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State estimation of linear dynamic system with unknown input and uncertain observation using dynamic programming

by

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Abstract: The paper is devoted to deriving a novel estimation algorithm for linear dynamic system with unknown inputs when observations contain outliers. The algorithm is derived for arbitrary input signals and does not require a priori statistical information concerning input signals. The filtering problem is considered as a control problem in which the unknown input is regarded as a controlling signal for system dynamics, which is described by Kalman equations. In this case, optimal control using Bellman dynamic programming can be calculated. The problem is complicated by the presence of outliers in the observations. To cope with this problem the Lainiotis' partitioning theorem has been used. The nonlinear algorithm of state estimation is obtained. Presented approach can be used both in control systems and decision procedures in tracking systems.

Keywords: state estimation, optimal control, Bellman principle.

1. Introduction

The problem of state estimation of linear dynamic system when the input is unknown is frequently met in practice. For instance in fault diagnosis or tracking systems the appearing changes can be modeled as an input of dynamic system. From the plant point of view the input has usually deterministic character with known kind of change and known onset moment. Unfortunately observer does not have this information. Directly incorporating all kinds of possible changes in estimation model leads to high estimation error and numerically inefficient estimation algorithm. In recent years a great deal of new state estimation algorithms for dynamic systems with different inputs were proposed. Among them there are such algorithms as those which use the input estimation (IE) technique, the variable dimension (VD) filtering, the multiple hypothesis tracking (MHT) and the interacting multiple model (IMM) approach (Bar-Shalom, Fortmann, 1988; Blackman, Popoli, 1999; Katayma, Sugimoto, 1997; Grishin, 1994; Janczak, Grishin, 1996; Mazor et al., 1998). All the above methods need *a priori* assumed input models and they are accurate for changes included in the implications of the assumed input models. The method proposed in the paper is derived for arbitrary input signals and does not require a priori statistical information concerning input signals. Such a method is superior in robust control systems (fault-tolerant). This approach allows real-time robust estimation process, which is independent of the fault model.

In practical realizations of control, measurement and telecommunication systems the problem of uncertain observations can be met. One of the problems widely encountered are outliers. The outlier is an abnormal measurement usually caused by electromagnetic disturbances or another process specific physical phenomena. Outlier phenomenon can highly deteriorate the accuracy of typical estimation algorithms and should be especially addressed. Usually it can be accepted that the outlier probability density function (pdf) has the same form as the normal measurement, but much larger variance.

The objective of the paper is to present a new state estimation algorithm based on a dynamic system model that incorporates both random and deterministic character of the changes, including non-linear. Such a model enables to work out relatively simple recursive adaptive algorithm. Besides, this model and the estimation algorithm are especially suitable for fault detection and diagnosis in non-stationary dynamical systems.

2. The system model

Mathematical model of a dynamic system that incorporates both random and deterministic character of changes can be based on the idea of the dynamical systems with unknown input signal. The output of such systems can be treated as the Gauss-Markov process, which is additive with respect to the state or observation equation. Such models can describe wide range of non-stationary effects, such as abrupt changes of state vector elements, state vector dimension, covariance matrices of the system and observation noises, or observation matrix. For the sequences additive to the state we obtain the state equation in the form (Bar-Shalom, Fortmann, 1988; Blackman, Popoli, 1999; Grishin, 1994; Janczak, Grishin, 1996):

$$x_{k+1} = \Phi_{k+1,k} x_k + G_{k+1,k} w_k + B_{k+1,k} u_{k,t_i} \tag{1}$$

where x_k is the system state vector, $\Phi_{k+1,k}$, $G_{k+1,k}$, $B_{k+1,k}$, are known transition and input matrices, w_k is the system zero mean white Gaussian noise with covariance matrix Q_k, $u_{k,ti}$ is an unknown input signal arising at the random time t_i.

When conditions of observation are nominal the measurement equation can be written in the form:

$$y_k = H_k x_k + v_k \ , \tag{2}$$

where y_k is the observation vector, H_k is the observation matrix, v_k is the observation zero mean white Gaussian noise with covariance matrix R_k .

When measurements contain the outliers the observation equation can be written as follows:

$$y_k = H_k x_k + \gamma_k v_k \ , \tag{3}$$

where γ_k is the random value (in a general case the Markov chain) which is equal to 1 when the outliers are absent and $\gamma_k \gg 1$ when they are present.

The Markov chain approach makes it possible to describe the outliers with correlation in time. The initial probabilities of the Markov chain and its transition matrix can be known or not depending on problem formulation. In both cases the problem can be solved in the same way but with more complicate calculations.

3. State estimation with unknown input and absence of outliers

Let us consider first the dynamic system described by the equations (1) and (2), that is the system when the outliers are absent. Then the optimal system state vector estimation, assuming that the input signal is known, can be calculated using the Kalman filter algorithm (Sorenson, 1985) described by equations (4)÷(8).

Estimation equation:

$$\hat{x}_{k/k} = \hat{x}_{k/k-1} + K_k[y_k - H_k \hat{x}_{k/k-1}] \ , \tag{4}$$

prediction equation:

$$\hat{x}_{k/k-1} = \Phi_{k,k-1} \hat{x}_{k-1/k-1} + B_{k,k-1} u_{k-1} \ , \tag{5}$$

gain matrix:

$$K_k = P_{k/k-1} H_k^T [H_k P_{k/k-1} H_k^T + R_k]^{-1} \ , \tag{6}$$

covariance matrix of prediction error:

$$P_{k/k-1} = \Phi_{k/k-1} P_{k-1/k-1} \Phi_{k/k-1}^T + G_{k/k-1} Q_k G_{k/k-1}^T \ , \tag{7}$$

covariance matrix of filtering error:

$$P_{k/k} = P_{k/k-1} - K_k H_k P_{k/k-1} \ . \tag{8}$$

Kalman filter is a linear filter optimal in the sense of root square mean error for linear dynamic systems with Gaussian noises and initial conditions. However, the equations (4) ÷ (8) of the Kalman filter can not be directly used because of unknown input u_k. But the problem can be reversed as follows. The prediction equation (5) can be treated as an equation describing dynamical system, for which an optimal control u_k should be calculated. The optimality criterion should ensure the minimal error of predicted estimates $\hat{x}_{k/k-1}$. For technical realization purposes minimizing the value of control u_k should also be taken into consideration. Finally, optimality criterion of the following form can be chosen:

$$J = \sum_{i=1}^{k} \{[y_i - H_i \hat{x}_{i/i-1}]^T V_i [y_i - H_i \hat{x}_{i/i-1}] + u_{i-1}^T W_{i-1} u_{i-1}\}, \tag{9}$$

where V_i and W_i are the positive symmetric matrices determining relative weights of the corresponding errors. Calculating control in such a way enables obtaining estimates of $\hat{u}_{k/k}$. Optimization process up to the current step $i = k$ requires minimization of criterion at every step of control:

$$J_k = \min_{u_0} \min_{u_1} \ldots \min_{u_{k-1}} \sum_{i=1}^{k} \{[y_i - H_i \hat{x}_{i/i-1}]^T V_i [y_i - H_i \hat{x}_{i/i-1}] + u_{i-1}^T W_{i-1} u_{i-1}\} \; . \tag{10}$$

At the beginning the control at the step $k = 1$ can be obtained. Optimality criterion at $k = 1$ can be written down as:

$$J_1 = \min_{u_0} \{[y_1 - H_1 \hat{x}_{1/0}]^T V_1 [y_1 - H_1 \hat{x}_{1/0}] + u_0^T W_0 u_0\} \; . \tag{11}$$

Taking into consideration that:

$$\hat{x}_{1/0} = \Phi_{1,0} \hat{x}_{0/0} + B_{1,0} u_0 = \Phi_{1,0} \hat{x}_0 + B_{1,0} u_0 \; , \tag{12}$$

and that y_k, $\hat{x}_{k/k-1}$, u_k are vectors of dimension $s \times 1$, $n \times 1$, $r \times 1$ respectively after simple calculations the value of J_1 can be presented in the form:

$$\begin{aligned} J_1 = \min_{u_0} \{ & y_1^T V_1 y_1 - 2 y_1^T V_1 H_1 \Phi_{1,0} \hat{x}_0 + \\ & + \hat{x}_0^T \Phi_{1,0}^T H_1^T V_1 H_1 \Phi_{1,0} \hat{x}_0 - 2 y_1^T V_1 H_1 B_{1,0} u_0 + \\ & + 2 \hat{x}_0^T \Phi_{1,0}^T H_1^T V_1 H_1 B_{1,0} u_0 + u_0^T (B_{1,0}^T H_1^T V_1 H_1 B_{1,0} + W_0) u_0 \} \; . \end{aligned} \tag{13}$$

Next, the value of control u_0 which minimizes J_1 should be obtained. Calculating first derivative $\frac{dJ_1}{du_0} = 0$ leads to:

$$-2 y_1^T V_1 H_1 B_{1,0} + 2 \hat{x}_0^T \Phi_{1,0}^T H_1^T V_1 H_1 B_{1,0} + 2 u_0^T (B_{1,0}^T H_1^T V_1 H_1 B_{1,0} + W_0) = 0 \; . \tag{14}$$

Finally, with the assumption that V_1 and W_0 are positive symmetric matrices, the value of control $u_0^\star$ can be obtained:

$$u_0^\star = (B_{1,0}^T H_1^T V_1 H_1 B_{1,0} + W_0)^{-1} B_{1,0}^T H_1^T V_1 (y_1 - H_1 \Phi_{1,0} \hat{x}_0) \tag{15}$$

and as in these conditions the second derivative at the point $u_0^\star$ is $\frac{d^2 J_1}{du_0^2}\Big/_{u_0^\star} > 0$, $u_0^\star$ minimizes J_1, and so it is the optimal value of control. The optimal value of control (15) can be written in the following clear form:

$$u_0^\star = \ S_0 (y_1 - H_1 \Phi_{1,0} \hat{x}_0) \ , \tag{16}$$

where

$$S_0 = (B_{1,0}^T H_1^T V_1 H_1 B_{1,0} + W_0)^{-1} B_{1,0}^T H_1^T V_1 . \tag{17}$$

Then the optimality criterion J_1 value equals:

$$J_1 = (y_1 - H_1 \Phi_{1,0} \hat{x}_0)^T M_1 (y_1 - H_1 \Phi_{1,0} \hat{x}_0) \ , \tag{18}$$

where

$$M_1 = V_1 - 2 V_1 H_1 B_{1,0} S_0 + S_0^T (B_{1,0}^T H_1^T V_1 H_1 B_{1,0} + W_0). \tag{19}$$

Next the control at $k = 2$ can be calculated in the following way. The predicted estimates and the optimality criterion at $k = 2$ are:

$$\hat{x}_{2/1} = \Phi_{2,1} \hat{x}_{1/1} + B_{2,1} u_1 \ , \tag{20}$$

$$\begin{aligned} J_2 = \min_{u_0} \min_{u_1} \Big[& \big\{ [y_2 - H_2 \hat{x}_{2/1}]^T V_2 [y_2 - H_2 \hat{x}_{2/1}] + u_1^T W_1 u_1 \big\} + \\ & + \big\{ [y_1 - H_1 \hat{x}_{1/0}]^T V_1 [y_1 - H_1 \hat{x}_{1/0}] + u_0^T W_0 u_0 \big\} \Big] \ . \end{aligned} \tag{21}$$

Using Bellman optimality principle (Bellman, Dreyfus, 1962; Bertsekas, 1987) the above problem can be reformulated into the following one:

$$J_2 = \min_{u_1} \Big[\big\{ [y_2 - H_2 \hat{x}_{2/1}]^T V_2 [y_2 - H_2 \hat{x}_{2/1}] + u_1^T W_1 u_1 \big\} + J_1 \Big] \ . \tag{22}$$

Then, taking into consideration (20) and (18) the equation (22) can be recalculated into:

$$\begin{aligned} J_2 = \min_{u_1} \big\{ & y_2^T V_2 y_2 - 2 y_2^T V_2 H_2 \Phi_{2,1} \hat{x}_{1/1} + \\ & + \hat{x}_{1/1}^T \Phi_{2,1}^T H_2^T V_2 H_2 \Phi_{2,1} \hat{x}_{1/1} - 2 y_2^T V_2 H_2 B_{2,1} u_1 + \\ & + 2 \hat{x}_{1/1}^T \Phi_{2,1}^T H_2^T V_2 H_2 B_{2,1} u_1 + \\ & + u_1^T (B_{2,1}^T H_2^T V_2 H_2 B_{2,1} + W_1) u_1 + \\ & + (y_1 - H_1 \Phi_{1,0} \hat{x}_0)^T M_1 (y_1 - H_1 \Phi_{1,0} \hat{x}_0) \big\} \ . \end{aligned} \tag{23}$$

Performing the same steps as in (13) ÷ (17) we can show that optimal control at $k = 2$ is:

$$u_1^\star = S_1(y_2 - H_2\Phi_{2,1}\hat{x}_{1/1}) \ , \tag{24}$$

where

$$S_1 = (B_{2,1}^T H_2^T V_2 H_2 B_{2,1} + W_1)^{-1} B_{2,1}^T H_2^T V_2. \tag{25}$$

Applying Bellman optimality principle for the next steps $k = 3, 4, \ldots, k$ and using the same operations as above, the optimal control $u_k^\star$ at the k-th step is derived:

$$u_k^\star = S_k[y_{k+1} - H_{k+1}\Phi_{k+1,k}\hat{x}_{k/k}] \ , \tag{26}$$

where

$$S_k = [B_{k+1,k}^T H_{k+1}^T V_{k+1} H_{k+1} B_{k+1,k} + W_k]^{-1} B_{k+1,k}^T H_{k+1}^T V_{k+1} \ . \tag{27}$$

Incorporating (5), (26), (27) to (4) yields the expression of estimation equation with unknown input:

$$\begin{aligned} \hat{x}_{k/k} = {} & \Phi_{k,k-1}\hat{x}_{k-1/k-1} + \\ & + [B_{k,k-1} - K_k H_k B_{k,k-1}] S_{k-1}[y_k - H_k\Phi_{k,k-1}\hat{x}_{k-1/k-1}] + \\ & + K_k[y_k - H_k\Phi_{k,k-1}\hat{x}_{k-1/k-1}] \ . \end{aligned} \tag{28}$$

4. Estimation in presence of outliers

Outliers can significantly deteriorate the accuracy of typical estimation algorithms and should be especially addressed. When the results of observations contain the outliers (3), then for calculating the system estimation it is necessary to use a general approach. It can be based on the Lainiotis' partitioning theorem (Lainiotis, Park, 1973). In this case the dynamic system state vector estimation can be found as a conditional mean of the following form:

$$\hat{x}_{k/k} = E[x_k/Y_1^k] = \sum_{i\in\{1,\ldots,2^k\}} \hat{x}_{k/k}^i P(\bar{\Gamma}_k^i/Y_1^k) \tag{29}$$

where $Y_1^k = \{y_1, y_2, ..., y_k\}$ is the sequence of observations, $\bar{\Gamma}_k^i = \{\gamma_1^i, \gamma_2^i, ..., \gamma_k^i\}$ denotes the sensor state sequence, $\hat{x}_{k/k}^i = E[x_k/Y_1^k, \bar{\Gamma}_k^i]$ is the partial state vector estimate calculated on the basis of (28) depending on observations and sensor state sequences.

The probability density function of the estimates (29) can not be defined exactly. That is why for calculating the probability density function of $f(x_k/Y_1^k)$ it is necessary to use the Gaussian approximation approach (Grishin, 1994; Grishin, Kazarinov, 1985). In such an approach the state vector estimates $\hat{x}_{k/k}$

can be expressed as the weighted sum of the partial estimates $\hat{x}^i_{k/k}$ corresponding to presence and absence of the outliers in the measurements on the current time step using predicted values,

$$\hat{x}_{k/k} = \sum_{i\in\{1,\sigma\}} \hat{x}^i_{k/k} P(\gamma_k = i/Y_1^k) \tag{30}$$

The $P(\gamma_k = i/Y_1^k)$ is a posterior probability of measurement channel state. It depends on the outlier stochastic characteristics. If the outliers are statistically independent the probability $p_{1/k} = P(\gamma_k = 1/Y_1^k)$ of the outlier absence can be found using the Bayes rule (Katayma, Sugimoto, 1997):

$$p_{1/k} = \frac{f(y_k/\gamma_k = 1, Y_1^{k-1})P_1}{\sum_{i\in\{1,\sigma\}} f(y_k/\gamma_k = i, Y_1^{k-1})P_i} \ , \tag{31}$$

where $P_1 = P(\gamma_k = 1)$ and $P_\sigma = P(\gamma_k = \sigma)$ are a priori probabilities of measurement channel state, and $f(y_k/\gamma_k = i, Y_1^{k-1})$ denotes the Gaussian density function of the predicted estimates (Sorenson, 1985):

$$f(y_k/\gamma_k = i, Y_1^{k-1}) = N\Big[H_k\hat{x}_{k/k-1}, \ \ H_k P^i_{k/k-1} H_k^T + i^2 R_k\Big], \ i \in \{1,\sigma\}, \tag{32}$$

where $P^i_{k/k-1}$ is the corresponding covariance matrix.

The state vector estimates (30) can be expressed as:

$$\hat{x}_{k/k} = p_{1/k}\hat{x}^1_{k/k} + (1 - p_{1/k})\hat{x}^\delta_{k/k}. \tag{33}$$

Partial state estimates $\hat{x}^i_{k/k}$ are to be calculated according to the estimation equation (28) for the unknown input case,

$$\begin{aligned}\hat{x}^i_{k/k} =& \Phi_{k,k-1}\hat{x}_{k-1/k-1} + K^i_k[1 - H_k B_{k,k-1} S_{k-1}] z_{k/k-1} + \\ &+ B_{k,k-1} S_{k-1} z_{k/k-1}, \quad i \in \{1,\sigma\},\end{aligned} \tag{34}$$

where K^i_k are gain matrices of partial state estimators, $z_{k/k-1}$ is the innovation process.

$$z_{k/k-1} = [y_k - H_k \Phi_{k,k-1}\hat{x}_{k-1/k-1}].$$

Partial gain matrices K^i_k can be calculated as follows:

$$K^i_k = P_{k/k-1} H^T_k (H_k P_{k/k-1} H^T_k + i^2 R_k) \ , \quad i \in \{1,\sigma\} \ . \tag{35}$$

On the basis of (26), (27), (28), (33) and (34) the final state and control estimation equations can be found:

$$\begin{aligned}\hat{x}_{k/k} =& \Phi_{k,k-1}\hat{x}_{k-1/k-1} + \\ &+ \big[p_{1/k}(K^1_k - K^\delta_k) + K^\delta_k\big][1 - H_k B_{k,k-1} S_{k-1}] z_{k/k-1} + \\ &+ B_{k,k-1} S_{k-1} z_{k/k-1} \ ,\end{aligned} \tag{36}$$

$$\hat{u}_k = S_k[y_{k+1} - H_{k+1}\Phi_{k+1,k}\hat{x}_{k/k}] \ , \tag{37}$$

where

$$S_k = [B^T_{k+1,k} H^T_{k+1} V_{k+1} H_{k+1} B_{k+1,k} + W_k]^{-1} B^T_{k+1,k} H^T_{k+1} V_{k+1} \ . \tag{38}$$

The estimation covariance matrix can be derived based on the approach presented in Lainiotis, Sims (1970) as follows:

$$\begin{aligned} P_{k/k} = & p_{1/k}\big\{P^1_{k/k} + [\hat{x}^1_{k/k} - \hat{x}_{k/k}][\hat{x}^1_{k/k} - \hat{x}_{k/k}]^T\big\} + \\ & + [1 - p_{1/k}]\big\{P^\delta_{k/k} + [\hat{x}^\delta_{k/k} - \hat{x}_{k/k}][\hat{x}^\delta_{k/k} - \hat{x}_{k/k}]^T\big\} \ . \end{aligned} \tag{39}$$

Applying partial estimation covariance matrices of the form:

$$P^i_{k/k} = P_{k/k-1} - \big\{K^i_k[1 - H_k B_{k,k-1} S_{k-1}] + B_{k,k-1} S_{k-1}\big\} H_k P_{k/k-1} \ , \quad i \in \{1, \sigma\} \ , \tag{40}$$

and using (39) and (40), after some calculations the estimation covariance matrix can be obtained:

$$\begin{aligned} P_{k/k} = & P_{k/k-1} - p_{1/k}\widetilde{K_k} H_k P_{k/k-1} + p_{1/k}[1 - p_{1/k}]\widetilde{K_k} z_{k/k-1} z^T_{k/k-1}\widetilde{K^T_k} - \\ & - \big\{K^\delta_k[1 - H_k B_{k,k-1} S_{k-1}] + B_{k,k-1} S_{k-1}\big\} H_k P_{k/k-1} \ , \end{aligned} \tag{41}$$

where $\widetilde{K_k} = (K^1_k - K^\delta_k)[1 - H_k B_{k,k-1} S_{k-1}]$.

Thus, equations (36), (41) describe the final formulas for state estimates and the corresponding covariance matrix for state estimation of linear dynamic system with unknown input and with presence of outliers in observations. The input estimates are obtained in the form of equation (37).

5. Simulation results

In this section we present simulation results to illustrate the performance of the proposed algorithm. In the following experiments we simulated a simple scenario where the unknown input was modelled by the pulse and sinusoidal signals. For description of the system and observations the first order model with $Q_k = 0.01$, $R_k = 0.49$, $\gamma = 10$ has been used. In Figs. 1 and 2 the examples of step and sinusoidal inputs ($30 \leq k \leq 60$) and their estimates obtained in the absence of outliers are presented. The basic algorithm (26)÷(28) revealed good estimation ability for inputs of different dynamics, but it was not robust with respect to outliers, what can be seen in Fig. 3. Outliers were present at $k = 10, 50, 70$.

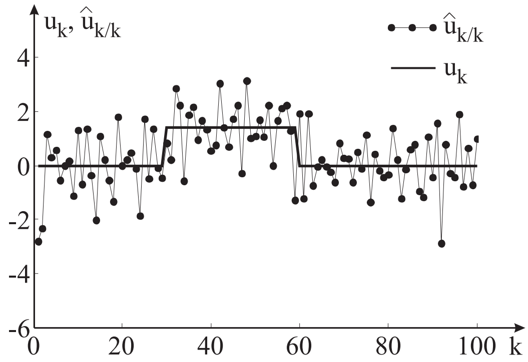

Figure 1. An example of step input and its estimate

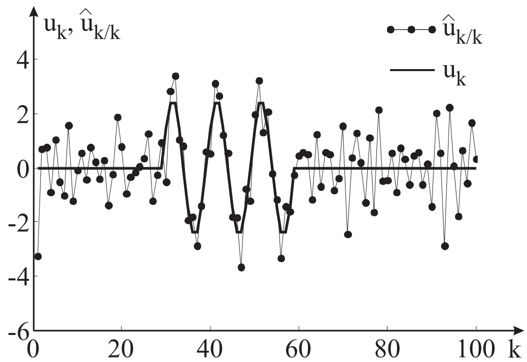

Figure 2. An example of sinusoidal input and its estimate

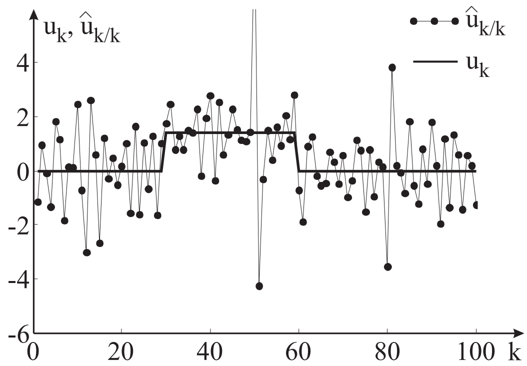

Figure 3. An example of step input and its estimate in presence of outliers

In the presence of outliers for the step input the robust algorithm (31)÷(41) was investigated. In Fig. 4 posterior probabilities of the outlier absence calculated according to (31) are shown.

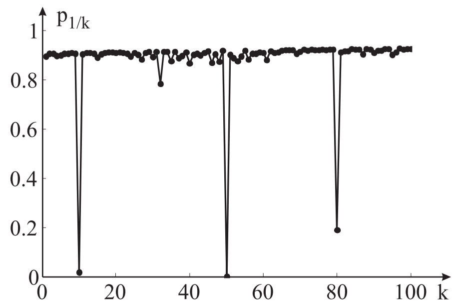

Figure 4. Posterior probability of the outlier absence

As can be seen from the schedule, the procedure of outlier detection (31) is rather reliable. The root mean square error of $\hat{u}_{k/k}$ estimates for the proposed filter is presented in Fig. 5.

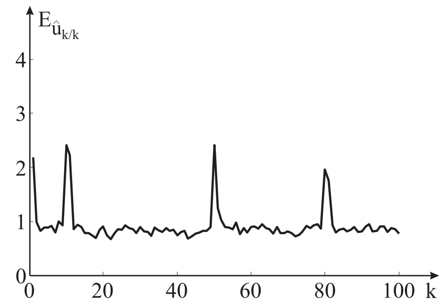

Figure 5. The root mean square error of $\hat{u}_{k/k}$

In Fig. 6 the root mean square errors of the state estimates for the proposed and Kalman filter algorithms are shown.

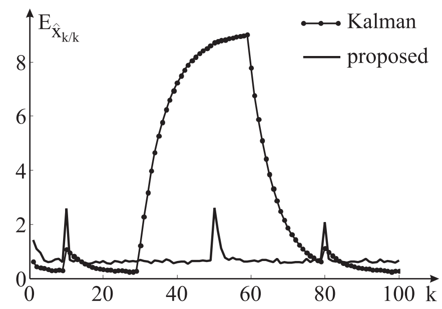

Figure 6. Comparative performance of the proposed algorithm and the Kalman filter

As one can see, the proposed algorithm revealed better performance in the presence of the input jump signal and outliers in observations in comparison with the Kalman filter.

6. Conclusions

The paper presents a new method of state estimation of linear dynamic system with unknown input in the presence of outliers in observations. The problem has been solved using the discrete time dynamic programming principle and the Gaussian approximation approach to non-linear filtering. The robust estimation algorithm, which can estimate the unknown input function and the system state vector has been developed. The proposed algorithm has a recursive structure and can be easily implemented with limited computational burden. Such an algorithm can also be used for fault detection and identification in industrial control processes, in state estimation of dynamic system and decision procedures in radar tracking systems. The specific feature of the developed method is a simultaneous estimation of system state vector as well as unknown input which makes it possible to solve the classification problem in a particular implementation.

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