Control and Cybernetics

vol. **35** (2006) No. 2

Book review:

VARIATIONAL METHODS IN SHAPE OPTIMIZATION PROBLEMS

by

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Shape optimization problem is a minimization problem of the form

$$\min\{F(A) \; : \; A \in \mathcal{A}\},$$

where $\mathcal{A}$ is a class of admissible domains in R^N, $N = 1, 2, 3$, and F denotes a cost functional. Usually $F(A) = F(u(A))$ where u is a solution to an ordinary or partial differential state equation in the domain A. The shape of the domain A is a control variable subject to optimization. Numerous theoretical investigations and numerical experiments indicate that unless some geometrical constraints are imposed on the admissible sets or some very special cases of the cost functionals are considered the existence of an optimal domain may fail. To ensure the existence of solutions to shape optimization problems different relaxation techniques are applied.

The book deals with the existence of solutions to shape optimization problems for systems governed by elliptic boundary value problems. In seven chapters the main assumptions and techniques ensuring the existence of solutions to these optimization problems are covered.

Chapter 1 recalls a few relevant examples of the shape optimization problems. These examples include: the isoperimetric problem, the Newton problem of optimal aerodynamical profiles, the optimal distribution of two different media in a given region, optimal shape design problem of a thin insulator around a given conductor. The existence of optimal solutions to these problems is shown.

Chapter 2 concerns shape optimization problems whose class of admissible domains is made of convex sets. The problems with the cost functionals equal either to the integral functionals on the fixed convex domain containing all admissible sets or the boundary integral functionals are considered. In both cases the convexity assumption provides an extra compactness which implies the existence of an optimal domain under very mild conditions on the integrand function.

In Chapter 3 the shape optimization problems are considered as a class of general optimal control problems where an admissible shape plays the role of an admissible control and the corresponding state variable is the solution of a partial differential equation on the control domain. The relaxation theory is recalled. This theory provides a general way to construct relaxed solutions through Γ - convergence methods. This approach is based on suitable enlarging the set of admissible controls and constructing the larger compact set of the relaxed controls.

This relaxation method is employed in Chapter 4 to show the existence of solutions to shape optimization problems for Laplace equation with Dirichlet condition on the free boundary. These solutions belong to a class of nonnegative Borel measures. Moreover, necessary optimality conditions are provided. The continuity results of the solutions to the state equations with respect to the domain perturbation in Hausdorff complementary topology are provided.

Chapter 5 deals with the shape optimization problems where suitable geometrical constraints are imposed on the admissible domains or the cost functional is monotonic. Under these assumptions the existence of classical, i.e., unrelaxed solutions to shape optimization problems, is shown. The general approach presented in Chapter 5 is applied in the next Chapter 6 to show the existence of solutions to very special class of shape optimization problems where the cost functional depends on the eigenvalues of an elliptic operator with Dirichlet condition on the free boundary. Assuming the lower semicontinuity and monotonic increasing of the eigenvalue dependent cost functional as well as the constrained volume and quasi - openess of the admissible sets the existence of solutions to these optimization problems is shown. Moreover, the extension of the existence result for problems with unbounded design domains or nonmonotonic cost functionals is discussed.

The existence of solutions to the shape optimization problems for systems governed by Laplace equation with the homogeneous Neumann boundary condition is discussed in Chapter 7. The boundary variations and the shape stability results for Neumann problems are recalled. The existence of solutions to the optimal cutting problem in a membrane is shown. The continuity of the eigenvalues of the Laplace equation with Neumann boundary condition with respect to the nonsmooth variations of the boundary of the geometrical domain is investigated. The conditions ensuring the existence of solutions to the shape optimization problems of the eigenvalues of the Laplace equation with Neumann boundary condition are discussed.

The list of references consists of 197 items and contains both the classical and the recent monographs and papers dealing with the subject.

The book summarizes the most important results concerning the existence of solutions to the shape optimization problems. The known existence results for smooth admissible sets and perturbations are generalized and extended to nonsmooth admissible sets and perturbations.

Since the book is based on the lecture notes from two courses addressed to Ph.D. students the style of the book remains informal and follows the lectures as given. The reader is assumed to know standard background of functional analysis and of function spaces such as Sobolev or bounded variations spaces.

The book can serve as a text for graduate or Ph.D. students in pure and applied mathematics, for applied mathematicians interested in functional shape optimization problems and working in the field of computational methods of optimal design and optimization as well as for engineers requiring a solid mathematical foundation for the solution of practical problems.

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Dorin Bucur and Giuseppe Butazzo, *Variational Methods in Shape Optimization Problems.* Birkhäuser Verlag AG, Basel-Boston-Berlin 2005, 216 pages, ISBN 0-8176-4359-1, EUR 62.06 (hardcover).