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Sudden death testing versus traditional censored life testing. A Monte-Carlo study

by

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Abstract: This paper considers two competing methods intended to shorten lifetime tests. The first method, due to L.G. Johnson, is known in reliability engineering as "sudden death testing". Its competitor is a widely known time-terminated, right-censored test.

Times of tests carried out according to these methods are set equal. Then, methods are compared in terms of variances and biases of lifetime parameter estimators. In addition, median, mode, skewness and kurtosis of estimator distributions are also calculated and compared. All data needed came from a large-scale Monte-Carlo numerical experiment.

Keywords: sudden death testing, time terminated life test, Weibull distribution, Monte Carlo method.

1. Introduction

Since years one of the most wanted solutions that reliability engineers wait for is any method of shortening reliability tests. Accelerated testing through overstressing was a dream that hightailed since VLSI components have been commonly applied. One has two ways to shorten life tests without overstressing. The first way is to apply sudden death testing method (SDTM) originally proposed by L.G. Johnson (1964) described in O'Connor (2002) and applied in Chi-Hyuck, Balamuurali, Sang-Ho (2006), Pascual, Meeker (1996), Vlecek, Hendricks (2004), Suzuki et al. (1992). According to Johnson a sample of n items is divided into k sub-samples of m items each. Each sub-sample is tested until the first failure occurs. The second way is a traditional time-terminated life test (TTLT) carried-out on the whole sample of n items. The SDTM is especially applicable when lifetimes follow the Weibull distribution. Remember that the Weibull distribution has the reliability function of the form

$$R(t) = \exp\left[-\left(\frac{t}{a}\right)^b\right], \tag{1}$$

where a, b are the scale and shape parameters, respectively.

Time to the first failure in the sub-sample is the random variable that also follows the Weibull distribution. In fact it is the leftmost order statistics in the sample of m.

$$F_1(t) = 1 - \left\{\exp\left[-\left(\frac{t}{a}\right)^b\right]\right\}^m = 1 - \exp\left[-\left(\frac{m^{1/b}\cdot t}{a}\right)^b\right] =$$
$$= 1 - \exp\left[-\left(\frac{t}{a_1}\right)^b\right]; \quad a_1 = \frac{a}{m^{1/b}}\,. \tag{2}$$

An interpretation that appeals to imagination is that SDTM makes test run $m^{1/b}$ times faster. Lifetimes of most active and passive electronic components satisfactorily fit the Weibull distribution with $b \approx 0.5$. Even a small sub-sample of $m = 10$ gives an exciting figure of $10^{1/0.5} = 100$ times!

Time of testing T_{SDTM} is equal to the rightmost order statistics $t_{(k)}$ in the sample of k. An appropriate cumulative distribution function has the form

$$F_2(T_{\mathrm{SDTM}}) = \left\{1 - \exp\left[-\left(\frac{T_{\mathrm{SDTM}}}{a_1}\right)^b\right]\right\}^k. \tag{3a}$$

Time of testing T_{TTLT} is equal to the rightmost order statistics $t_{(n)}$ in the sample of n. An appropriate cumulative distribution functions has the form

$$F_3\left(T_{\mathrm{TTLT}}\right) = \left\{1 - \exp\left[-\left(\frac{T_{\mathrm{TTLT}}}{a}\right)^b\right]\right\}^n. \tag{3b}$$

Both times are random variables. The following quantile-based shortening coefficient patterned after definition of the confidence interval.

$$\mathrm{SC} = q_{\mathrm{TTLT},0.95}/q_{\mathrm{SDTM},0.95} \tag{4}$$

was proposed in Motyka (2006). In the case of the Weibull distribution $\mathrm{SC} = m^{1/b}$. It is noteworthy that SC does not depend on the number of sub-samples k.

2. Comparison rules

This paper aims at comparing SDTM to TTLT objectively with respect to properties of parameter estimates that SDTM and TTLT produce. For the comparison to be objective two main properties of the methods in question, namely time of testing and sample size were intentionally set the same. This made estimators' properties comparable. Figuratively speaking, two anchors were dropped and the methods can differ in sample arrangement only. The SDTM produces non-censored data. In contrast, TTLT produces right-censored data. Presumably, this differentiates estimators' properties. Table 1 lists main properties of SDTM and TTLT.

Table 1.

Property	**SDTM**	**TTLT**
Number of samples	k	1
Sample size	m	$n = m * k$
Number of failures	m	Random
Time of testing	The same for both methods and equal to $t_{(m)}$	
Number of items tested	The same for both methods and equal to n	

Producing non-censored data was a great advantage of SDTM in the pre-computer era, when the method was put forward. Both goodness-of-fit test and parameter estimation could be performed with the probability paper and hand-held calculator. An old-fashioned estimation method by fitting a straight line to points of empirical cumulative failure function plotted on the Weibull probability paper is still commonly used. The method was on purpose built into numerical experiment as the third anchor.

3. An outline of numerical results

The entire numerical experiment was composed of seven component experiments that differ in sample arrangement, as it is shown in Table 2. The component experiments are ordered according to tendency of making test shorter and shorter. This can be achieved by splitting the sample into a small number of large sub-samples as it is shown in Table 2 where moderate and extreme arrangements were distinguished.

Table 2.

Arragement type	**Number of samples**	**Sample size**	**Arrangement code**
Moderate	25	4	25X4
	20	5	20X5
	14	7	14X7
Extreme	10	10	10X10
	7	14	7X14
	5	20	5X20

Each component experiment was performed in three phases. Particular phases consisted of one or more steps as it is shown in Table 3.

Table 3.

Phase		Consists of Steps
Number	***Description***	
1	Generating data	1
2	Modelling SDTM	2, 3, 4
3	Modelling TTLT	5, 6
4	Parameter estimation	7,8

The experiments were performed assuming the scale and shape parameters equal to 1.

Each component experiment repeats the following sequence 10,000 times!

Step 1: Create two sets of n=100 pseudo-random numbers coming from the Weibull general population. These numbers denoted $t^*_{(i)}$ will then be treated as observed lifetimes of items tested in this virtual reliability test. Denote the sets WRN1 and WRN2.

Step 2: Split WRN1 into k sub-samples of m items each. Denote corresponding lifetimes as $t^{\mathrm{SD}}_{i,j}$, $i = 1, 2, ..., m,$, $j = 1, 2, ..., k$.

Step 3: Find the shortest lifetime in each sub-sample. Denote it $t^{\mathrm{SD}}_{(1),j}$, $j = 1, 2, ..., k$.

Step 4: Determine how long it took SDTM to find $t^{\mathrm{SD}}_{\max} = \max_j \{t^{\mathrm{SD}}_{(1),j}\}$.

Step 5: Set time of TTLT as $t^{\mathrm{TT}} = t^{\mathrm{SD}}_{\max}$.

Step 6: Determine the number of failures NoF by counting all TTD (time terminated data) $t^*_{(i)} \leqslant t^{\mathrm{TT}}$, $i = 1, 2, ..., n$.

Step 7: Treat $t^{\mathrm{SD}}_{(1),j}$ as the non-censored sample. Determine the empirical reliability function. Place the points on the Weibull probability paper. Fit the straight line to the points with the least square method. Calculate estimates a^{SD^*}, b^{SD^*} of unknown values of the scale parameter a and shape parameter b.

Step 8: Treat members of WRN2 as members of right-censored sample. Select these $t^*_{(i)} \leqslant t^{\mathrm{TT}}$. Determine the left-hand segment of the empirical reliability function. From this point repeat Step 7. The results are estimates denoted a^{TT^*} and b^{TT^*}.

4. The results obtained

Figs. 1 and 2 show how test arrangements influence medians and extreme order statistics. It is readily seen in terms of order statistics how estimate distributions spread-out dramatically when one strives to increase the shortening coefficient. Figs. 3 and 4 complement previous figures in terms of basic sample moments. In turn, Figs. 5 and 6 show estimate distributions obtained with computer implementation of Parzen's idea of kernel density estimation, Drapella (2002).

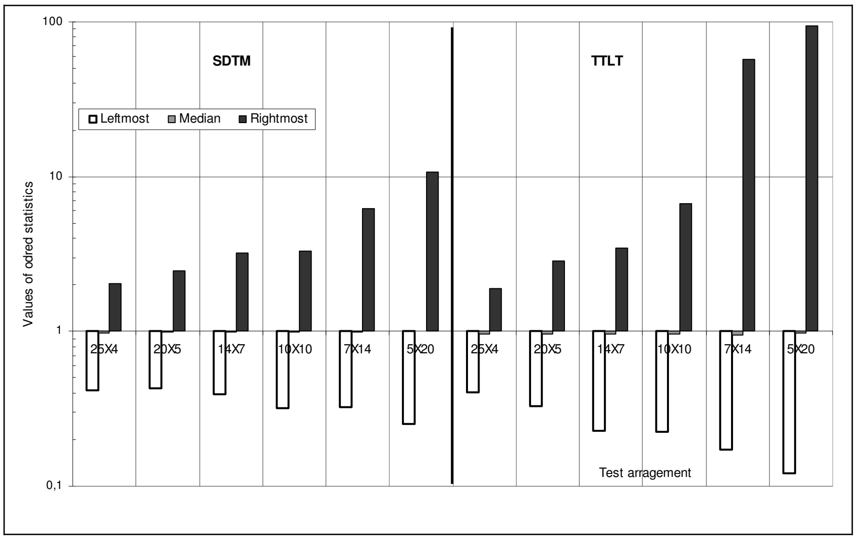

Figure 1. Order statistics of the shape parameter estimates.

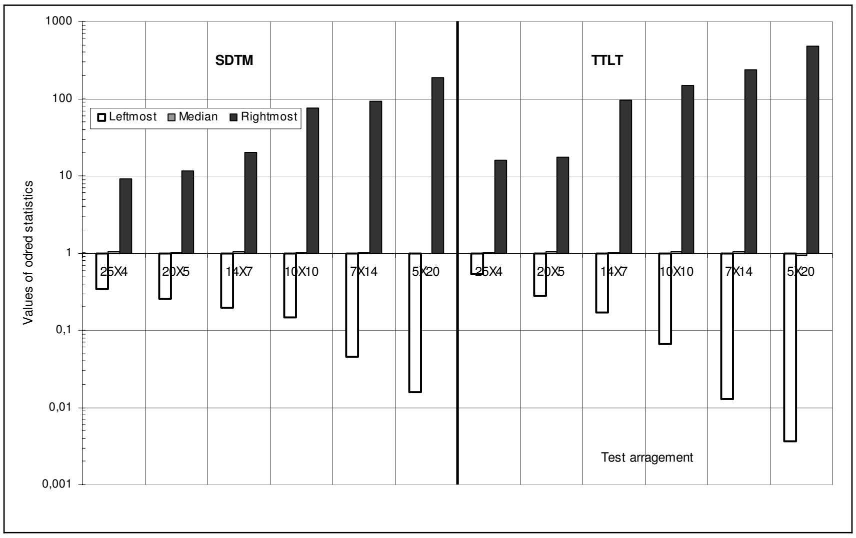

Figure 2. Order statistics of the scale parameter estimates.

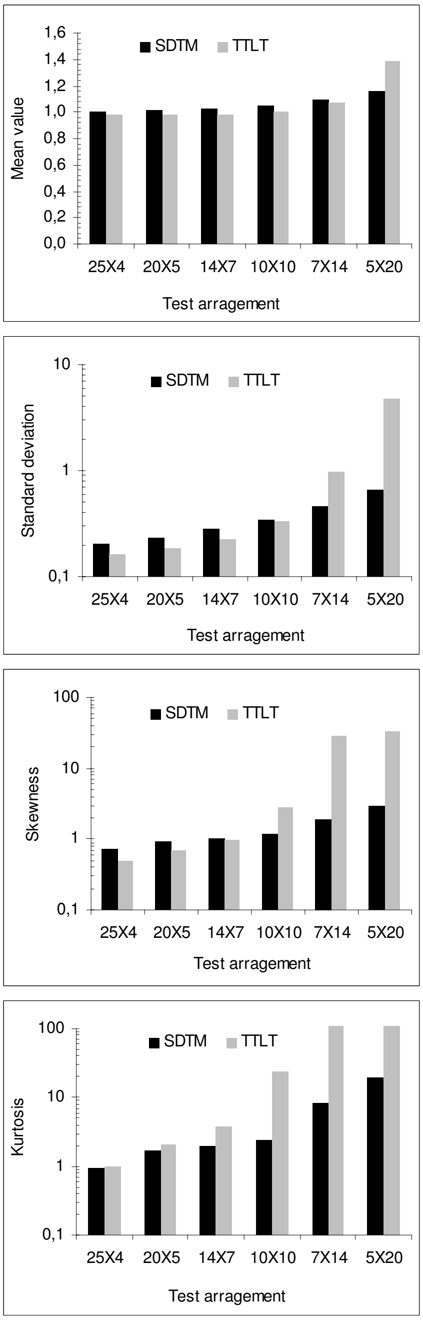

Figure 3. Moments of shape estimates.

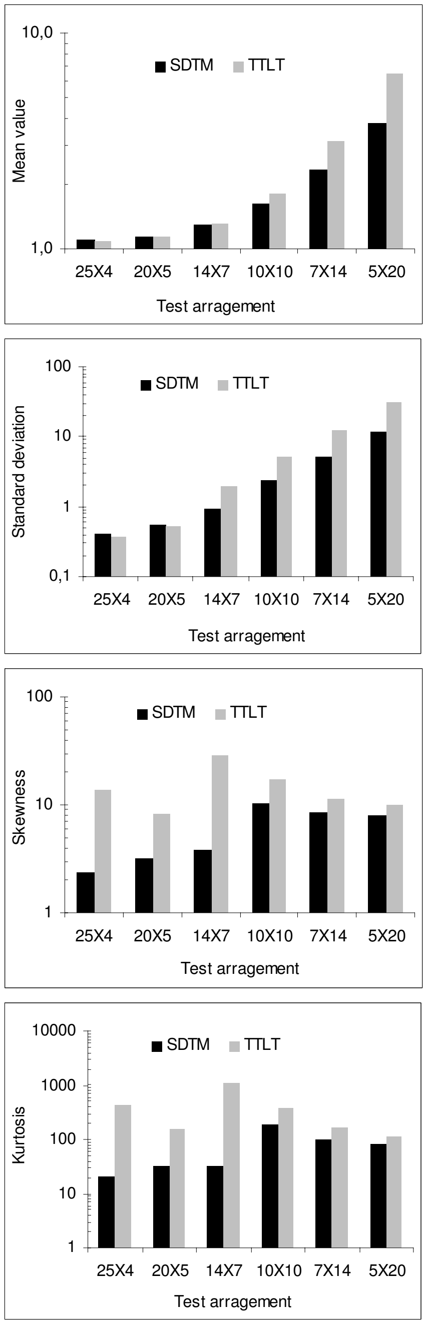

Figure 4. Moments of scale estimates

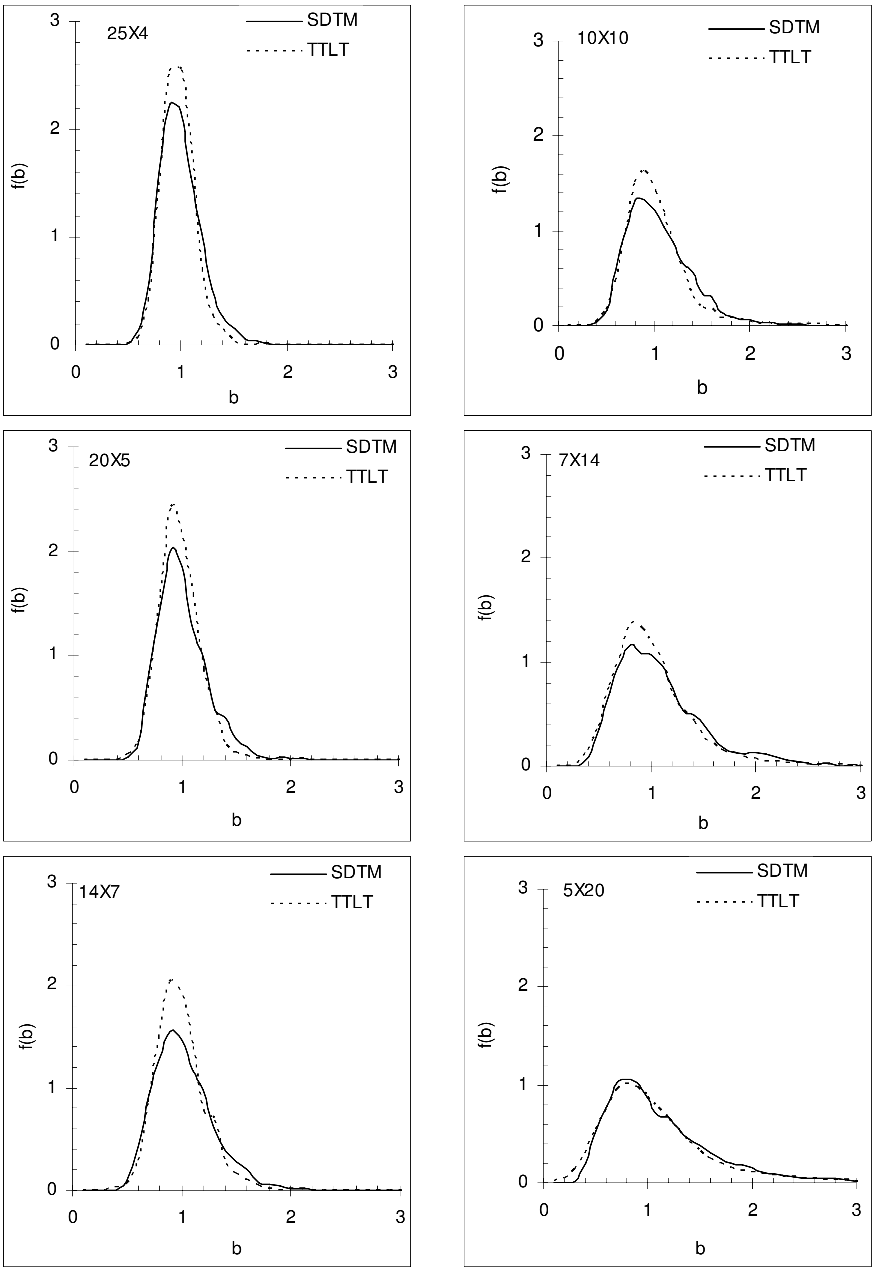

Figure 5. Densities of shape estimates.

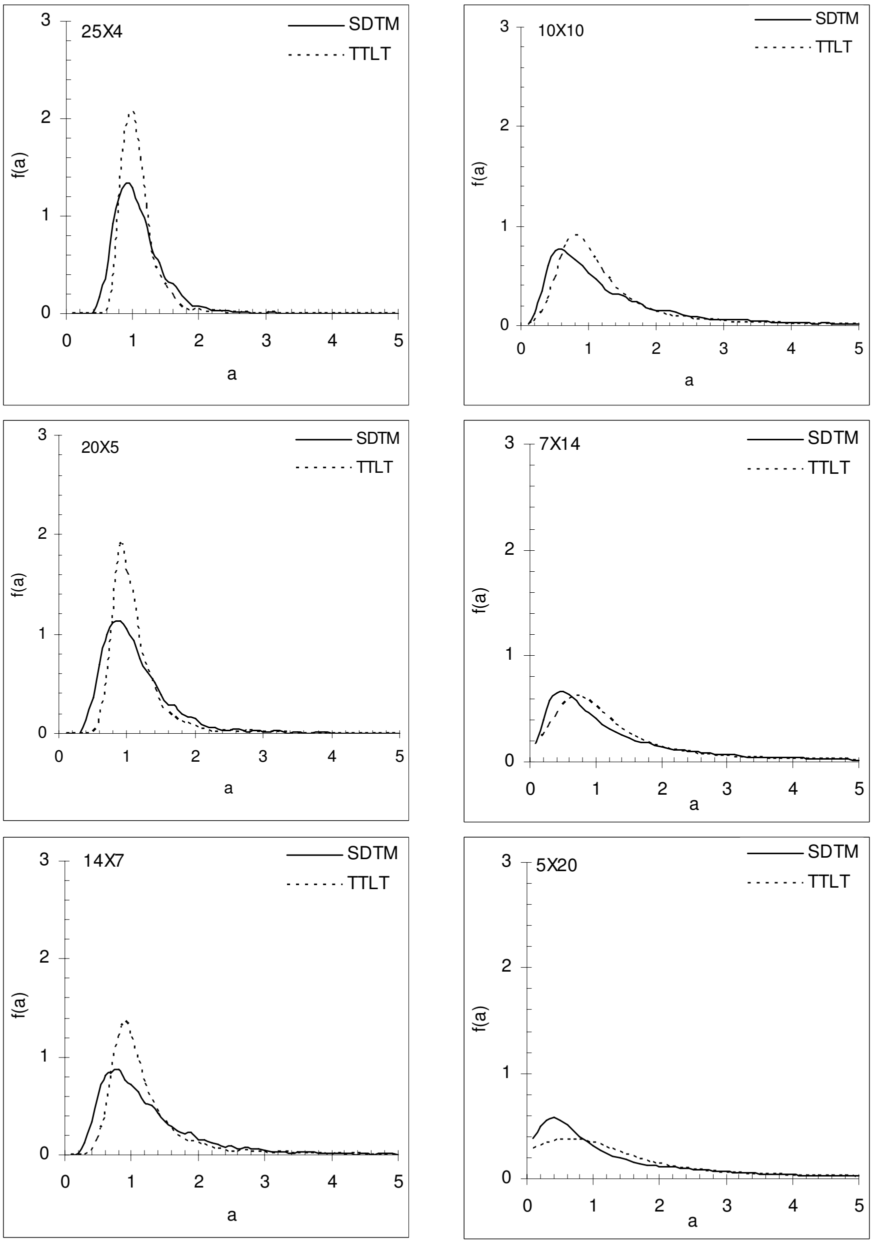

Figure 6. Densities of scale estimates.

5. Conclusions

Let us consider all the test arrangements and compare SDMT and TTLT in different terms.

Conclusions we come to fall into two categories: conclusions of general nature (I) and conclusions strictly related to SDTM and TTLT (II):

Category I:

- A concept of estimation bias was intended to measure departure of a point, around witch distribution of parameter estimator is concentrated, from actual parameter value. A classic definition of bias involves the mean value. Figs. 1-2 and 3-4 embarrass us, because the mean value is not a point of concentration. These evidently are the median and the mode. For moderate arrangements median and mode fit the actual parameter values. Stepping from moderate to extreme rearrangements cause mode move to the left, the mean value move to the right and leave the median unmoved.
- Regardless of whether compared in terms of moments or order statistics the shape parameter estimator has better properties than the scale parameter estimator. Moreover the scale parameter estimator is much sensitive to rearrangement than that of shape.

Category II:

- Two separate comparisons can be made, namely in terms of moments and in terms of order statistics.
- If test methods are compared in terms of order statistics, SDTM is to be recommended for all arrangements.
- If test methods are compared in terms of moments, only extreme SDTM arrangements are to be recommended.
- Tests arranged in an extreme manner have a short duration. However, extreme arrangements may put a producer in danger of offering product with specified MTTF even several times greater than the actual. It means that the arrangements are only for risk-takers, when time factor is of importance rather than the estimation precision factor.

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