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# On the real-time emission control – case study application

by

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**Abstract:** The paper addresses the problem of real-time emission control in a given set of air pollution sources. The approach applied utilizes the optimal control technique for distributed parameter systems. A set of pointwise emission sources with a predefined location and emission characteristics is considered as the controlled object. The problem is formulated as on-line minimization of an environmental cost function, by the respective modification of emission level in the controlled sources, according to the changing meteorological conditions (e.g. wind direction and velocity). Dispersion of atmospheric pollution is governed by a multi-layer, dynamic model of $SO_x$ transport, which is the main forecasting tool used in the optimization algorithm. The objective function includes the environmental damage related to air quality as well as the cost of the controlling action. The environmental cost index depends on the current level of $SO_x$ concentration and on the sensitivity of the area to this type of air pollution. The adjoint variable, related to the main transport equation of the forecasting model, is applied to calculate the gradient of the objective function in the main optimization procedure. The test computations have been performed for a set of major power plants in the industrial region of Upper Silesia (Poland).

**Keywords:** air pollution, mathematical modeling, emission control.

## 1. Air pollution forecasting model

The most common application of environmental models is the forecasting of dispersion of pollutants. Some air quality studies are also aimed at optimization, but numerous applications of optimization methods mainly occur in the design of monitoring networks. On the other hand, many important decisions in air pollution and environmental problems, which could be supported by the respective models, are directly made by decision makers. However, some optimization methods and recently developed environmental models (e.g. Haurie et al., 2004;

Holnicki, 2004) give the possibility of implementation of complex air pollution control strategies.

The long-term air pollution forecasting model was applied to evaluate the possible environmental consequences of alternative strategies of energy sector expansion in Poland (Ciechanowicz et al., 1996). The problem of the regional-scale strategy of emission abatement in a set of major power plants was discussed, e.g. by Chang (2000) or Holnicki and Kałuszko (2004). The solution of the last task is sought by the optimal selection of the desulphurization technologies for a given set of emission sources. From the viewpoint of mathematical formulation, the above tasks are static optimization problems.

This paper presents some possibilities of utilizing pollution transport models in the real-time control of air quality. The optimal control problem is formulated as on-line minimization of an environmental cost function, by modification of emission level in a set of the controlled sources, according to the changing meteorological conditions. In the case of linear, single-component transport models (Trojanowski and Michalewicz, 2000), pollution forecast can be considered as superposition of individual source contributions. The optimization process is based on the unit transfer matrices, which are calculated *off-line* for each emission source. Such a simplified approach is no longer applicable in case of the multi-species and nonlinear models, where chemical transformations are considered. The transport model – fully integrated with the optimization procedure – must then *on-line* generate pollution transport forecast. Statement of the problem and computational algorithm are presented below.

The aim is to formulate and implement the optimal control procedure, and verify effectiveness and performance of this formulation. The approach is based on the integration of the dispersion model with a general theory of the optimal control in distributed parameter systems (Lions, 1971). For this reason, it is assumed that the pollution transport process can be considered as a system governed by the set of transport equations. The model generates short-term forecasts of air pollution related to a specified, complex emission field. The implementation discussed in the sequel is illustrated by an example of the sulfur-oriented transport model, but the approach can be applied in a more general class of the multi-species, forecasting models.

Computation of the transport of sulfur pollution is carried out by Lagrangian type, three-layer trajectory model, as discussed by Holnicki et al. (2000). The mass balance of the pollutants is calculated for air parcels following the wind trajectories. The model takes into account two basic polluting components of sulfur cycle: primary – $SO_2$ and secondary – $SO_4^=$. Transport equations include chemical transformations $SO_2 \Longrightarrow SO_4^=$, dry deposition and the scavenging by precipitation.

The main output constitute the concentrations of $SO_2$ and $SO_4^=$, averaged over the space discretization element and the horizontal layer height. Mathematical representation has a form of two transport equations in each layer of the model. For presentation simplicity, we consider in the sequel one governing equation, expressed in an aggregated form, representing the main polluting component – ($SO_2$) in one vertical layer:

$$\frac{\partial c}{\partial t} + \vec{v}\,\nabla c - K_h \Delta c + \gamma c = Q \qquad \text{in} \qquad \Omega \times (0, T)\,, \tag{1}$$

along with the boundary conditions

$$c = c_b \;\; \text{on} \quad S^- = \{\partial\Omega \times \langle 0, T\rangle \mid \vec{v}\cdot\vec{n} < 0\} \;\; \text{– inflow to the domain,} \tag{1a}$$

$$K_h \frac{\partial c}{\partial \vec{n}} = 0 \;\; \text{on} \;\; S^+ = \{\partial\Omega \times \langle 0, T\rangle \mid \vec{v}\cdot\vec{n} \geq 0\} \;\; \text{– outflow from the domain,} \tag{1b}$$

and the initial condition

$$c(0) = c_0 \quad \text{in} \quad \Omega\,. \tag{1c}$$

Here we use the following notation:
$\Omega$ – domain considered, with the boundary $\partial\Omega = S^+ \cup S^-$,
(0,T) – time interval of the forecast,
$c$ – pollution concentration,
$\vec{v}$ – wind velocity vector,
$\vec{n}$ – normal outward vector of the domain $\Omega$,
$K_h$ – horizontal diffusion coefficient,
$\gamma$ – pollution reduction coefficient, due to deposition and chemical transformation,
$Q$ – total emission field.

The details concerning computer implementation are discussed by Holnicki et al. (2000). Here, the pollution reduction coefficient, $\gamma$ used in (1), represents in an aggregated form processes of dry and wet deposition as well as the chemical transformations. The turbulent exchange of pollutants between layers is usually parameterized by introducing the respective vertical diffusion coefficient.

The emission field on the right hand side of (1) is composed of two parts: controlled and uncontrolled (background) emission sources. It can be expressed as follows:

$$Q(x, y, t) = q(x, y, t) + \sum_{i=1}^{N} \chi_i(x, y) \cdot q_i(t)\,, \tag{2}$$

where
$q_i(t)$ – emission intensity of the controlled, *i-th* source,
$\chi_i(x, y)$ – characteristic function of the *i-th* source location,
$q(x, y, t)$ – background (uncontrolled) emission field.

Numerical algorithm is based on the discrete in time, linear finite element spatial approximation, combined with the method of characteristics (Holnicki, 1996; Holnicki et al., 2000). The uniform discretization step, $h = \Delta x = \Delta y$ is applied for space approximation. The mass balance of pollutants is calculated for air parcels following the wind field trajectories. The discrete computational points along the wind trajectory are determined according to the predefined time discretization interval – $\tau$.

## 2. The optimal control problem

Basing on the forecasts of the pollution dispersion model, the real-time emission control problem for the system of sources located in a given area can be formulated. The general idea of control consists in minimizing a predefined environmental cost function, according to the changing meteorological conditions, by modification of the emission intensity (supervised by the regional coordinating center for redistribution of the energy) within the set of the selected controlled sources. Certain economic and technological constraints are also taken into account.

To state the optimal control problem, we define the basic conditions. Assume that in a given domain $\Omega$ there are $N$ controlled emission sources described by certain spatial characteristics (location, stack height, etc.) and emission intensity. There is also a set of uncontrolled emission sources, which form the background pollution field.

**State equation**. We consider the layer-averaged concentration of the polluting factor $c(x, y, t)$, which satisfies the following transport equation

$$\frac{\partial c}{\partial t} + \vec{v}\,\nabla c - K_h \Delta c + \gamma c = q + \sum_{i=1}^{N} q_i \quad \text{in} \quad \Omega \times (0, T)\,, \tag{3}$$

with the boundary conditions (1b) and the initial condition (1c). Function $q(x, y, t)$ represents an uncontrolled emission field (background emission). Emission characteristics of the controlled sources are represented by the product

$$q_i(x, y, t) = \chi_i(x, y) \cdot F_i(u_i(t)) \quad \text{for} \quad i = 1, \ldots, N\,,$$

where $\chi_i(x, y)$ describes the spatial location of the source, while $F_i(u_i(t))$ is the temporal characteristics of emission intensity. Vector function $\vec{u} = [u_1, \ldots, u_N]^T$ denotes here the control and represents the production level (e.g. energy generated by the power plant). Functions $F_i(\cdot)$, $(i = 1, \ldots, N)$ relate the energy production level of the respective plant, to the emission intensity, which forms the right-hand side of the state equation.

**Cost functional** to be minimized is the weighted sum of two components: environmental cost function (air quality damage) and the control cost. It is defined as follows

$$J(q) = \frac{\alpha_1}{2}\int_0^T \int_\Omega w \cdot [\max(0, c(\vec{u}) - c_{ad})]^2 \, d\Omega \, dt + \frac{\alpha_2}{2}\int_0^T \sum_{i=1}^{N} \beta_i \, (u_i(t) - u_i^\star)^2 \, dt \,. \tag{4}$$

Here the coefficients $\alpha_1,\ \alpha_2,\ \beta_i\,,\ \ (i = 1, \ldots, N)$ are given (experimentally determined) constants, where $\alpha_1 \geq 0\,,\ \alpha_2 \geq 0\,,\ \beta_i > 0\,.$ The area sensitivity function satisfies the inequality $0 \leq w(x, y) \leq 1$, and $c_{ad}$ is a constant, admissible level of concentration. The nominal production level of the controlled source is denoted by $u_i^\star\,,\ (i = 1, \ldots, N)\,.$

**Constraints** imposed on the production level of the controlled emission sources represent some technological and economic requirements, and are as follows

$$\underline{u}_i \leq u_i(t) \leq \overline{u}_i(t) \quad \text{for} \quad i = 1, \ldots, N\,, \tag{5a}$$

$$\sum_{i \in N_j} \delta_{ij} \, u_i(t) \geq d_j \quad \text{for} \quad j = 1, \ldots, M, \quad N_j \subset \{1, \ldots, N\}\,. \tag{5b}$$

Inequalities (5a) are lower and upper technological limits on the real production level of the plant under consideration. Conditions (5b) represent constraints of the total energy demand, which is imposed on the $j$-th subset of plants, with some coefficients $\delta_{ij}$ .

We denote by $U_{ad} \subset L^2(0, T; R^N)$ the set of admissible controls of the form

$$U_{ad} = \{\vec{u} \in L^2(0, T; R^N) \mid \vec{u} \text{ satisfies conditions (5) a.e. in (0,T)}\}\,. \tag{6}$$

It is known (Lions and Magenes, 1968) that the state equation (3) has a unique solution $c = c(\vec{u})$, determined for a given control $\vec{u} \in U_{ad}$ and for fixed, constant parameters $K_h,\ \gamma$, where $K_h > 0\,.$

**Optimal control problem (P)**. Find the element $\vec{u}^{\,o}$ which minimizes the cost functional (4) over the set of admissible controls,

$$J(c(\vec{u}^{\,o})) = \inf_{\vec{u} \in U_{ad}} J(\vec{u})\,,$$

where $c^o = c(\vec{u}^{\,o})$ satisfies the state equation (3).

We assume that there exists a constant $\sigma > 0$ such that the following inequality holds

$$\langle\, DJ(\vec{u}) - DJ(\vec{v}), \vec{u} - \vec{v}\,\rangle > \sigma \, ||\vec{u} - \vec{v}||^2_{H^1(0,T;R^N)} \quad \forall \vec{u}, \vec{v} \in U_{ad}\,, \tag{7}$$

where $DJ(\vec{u})$ denotes the gradient of the functional (4). The last inequality is satisfied, e.g. for $F_i(u_i) = u_i$ and $\alpha_i > 0,\, \beta_i > 0,\, \delta > 0\,.$ According to Lions

(1971) or Martchuk (1995), condition (7) ensures the uniqueness of the optimal solution to Problem (P).

**The optimality system for Problem (P)**. It is known (Lions, 1971) that Problem (P) can be characterized as follows. Find $(\vec{u}^o, c^o, p^o)$, where $\vec{u}^o = [u_1^o, \dots, u_N^o] \in U_{ad}$, such that

$$\frac{\partial c^o}{\partial t} + \vec{v}\,\nabla c^o - K_h \Delta c + \gamma c^o = q + \sum_{i=1}^{N} \chi_i\, F_i(u_i^o) \quad \text{in} \quad \Omega \times (0,T)\,, \tag{8}$$

$$c^o = c_b^o \quad \text{on} \quad S^- \tag{8a}$$

$$K_h\, \frac{\partial c^o}{\partial \vec{n}} = 0 \quad \text{on} \quad S^+ \tag{8b}$$

$$c^o(0) = c_0^o \quad \text{in} \quad \Omega\,. \tag{8c}$$

$$-\frac{\partial p^o}{\partial t} - \vec{v}\,\nabla p^o - K_h \Delta c + \gamma p^o = \alpha_1\, w \max[0, c^o - c_{ad}] \quad \text{in} \quad \Omega \times (0,T)\,, \tag{9}$$

$$p^o = 0 \quad \text{on} \quad S^- \tag{9a}$$

$$K_h\, \frac{\partial p^o}{\partial \vec{n}} + \vec{v} \cdot \vec{n}\, p^o = 0 \quad \text{on} \quad S^+ \tag{9b}$$

$$p^o(T) = 0 \quad \text{in} \quad \Omega\,. \tag{9c}$$

$$\begin{aligned} &\sum_{i=1}^{N} \{\alpha_1 \int_0^T \int_\Omega \chi_i\, F_i^{'}(u_i^o)\, p^o\, (u_i - u_i^o)\, d\Omega\, dt \\ &\qquad + \alpha_2 \int_0^T \beta_i\, (u_i^o - u_i^\star)\, (u_i - u_i^o)\, dt\} \geq 0 \quad \forall\, \vec{u} = [u_1, \dots, u_N] \in U_{ad}\,. \end{aligned} \tag{10}$$

Finite dimensional approximation of the problem (P) can be numerically solved by any gradient method. It follows from the above optimality conditions (Lions, 1971) that the gradient of the cost functional (4) depends on the adjoint variable $p^o$ and can be expressed as follows

$$\begin{aligned} D_i J(\vec{u}) = {} & \alpha_1 \int_0^T \int_\Omega \chi_i\, F_i^{'}(u_i^o)\, p^o\, d\Omega\, dt \\ & + \alpha_2 \int_0^T \beta_i\, (u_i^o - u_i^\star)\, dt \qquad (i = 1, \dots, N). \end{aligned} \tag{11}$$

To calculate the gradient components according to (11), the following steps have to be performed in the consecutive iterations of the optimization procedure:

- solve the transport equation – problem (8),
- solve the adjoint equation – problem (9) – for the reversed time and the wind direction,
- substitute the adjoint variable $p^o$ to (11) and calculate gradient components of environmental cost functional (4).

The next section presents the results of test computations performed for the real-data case study. The computational domain is a selected industrial region with the set of the major power plants, considered as the controlled emission sources.

## 3. The real-data case study

The general approach presented in Section 2 has been implemented and tested in a real data case. The test calculations have been performed for the set of the major power plants of the selected industrial region of Poland. To formally state the optimal control problem, which is to be solved, certain simplifications have been introduced in the general formulations discussed above.

We assume that the set of admissible controls $U_{ad}$ is given by

$$U_{ad} = \{\vec{u} \in L^2(0,T;R^N) \mid \vec{u}(t) \text{ satisfies (5a–b)} \quad \text{for a. a.} \quad t \in (0,T)\}, \tag{12}$$

where condition (5b) has a form of a total energy demand

$$\sum_{i \in N} \delta_i \, u_i(t) \geq d\,. \tag{13}$$

Furthermore, we assume for simplicity that the function, which relates emission to production level, $F_i$ in (3), is identity

$$F_i(u_i) = u_i\,, \quad i = 1,\ldots,N\,. \tag{14}$$

The above relation means that we directly consider emission intensity of the source as the controlling function. The cost functional $J(\vec{u})$ is defined by (4) for $\vec{u} \in L^2(0,T;R^N)$. The weight coefficients $\alpha_1$, $\alpha_2 > 0$ and $\beta_i = \beta = 1$ for $i = 1,\ldots,N$, are experimentally established, to obtain the comparable contribution of two basic components in (4).

The test calculations have been performed for the selected region of Upper Silesia (Poland) and the set of 27 power plants, considered as the controlled emission sources. They represent the dominating power stations located within the region. Fig. 1 presents the domain considered and the location of the controlled emission sources.

The nominal emission intensity of the controlled sources, taken into account in the optimization procedure, refer to the winter season values. Grid coordinates and the main emission parameters of the controlled sources are shown in Table 1.

Table 1. List of the controlled $SO_2$ emission sources

| No. | Source | Source coordinates | Stack height [m] | Emission – winter [kg/h] | Emission – summer [kg/h] |
|---|---|---|---|---|---|
| 1. | Bielsko-Biala | (14,2) | 160 | 426.91 | 256.15 |
| 2. | Będzin A | (18,31) | 95 | 94.89 | 63.25 |
| 3. | Będzin B | (18,31) | 135 | 132.82 | 31.63 |
| 4. | Bielsko-Komorowice | (15,1) | 250 | 426.91 | 189.74 |
| 5. | Chorzów | (12,27) | 100 | 363.66 | 180.25 |
| 6. | Halemba | (8,25) | 110 | 569.24 | 379.48 |
| 7. | Jaworzno I | (20,23) | 152 | 284.61 | 158.12 |
| 8. | Jaworzno IIA | (21,24) | 100 | 573.60 | 379.48 |
| 9. | Jaworzno IIB | (21,24) | 120 | 664.08 | 426.91 |
| 10. | Jaworzno III | (21,24) | 300 | 6324.60 | 4743.45 |
| 11. | Katowice | (13,25) | 250 | 1106.81 | 790.58 |
| 12. | Łagisza A | (18,31) | 160 | 948.69 | 695.71 |
| 13. | Łagisza B | (18,31) | 200 | 1359.79 | 1011.94 |
| 14. | Łaziska I | (8,20) | 200 | 1660.21 | 1185.86 |
| 15. | Łaziska II | (8,20) | 160 | 758.95 | 505.97 |
| 16. | Łaziska III | (8,20) | 100 | 727.95 | 505.97 |
| 17. | Łęg | (46,12) | 260 | 1106.81 | 790.58 |
| 18. | Miechowice | (14,17) | 68 | 161.28 | 117.01 |
| 19. | Rybnik | (1,20) | 300 | 4711.83 | 3510.15 |
| 20. | Siersza A | (30,23) | 150 | 1929.00 | 1423.04 |
| 21. | Siersza B | (30,23) | 260 | 2055.49 | 1739.27 |
| 22. | Skawina | (43,11) | 120 | 1992.25 | 1296.55 |
| 23. | Szombierki A | (9,31) | 110 | 164.44 | 113.84 |
| 24. | Szombierki B | (9,31) | 120 | 170.76 | 110.68 |
| 25. | Tychy | (13,19) | 120 | 240.33 | 177.09 |
| 26. | Zabrze A | (2,29) | 60 | 205.55 | 158.12 |
| 27. | Zabrze B | (2,29) | 120 | 221.36 | 145.47 |

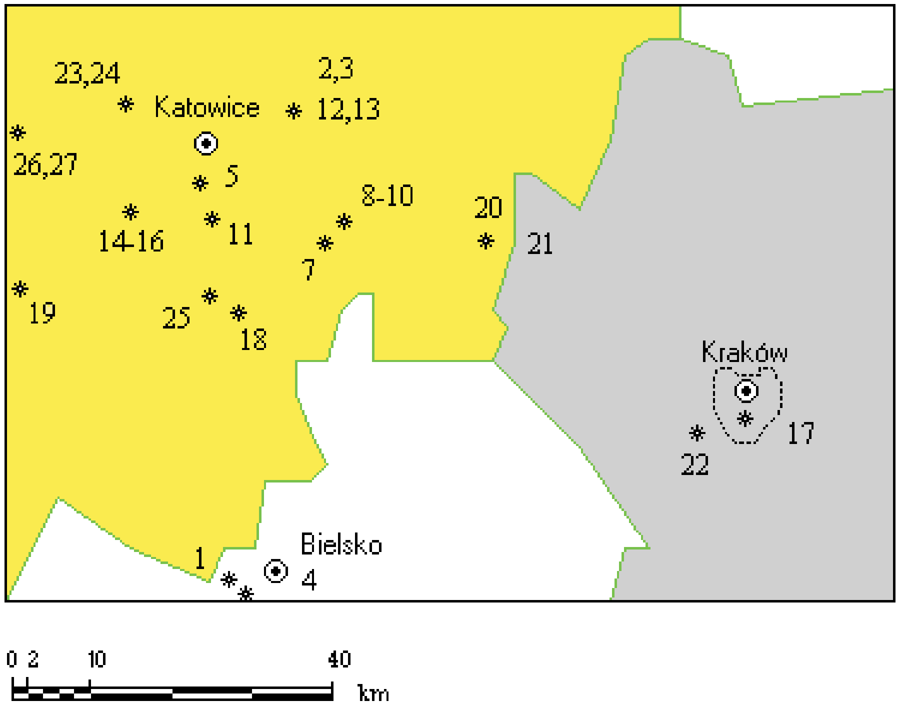


Figure 1. The computational domain and the emission sources

The computational domain constitutes a rectangle of 110 km×74 km, which was discretized with the homogeneous grid, for the space discretization step, $h =$ 2 km. This gives the discrete domain of dimensions $55 \times 37$. The area weight coefficient, $w(x, y)$, in the objective function (4) was defined in a specific way, to get a more illustrative form of the optimal solution and to obtain relatively simple interpretation of the adjoint variable, $p^o$. The surroundings of Kraków (indicated by the dashed line in Fig. 1) was defined as a region of high sensitivity, with the respectively higher value of the area weight index. It is defined as follows:

$$w(x, y) = \begin{cases} 1 & \text{for } (x, y) \text{ in Kraków area,} \\ 0 & \text{for } (x, y) \text{ outside this area.} \end{cases} \tag{15}$$

Computational results presented in the sequel relate to the real-time emission control task for one 12-h time interval and two selected meteorological scenarios. The meteorological data for two episodes differ with respect to the wind direction (with other parameters comparable), and are as follows:

- **case A** – the North-West moderate wind,
- **case B** – the West moderate wind.

Numerical implementation of the optimal control problem (P) discussed in Section 2 is based on the linearization method by Pschenitchny (1983). The FORTRAN 90 code of the optimization algorithm includes the forecasting model, the adjoint equation simulator and the optimization procedure. The computational experiments were performed on the UNIX platform server. The Winter season emission intensities of the controlled sources, as stated in Table 1, were assumed as the starting point for the optimization procedure. Computing time required to find the solution, in both scenarios considered, is below 1 minute. Some general results, concerning performance, number of iterations of the optimization procedure and the reduction factor of the quality function, are presented in Table 2.

Table 2. General results of 12-h simulation for two control tasks

| Case | Number of iterations | Quality index – initial | Quality index – final | Reduction factor |
|---|---|---|---|---|
| A | 4 | 82.2 | 70.0 | 15% |
| B | 3 | 10.5 | 7.2 | 31% |

Graphical presentation of the results for scenario A is shown in Figs. 2 – 3. Fig. 2 indicates the differences in the distribution of $SO_2$ concentration for the reference emission (no control) field and for the optimal control strategy, generated by the algorithm. The most noticeable differences can be observed in the surroundings of the protected area, according to definition (15), while the the reduction of the objective function is about 15%.

The correlation between the adjoint variable distribution and the dominating controlled sources are seen in Fig. 3(a). This figure also gives an interpretation of the adjoint variable. The area of high values of the adjoint variable indicates locations of those sources, which significantly contribute to the overall environmental cost function. These sources have the emissions respectively reduced, as the result of the optimization algorithm; the related changes in emission intensities are shown in Fig. 3(b). On the other hand, to satisfy the total demand constraints (5), production level (and emission) in some sources must increase. These are the sources located outside the area of high influence, which do not contribute significantly to environmental cost (see Fig. 3a–b). The respective graphical results for scenario B are shown in Figs. 4–5.

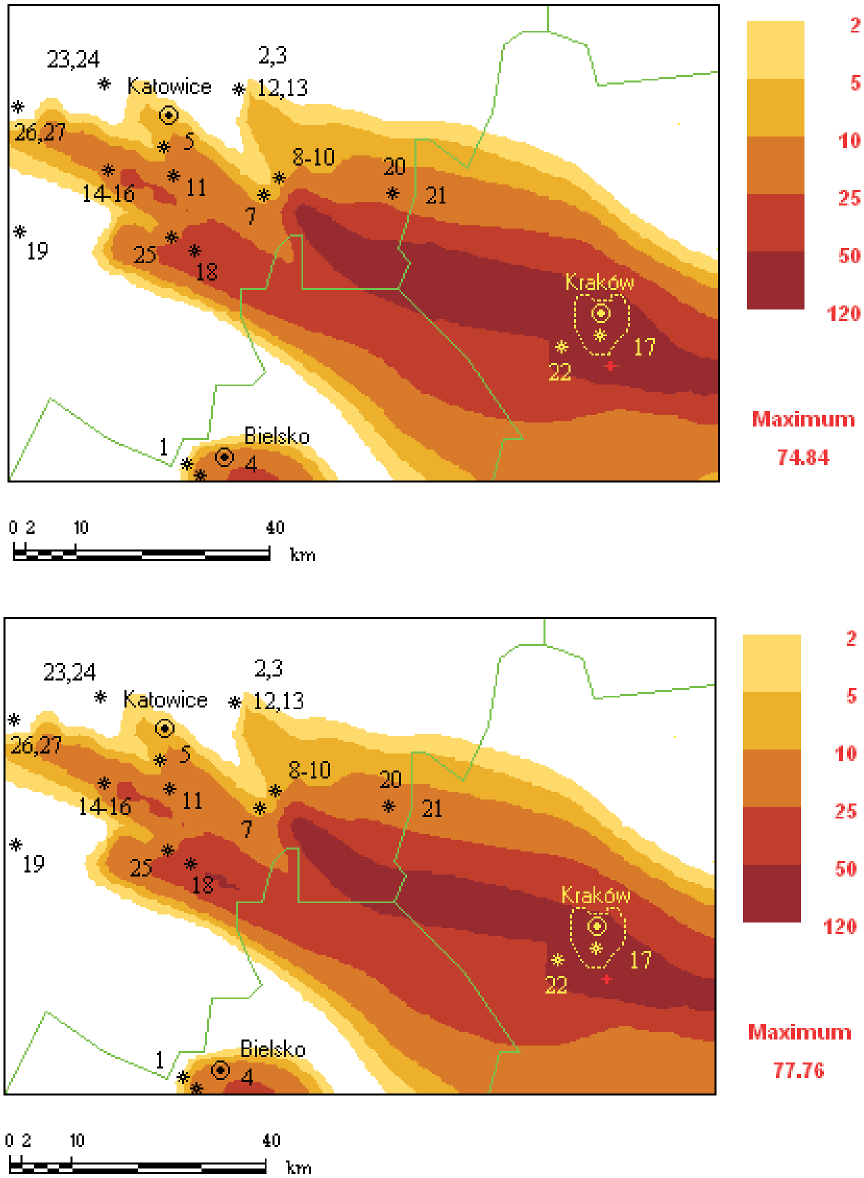


Figure 2. Map of $SO_2$ concentration [$\mu$g/m$^3$]; initial (top) and optimal (bottom) – scenario A

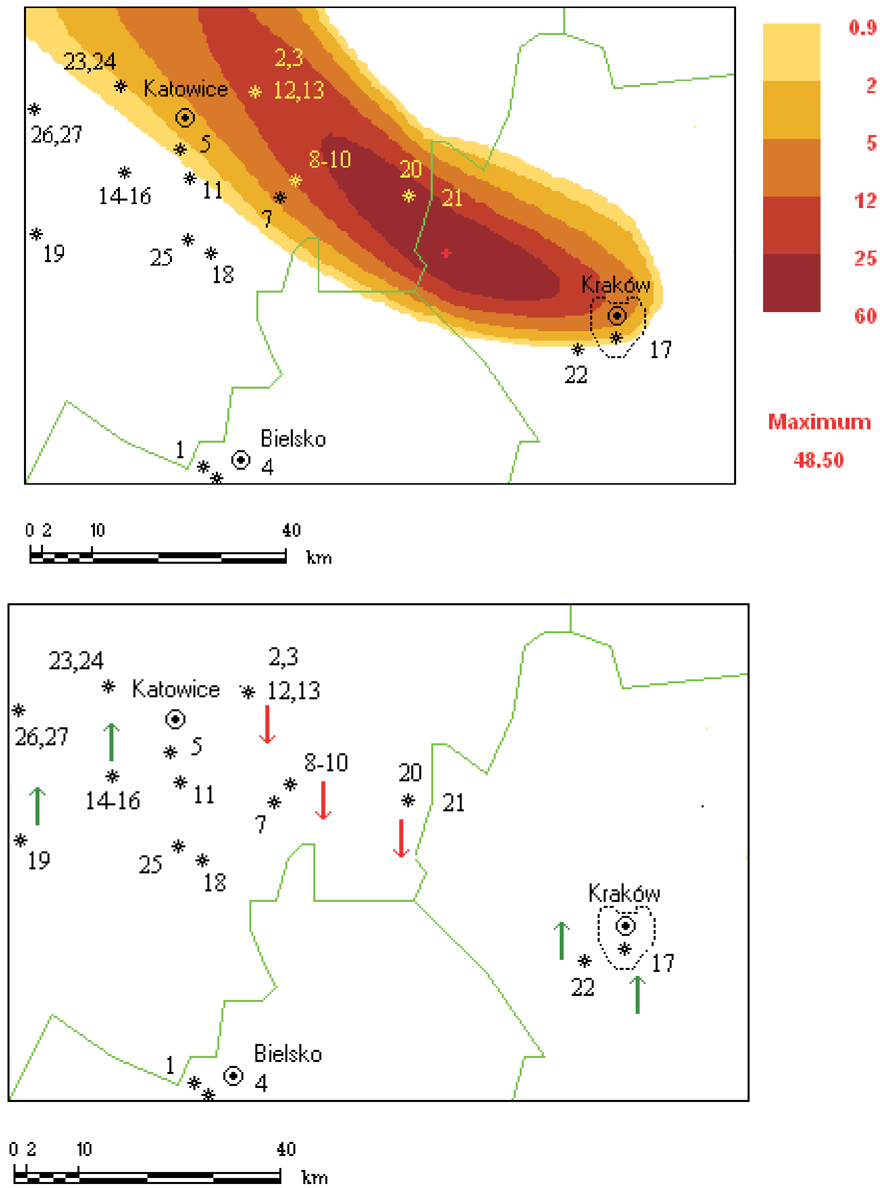


Figure 3. Distribution of adjoint variable (top); modification of emissions in controlled sources (bottom) – scenario A

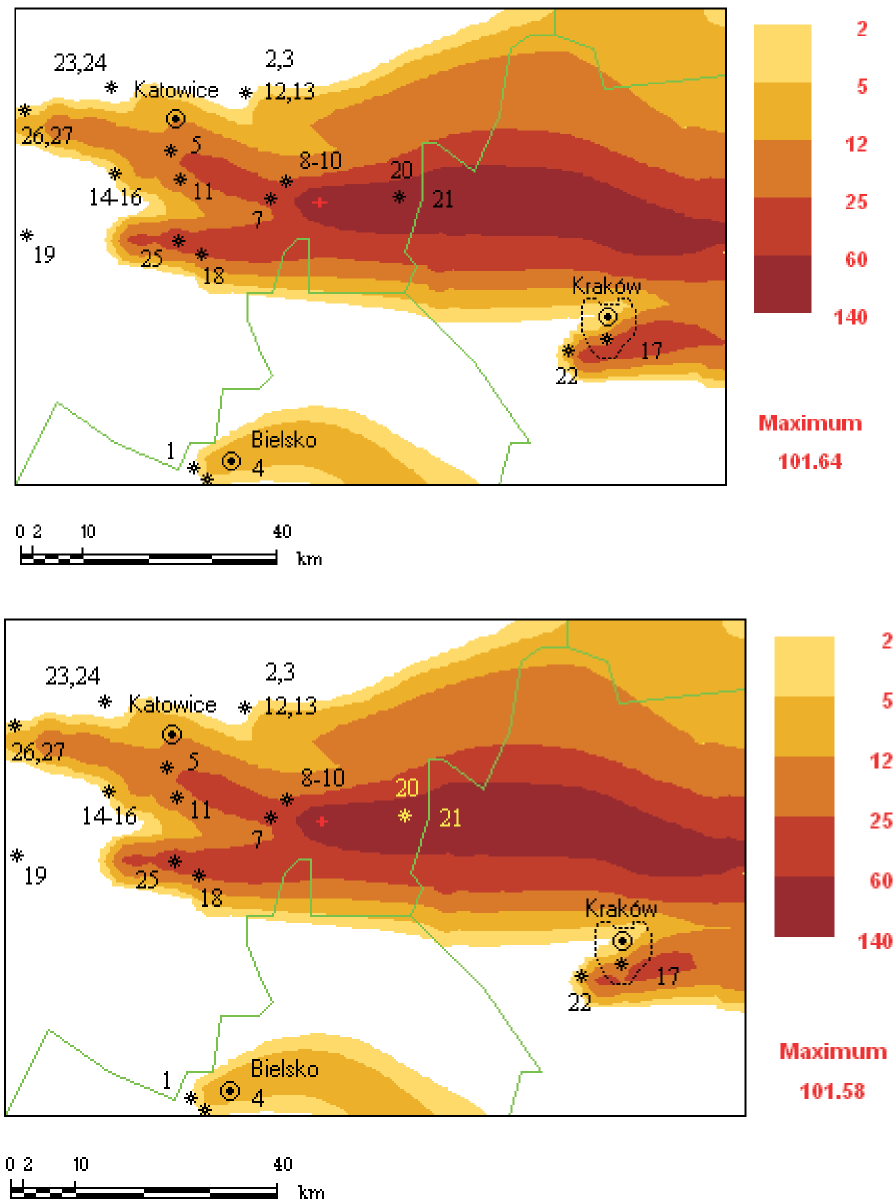


Figure 4. Map of $SO_2$ concentration [$\mu$g/m$^3$]; initial (top) and optimal (bottom) – scenario B

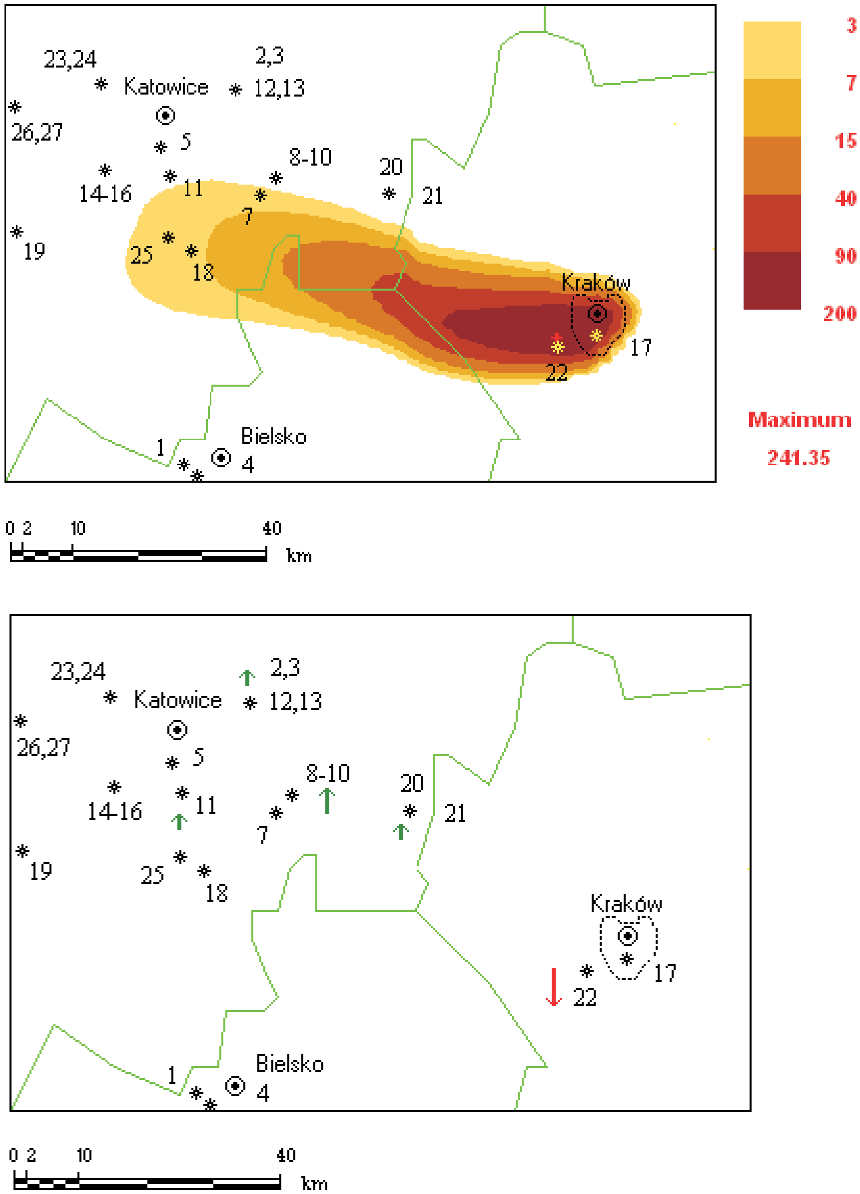


Figure 5. Distribution of adjoint variable (top); modification of emissions in controlled sources (bottom) – scenario B

The quantitative results of the optimal control procedure for both meteorological scenarios, related to the final modifications of the emission level in controlled sources – are shown in Table 3. They are expressed as the factors of relative change of the initial (or nominal) emission intensity of the selected sources (factor less than 1.0 means reduction of the emission, factor greater then 1.0 – increase, respectively).

Table 3. Optimal emission control – modifications of the controlled sources

| No. | Source | coordinates | height [ m ] | Emission [kg/h] | control factor case A | control factor case B |
|---|---|---|---|---|---|---|
| 1. | Bielsko-Biala | (14,2) | 160 | 426.91 | 1.00 | 1.00 |
| 2. | Będzin A | (18,31) | 95 | 94.89 | 1.00 | 1.00 |
| 3. | Będzin B | (18,31) | 135 | 132.82 | 1.00 | 1.00 |
| 4. | Bielsko-Komorowice | (15,1) | 250 | 426.91 | 1.00 | 1.00 |
| 5. | Chorzów | (12,27) | 100 | 363.66 | 1.00 | 1.00 |
| 6. | Halemba | (8,25) | 110 | 569.24 | 1.00 | 1.00 |
| 7. | Jaworzno I | (20,23) | 152 | 284.61 | 1.00 | 1.00 |
| 8. | Jaworzno IIA | (21,24) | 100 | 573.60 | 1.00 | 1.00 |
| 9. | Jaworzno IIB | (21,24) | 120 | 664.08 | 0.80 | 1.00 |
| 10. | Jaworzno III | (21,24) | 300 | 6324.60 | 0.80 | 1.04 |
| 11. | Katowice | (13,25) | 250 | 1106.81 | 1.10 | 1.01 |
| 12. | Łagisza A | (18,31) | 160 | 948.69 | 1.00 | 1.01 |
| 13. | Łagisza B | (18,31) | 200 | 1359.79 | 0.90 | 1.01 |
| 14. | Łaziska I | (8,20) | 200 | 1660.21 | 1.10 | 1.00 |
| 15. | Łaziska II | (8,20) | 160 | 758.95 | 1.00 | 1.00 |
| 16. | Łaziska III | (8,20) | 100 | 727.95 | 1.00 | 1.00 |
| 17. | Łęg | (46,12) | 260 | 1106.81 | 1.10 | 1.00 |
| 18. | Miechowice | (14,17) | 68 | 161.28 | 1.00 | 1.00 |
| 19. | Rybnik | (1,20) | 300 | 4711.83 | 1.25 | 1.00 |
| 20. | Siersza A | (30,23) | 150 | 1929.00 | 0.80 | 1.02 |
| 21. | Siersza B | (30,23) | 260 | 2055.49 | 0.80 | 1.02 |
| 22. | Skawina | (43,11) | 120 | 1992.25 | 1.10 | 0.82 |
| 23. | Szombierki A | (9,31) | 110 | 164.44 | 1.00 | 1.00 |
| 24. | Szombierki B | (9,31) | 120 | 170.76 | 1.00 | 1.00 |
| 25. | Tychy | (13,19) | 120 | 240.33 | 1.00 | 1.00 |
| 26. | Zabrze A | (2,29) | 60 | 205.55 | 1.00 | 1.00 |
| 27. | Zabrze B | (2,29) | 120 | 221.36 | 1.00 | 1.00 |

The initial value and reduction of environmental cost index significantly depend on the meteorological situation (see Table 2). Several sources in scenario A contribute to the quality function and the initial value of this index is high, while the reduction, which can be obtained for the optimal solution, is about 15%. The other scenario means the dominating contribution of one source (No 22) only. Thus, the initial value of quality index is much lower, but the relative reduction of this initial state – respectively higher, because only the abatement of emission level in this dominating source is selected in the optimal solution.

## 4. Conclusions

The paper concentrates on two basic tasks: i) formulation of the optimal control problem, which represents the real-time emission control of air pollution sources, and ii) testing the efficiency and accuracy of an implementation of the computational algorithm designed for solving the problem. As stated in Section 2, the emission field of the controlled sources and air pollution dispersion processes are considered as a distributed parameter system, which is governed by the respective set of transport equations. Consequently, the optimal control technique for distributed parameter systems (Lions, 1971) is utilized in definition of the real-time emission control problem.

The key module of the system is the numerical model of air pollution transport, and the quality of the respective finite-dimensional approximation scheme applied for solving the state and adjoint equations constitutes the basic problem. Both equations must be solved, at least once, in every iteration of any gradient optimization algorithm. Numerical solution of this type evolutionary equations is especially sensitive to the properties of the numerical scheme applied. It is known that the crucial role in the final accuracy of the method is played by monotonicity and positivity of approximation method, as discussed by Holnicki (1995, 1996). These properties are particularly important in an optimization process, since the solution of the state equation is entered as an input of the adjoint equation. For this reason, an effective, shape preserving scheme, based on a combination of the method of characteristics and the piecewise-quintic spatial interpolation (Holnicki, 1996), is used for simulation of air pollution transport.

The above numerical algorithm was used in analysis of the state and adjoint equations. Since in the application considered in the paper, the emission field is composed of the pointwise sources – the case is especially sensitive to shape-preserving properties of the numerical approximation scheme. The test computations, performed for two selected episodes, confirm good accuracy of the solution to transport equation as well as satisfactory integration of the method with the optimization algorithm. The obtained results also show that the method is computationally effective (the optimum reached in a few iterations – see Table 2) and the resulting accuracy of the optimal solution is sufficient, having in perspective future applications.

The utilization of the techniques discussed in the paper concentrates on the problem of the real-time emission control. Presented results show that some elements of this approach can also be applied in long-term analysis of regional scale environmental tasks, e.g. in sustainable development problems, as discussed by Chang (2000) or Haurie et al. (2004). The remark refers to the adjoint variable, which indicates the most influencing area, from the environmental perspective. Thus, in long-term analysis, distribution of this variable can also be an important factor in supporting decisions concerning the planned energy sector investments and their location within the region.

## References


Ciechanowicz, W., Holnicki, P., Kałuszko, A., Partyka, A., Uhrynowski, Z. and Żochowski, A. (1996) Energy and environment – problems of sustainable development. *Control and Cybernetics* **25**, 1261–1271.

Chang, M.E. (2000) New Directions: Sustainability in strategic air quality planning. *Atmosheric Environment* **34**, 2495 – 2496.

Haurie, A., Kubler, J., Clappier, A. and van den Bergh, H. (2004) A metamodeling approach for integrated assessment of air quality policies. *Environment Modeling and Assessment* **9**, 1–12.

Holnicki, P. (1995) A shape preserving interpolation: applications to semi-Lagrangian advection. *Monthly Weather Review* **123**, 862 – 870.

Holnicki, P. (1996) A Piecewise-Quintic Interpolation Scheme. *Journal of Computational Physics* **127**, 316 – 329.

Holnicki, P., Nahorski, Z. and Żochowski, A. (2000) *Modeling of Environmental Processes* (Polish). WSISiZ Publishers, Warsaw.

Holnicki, P. and Kałuszko, A. (2004) Decision support system for optimal emission reduction. *Management of Environmental Quality* **15**, 250 – 257.

Holnicki, P. (2004) Evaluation of Environmental Impact of Air Pollution Sources. *Cybernetics and Systems* **35**, 595 – 606.

Lions, J.L. (1971) *Optimal Control of Systems Governed by Partial Differential Equations*. Springer, New York.

Lions, J.L. and Magenes, E. (1968) *Problèmes aux limites non homogènes*. Dunod, Paris 1968.

Martchuk, G.I. (1995) . *Adjoint Equations and Analysis of Complex Systems*. Kluwer Academic Publishers, Dordrecht.

Pschenitchny, B.N. (1983) *Method of Linearization* (Russian). Nauka, Moscow.

Trojanowski, K. and Michalewicz, Z. (2000) Evolutionary optimization in non-stationary environments. *Journal of Computer Sciences and Technology* **1**, 93 – 124.