an integrated value of the attributes weights in efficiency evaluation techniques

Table 3. The values obtained for integrated and objective complex weights of the attributes

Weights of criteria	Criteria			
	R_1	R_2	R_3	R_4
Integrated weights	0.188	0.266	0.135	0.411
Objective complex weights	0.0473	0.1913	0.2772	0.4842

eliminates the above negative effect. When a system of equations (13) is solved, the accuracy factor f acquires the value of 0.000132 indicating that the accuracy of the integrated alternative weights is not considerably affected.

The values of q_i^* in this case are slightly different from the corresponding values obtained by another method (Fan, Ma and Tian, 1999). The calculations made revealed the need for further investigation in the area of the integrated weights of the efficiency attributes.

By applying TOPSIS, SAW and COPRAS approaches, the efficiency of the alternatives of office purchasing has been determined. The calculation results are given in Table 4.

Table 4. Data obtained by calculating the efficiency of the alternatives by various methods

Method used	Efficiency value of alternative			
	S_1	S_2	S_3	S_4
TOPSIS	0.505	0.338	0.595	0.605
SAW	0.748	0.656	0.786	0.766
COPRAS	96.44	86.04	98.08	100

The analysis of the obtained data yielded the following priorities for the alternatives considered (see Table 5).

Table 5. The priority order of the alternatives obtained by various methods

Method used	Efficiency value of alternative			
	B_1	B_2	B_3	B_4
TOPSIS	3	4	2	1
SAW	3	4	1	2
COPRAS	3	4	2	1

The data obtained by the application of Borda, Copeland and the average weight methods are presented in Table 6.

Table 6. The data obtained by Borda, Copeland and the average weight methods

Method used	Efficiency value of alternative			
	B_1	B_2	B_3	B_4
Borda method	3	4	2	1
Copeland method	3	4	1	2
The average weight method	3	4	2	1

Thus, it is hardly possible to rank the alternatives in the order of preference by Borda, Copeland and the average weight methods. To achieve this aim, matrix $P$is constructed by using formula (10) and the data from Table 4. Then, the calculations are made with the values of the efficient alternatives obtained in calculating a basic decision-making matrix X (Table 4). Matrix P is of the following form:

$$P = \begin{bmatrix} 0.505 & 0.748 & 96.44 \\ 0.338 & 0.656 & 86.04 \\ 0.595 & 0.786 & 98.08 \\ 0.605 & 0.766 & 100 \end{bmatrix} \atop \quad \max \quad\; \max \quad\; \max$$

The results of calculations are given in Table 7.

Table 7. The data obtained by the iterative decision method

Method used	Efficiency value of alternative			
	B_1	B_2	B_3	B_4
TOPSIS	0.641	0.004	0.951	0.954
SAW	0.917	0.752	0.988	0.992
COPRAS	92.04	75.05	99.60	100

The analysis of the obtained data yielded the following priorities for the alternatives considered (see Table 8).

Table 8. Data obtained by calculating the efficiency of the alternatives by various methods

Method used	Efficiency value of alternative			
	B_1	B_2	B_3	B_4
TOPSIS	3	4	2	1
SAW	3	4	2	1
COPRAS	3	4	2	1

Thus, at this stage, the order of preference may be determined for the alternatives analysed.

4. Conclusion

A comprehensive analysis of methods and techniques currently used by various researchers to determine the efficiency of investments was made. It has been found that currently used methods have definite drawbacks, therefore, their application can result in lower profitability of construction investment. Advantages and disadvantages of commonly used decision making methods have been described and it has been stated that the particular features of the above techniques should be assessed by applying them to actual problems of decision-making.

A quantitative multicriteria approach **MCDM-1** for determining investment efficiency based on precisely defined quantitative attributes has been developed. A technique for determining integrated weights of attributes has been developed within this approach. It can be used when a considerable difference between objective and subjective weights of attributes is observed. A method of pairwise comparison for determining the preference of criteria weights was refined. This method provides additional means of checking the concordance of experts' judgements and the reliability of their evaluation. The iterative multicriteria decision approach suggested by the authors yields more precise multicriteria evaluation, and provides a general description of the results obtained by decision making methods.

The developed methods were validated by solving actual problems of selecting the best options of construction and reconstruction investment projects. The investment projects selected were implemented.

The methods created were used in preparing the courses of lectures for training bachelors and masters of science.

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