Control and Cybernetics

vol. **35** (2006) No. 1

A qualitative trajectory calculus as a basis for representing moving objects in Geographical Information Systems

by

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Abstract: Qualitative formalisms, suited to express qualitative temporal or spatial relationships between entities, have gained wide acceptance as a useful way of abstracting from the real world. The question remains how to describe spatio-temporal concepts, such as the interaction between disconnected moving objects, adequately within a qualitative calculus and more specifically how to use this in geographical information systems. In this paper, the Basic Qualitative Trajectory Calculus (QTC_B) for representing and reasoning about moving objects is presented. QTC_B enables comparisons between positions of objects at different time points to be made. The calculus is based on few primitives (i.e., distance and speed constraints), making it elegant and theoretically simple. To clarify the way in which trajectories are represented within QTC_B, specific cases of movements (e.g. circular movement) are presented. To illustrate the naturalness of QTC, a "predator-prey" example is studied.

Keywords: moving objects, qualitative reasoning, qualitative representation, spatio-temporal modelling.

1. Introduction

For centuries, geographers have developed skills to store, present and analyse spatial and spatio-temporal information. They have developed their ways of studying interacting objects and phenomena defining the real world. It is therefore not surprising that geography is still of vital importance to the study of geographic information systems. A GISystem "*is a powerful set of tools for collecting, storing, retrieving at will, transforming and displaying spatial data from the real world for a particular set of purposes*" (Burrough, McDonnell, 1998). GISystems form the interface between the spatio-temporal real world and the way geographers and many other researchers and users interact with this reality. The research aiming to enhance GISystems is called Geographic Information Science. GIScience was introduced at beginning of the 1990s by Goodchild (Goodchild, 1992). The definition of GIScience is based on the term information science, which can be defined as "*the study according to scientific principles of the nature and properties of information*" (Goodchild et al., 1999, p.737). GIScience is "*the subset of information science that is about geographic information*" (Goodchild et al., 1999, p.737).

Humans usually prefer to communicate in qualitative categories supporting their intuition and not in quantitative categories, as exemplified below:

- "We hear thunder after we see lightning", and not: "given the speed of light and sound, and the distance between the clouds and our current position, we hear the thunder 2.24 seconds after we have seen the lightning".
- "The first car is moving faster than the second one", and not: "the first car is moving at a speed of 119 kilometres per hour and the second car at a speed of 116 kilometres per hour".

Therefore, qualitative relations are essential components of queries that people would like to run on a GISystem. Important work in the area of qualitative reasoning has been done. Temporal calculi, such as the Interval Calculus (Allen, 1983) and the Semi Interval Calculus (Freksa, 1992), have been proposed. In the domain of spatial reasoning, Randell, Cui and Cohn (1992) proposed the Region Connection Calculus (*RCC*) focusing on topological relationships between regions (see Fig. 1) – topological relationships being the geometric relations between spatial objects that are invariant under homeomorphisms such as translation, rotation and scaling (Bennett, 1997; Vieu, 1997).

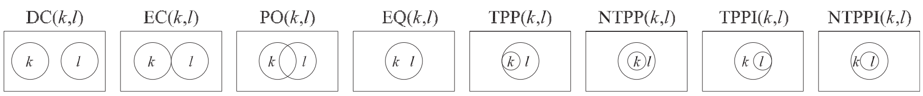

Figure 1. Relations in *RCC* (Randell, Cui and Cohn, 1992).

Simultaneously, within the research field of spatial databases, Egenhofer and Franzosa (1991) defined essentially the same set of relations in their 9 Intersection Model. These topological relationships, having a central place in GIScience, form the basis of most GISystems. The practical relevance of investigations of qualitative spatial relationships has been studied by the GIS community (Frank, Campari, 1993).

Notwithstanding the fact that motion is a key notion in our understanding of spatio-temporal relations, the question remains of how to describe motion, and more specifically the interaction between moving objects, adequately within a qualitative calculus. As a direct consequence, GISystems and GIScience do not cover the full range of space and time yet. Apart from some limiting cases when two objects actually touch, such as a car accident and a lion catching a zebra, mobile objects in a mereotopological calculus are represented by the use of the *RCC* relation *disconnected from (DC)* (see Fig. 1). However, this approach ignores some important aspects of reasoning about continuously moving physical objects. For example, given two trains on a railroad, it is of the utmost importance to know their movement with respect to each other, in order to detect whether or not they could crash in the near future. Thus, the problem with the Region Connection Calculus is that it puts all *DC*-relations into one undifferentiated set. One can also add other kinds of information, e.g. using an orientation relation calculus (such as in Isli and Cohn, 2000; Frank, 1996; Ligozat, 1998), but in all these systems the dynamics of the situation is not explicitly represented, except indirectly through the use of a *continuity network*, often called a *conceptual neighbourhood* (Cohn, Hazarika, 2001).

Another possibility is to represent and reason about the distance between objects. Several calculi in the field of qualitative reasoning could be used for this purpose, e.g. in Weld and Kleer (1990), the distance between objects can be demarcated by a finite set of intermediate landmark values and the transitions between these landmarks reasoned about. It is also possible to use these calculi to reason about the first derivative of these values to give a calculus of relative velocity, or acceleration if one uses a second derivative. However, no attempt was made in this previous work to derive a set of jointly exhaustive and pairwise disjoint relationships for a pair of objects, or to combine distance with orientation notions[1]. Therefore, a challenging question remains largely unaddressed: "how do we handle changes between moving objects, if there is no change in the topological relationship?"

With this in mind, the Qualitative Trajectory Calculus (*QTC*) is presented, being a theory for representing and reasoning about movements of objects in a qualitative framework, differentiating groups of disconnected objects. Depending on the level of detail and the number of spatial dimensions, different types of *QTC* are defined and studied in detail in Van de Weghe (2004), all belonging to *QTC* - Basic (QTC_B) or *QTC* - Double Cross (QTC_C). In Van de Weghe et al. (2005), QTC_C is presented and its usefulness is illustrated by means of an example: the overtake event. In this paper, we present the Qualitative Trajectory Calculus - Basis (QTC_B). For a description and an illustration of how *QTC* can be extended to movements along (road) networks, we refer to Van de Weghe (2004), Bogaert, Van de Weghe and De Maeyer (2004).

[1] One exception which combines orientation with a limited notion of distance is the double-cross calculus (Zimmermann, Freksa, 1996.)

Just as a theory about topological relations is at the basis of every GISystem, a qualitative theory concerning moving objects would usefully be at the basis of a temporal GISystem. Additionally, it is clear that continuously moving objects are prevalent in many domains. We especially think of recent applications that emerge from new technologies such as location-based services (mobile phones, GPS) that produce huge collections of spatio-temporal data.

The paper is organised as follows. After this introduction, a brief overview of the fields examined in this work is given in Section 2. Section 3 presents the Qualitative Trajectory Calculus - Basic (QTC_B), being a qualitative spatio temporal representation for moving objects based on describing their trajectories with respect to each other. To clarify the way in which trajectories are represented within QTC_B, Section 4 considers some particularly interesting cases of movements. Section 5 illustrates the applicability and naturalness of QTC_B on an example situation. Finally, Section 6 concludes the paper with a discussion of the major results and an outline of further research.

2. Qualitative representation and reasoning about space and time

Reasoning can be performed on quantitative as well as on qualitative information. Typically when working with quantitative information, a predefined unit of a quantity is used (Goyal, 2000). For example, one could say that the distance between Ghent and Brussels is 55 kilometres. In the qualitative approach, continuous information is qualitatively discretised by landmarks separating neighbouring open intervals, resulting in discrete quantity spaces (Weld, de Kleer, 1990). The major idea in the qualitative approach is that a distinction is introduced only if it is relevant to the current context (Cohn, 1996; Clementini, 1997). Thus, qualitative reasoning only studies the essence of information, represented as a small set of symbols such as the quantity space $\{-, 0, +\}$ consisting of the landmark value 0 and its neighbouring open intervals $]-\infty, 0[$ represented by the symbol $-$ and $]0, \infty[$ represented by the symbol $+$. For example, if one does not know the precise speed of a car and a bicycle, but knows that the speed of the car is higher than the speed of the bicycle, one can label this with the qualitative value $+$, meaning that the car is moving faster than the bicycle. One could also say that the bicycle is moving slower than the car, by giving the qualitative value $-$ to this converse relation. Finally, both objects can also move at the same speed, resulting in a qualitative value 0 of both relations. One thing is certain: the speed of a car cannot change from being higher than the speed of the bicycle to being lower than the speed of the bicycle, without passing the value 0. This idea of 'continuity' is of vital importance to qualitative reasoning (Cohn, Hazarika, 2001).

Spatial reasoning goes from simple calculation of spatial attributes (e.g. the area of a region) and the description of spatial relations (e.g. two polygons are disjoint) to all operations of map algebra as presented in Tomlin (1990). More complex tasks, such as planning of routes, computing shortest path problems, and allocation of resources, can be categorised in the broad area of spatial reasoning as well (Yang, 2001). In fact, spatial reasoning can be seen as any deduction of knowledge from a situation having spatial properties (Timpf, Frank, 1997). In most geographical software packages, the absolute positions of spatial entities are represented by sets of coordinates in the Euclidean space, and information is extracted by means of arithmetic and trigonometric computations. Numerical representations may be well suited, in particular, where precise spatial information of a definite situation is available, and if the output required from the system is itself primarily numerical (Bennett, 1997). However, such quantitative information is often too detailed for the given spatial context. For example, if we show a person the way to get to the post office, we do not need to be more precise than the streets he has to follow (Hobbs, Narayanan, 2002). As in this example, spatial reasoning in our every day interaction with the physical world is mostly driven by qualitative abstractions of the (too precise) quantitative space (Cohn, Hazafika, 2001). Inferring new knowledge from this qualitative spatial information is called qualitative spatial reasoning (Cohn, 1996; Goyal, 2000). Humans may benefit from representing and describing the actual world qualitatively, but their interaction with information systems can also benefit from qualitative approaches (Clementini, di Felice, Hernandez, 1997). Qualitative information tends to be less expensive to obtain and reason about, and more available than its quantitative counterpart (Freksa, 1992). In addition, the major goal of many reasoning processes is being able to take a decision being rather qualitative than quantitative (Freksa, 1992), which can be stated by the following example taken from Clementini, di Felice and Hernandez (1997, p.318): "*Saying that Alaska is 1 518 800 km*^{2} *is sufficiently exact quantitative information about size and distances in Alaska but very likely it is not meaningful in relation to the spatial knowledge of the average listener. On the other hand saying that Alaska alone is bigger than all the states of the East coast from Maine to Florida is cognitively more immediate*".

Motion or movement is a critical concept in several research areas ranging from philosophy (starting with Aristotle) and physics (from Newton to Einstein) to artificial intelligence (Muller, 1998). "*The phenomenon of movement arises whenever the same object occupies different positions in space at different times*" (Galton, 1995). People have their own intuitive idea of continuous motion. Think about the path of a thrown ball (Mortensen, 1999); a motion is often represented by a trajectory, which is a connected non branching continuous line having a certain shape and direction (Eschenbach, Habel and Kulik, 1999). Between two points of a trajectory, one can always find, or at least imagine, another point. This means that one cannot identify two points next to each other. One could state that the concept of continuity allows no nextness (Mortensen, 1999). A variety of research communities have been studying movements of objects. The remainder of this section presents a brief overview of those areas that are most important from the point of view of our research.

A move in the direction to study mobile disconnected objects is the extension of qualitative physics to handle relative positions of objects in 2D in Rajagopalan (1995). Although this calculus does make use of a qualitative quantity space, it relies on projecting positions to x and y axes and does not provide a calculus with a set of jointly exhaustive and pairwise disjoint relationships. In Muller (1998), space and time are combined to achieve a theory of spatio-temporal entities. The expressive power of this qualitative theory of motion allows for the definition of complex motion classes such as those expressed by 'motion verbs' in natural language (e.g. leave, reach, hit, and cross). Another formalisation of 4D spatio-temporal regions has been worked out in Hazarika and Cohn (2002) allowing complex motion classes as well. Both theories are limited to topological relationships.

In Erwig et al. (1999), spatio-temporal databases are essentially considered as being databases about moving objects. These moving points and moving regions are presented via abstract data types that can be integrated into relational, object oriented or other database models. A promising application domain for moving object databases is the field of car navigation systems. The critical issues here are real time queries and permanently updated location information, as well as handling the inherently imprecise location information of moving objects (Laube, 2001). Current work can be found for example in Wolfson (1998), de Caluwe (2004), Petry et al. (2005). In Moreira, Ribeiro and Saglio (1999), a data model for representing moving points by a decomposition of a trajectory in several sections is specified. By use of regression techniques, movement within each section is described by a variability function. This way each section is represented by a triplet containing the interval during which the function holds, the location of the object at the beginning of the interval, and a function representing the object's movement during the specified interval. Typical for these database approaches is that little attention is paid to the reasoning aspect.

Modelling of moving objects has become a topic of increasing interest in the area of video databases, as has been studied for example in Koprulu, Cicekli and Yazici (2004), where a video data model that supports spatio-temporal querying in videos, focused on the semantic content of video streams, has been presented. In Li (1997), a way of modelling single object trajectories and relative spatio temporal relations between multiple moving objects based on the Interval Calculus is presented. They consider twelve directional relationships in 2D space. Their work is not strictly qualitative, neither can it make a differentiation in the group of disconnected moving objects. A move in the direction of a calculus for relative motion only containing jointly exhaustive and pairwise disjoint relationships is presented in Fernyhough, Cohn and Hogg (2000). They analysed traffic movements from visual input, categorising the motion of pairs of vehicles using a qualitative representation, which distinguished between movements in the same, opposite or orthogonal directions, and the relative orientation of two objects expressed in an eight relation calculus. The most complex sequence of vital importance, the complete overtake event (i.e., an object starts behind a second, and pulls out and all the way around to finish in front of the other vehicle) was not detected since the observed area was not large enough (Fernyhough, Cohn and Hogg, 2000).

In the domain of robotics, there has been some work concerning moving objects. Unfortunately, most robot control systems rely on algorithms, which do not usually have a well specified semantics (Bennett, 1997). A good example is Stolzenburg (2002), just considering an example of ball interception of a soccer player running towards a moving ball. They use qualitative directions and qualitative descriptions of velocity, without making use of typical important qualitative reasoning approaches and techniques.

3. The qualitative trajectory calculus - Basic (QTC_B)

Many qualitative calculi are based on representing the relationships between pairs of entities: temporal relations between intervals in the Interval Calculus (Allen, 1983), temporal relations between semi-intervals in the Semi Interval Calculus (Freksa, 1992), topological relations between regions in the Region Connection Calculus (Randell, Cui and Cohn, 1992), etc. One might represent the motion of objects by describing the distance between pairs of objects changing over time. This is the central part of the task we have set ourselves in this work. We call the presented calculus the Qualitative Trajectory Calculus - Basic (QTC_B), which is thus a calculus for representing and reasoning about movements of objects in Euclidean space in a qualitative framework. We assume continuous time for QTC_B. QTC_B will be presented in 1D (QTC_{B1D}) and in 2D (QTC_{B2D}).

3.1. QTC - Basic in One Dimension (QTC_{B1D})

In this section, we present an informal account of the Qualitative Trajectory Calculus - Basic in one dimension (QTC_{B1D})[2], which handles the qualitative movement of a pair of point objects restricted to 1D, and having the relation *disconnected from* (DC) during the whole study period. Let us consider two objects k and l. Because the movement is restricted to 1D, the velocity vector of an object has only two possible directions, with the intermediate case where the object stands still. Hence, the direction of the movement of each object can be described by one single qualitative variable. Both degrees of freedom can be further subdivided according to the relative speed of the objects. This subdivision results in redundant information because the relative speed of k with respect to l is the inverse of the relative speed of l with respect to k. By reducing the continuum to the qualitative values $-$, 0 and $+$, the underlying continuous system can be described discretely. Consider also Fig. 2 in order to

[2]For a formal axiomatisation of QTC_B, we refer to Van de Weghe (2004).

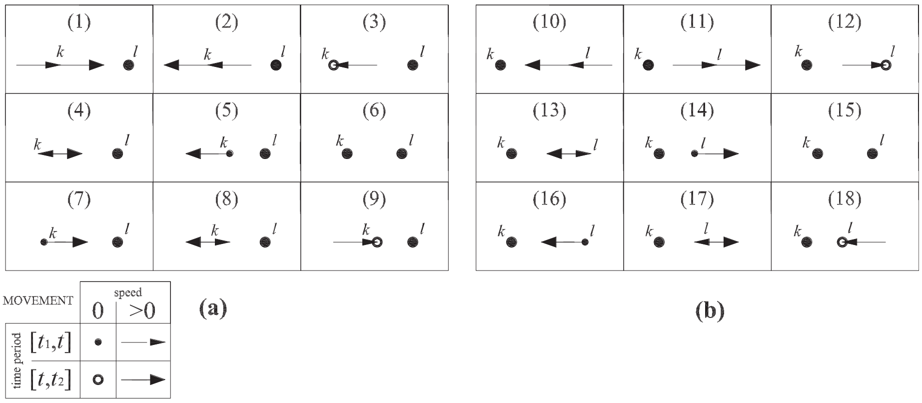

Figure 2. Different cases of motion in QTC_{B1D}.

clarify what we mean by *is moving towards, is moving away from* and *is stable with respect to*[3].

We introduce the following notation for QTC_B:

$x|t$ denotes the position of an object x at time t,
$d(u, v)$ denotes the distance between two positions u and v,
$v_x|t$ denotes the speed of x at time t,
$t_1 \prec t_2$ denotes that t_1 is temporally before t_2.

We obtain $-$ if there is a decrease in distance, $+$ if there is an increase in distance, and 0 if the distance remains the same. Hence, a 1D movement is presented using the following conditions:

Assume: objects k and l

1. Movement of the first object k, with respect to the position of the second object l at time point t (distance constraint):

   $-$: k is moving towards l[4]:

$$\exists t_1(t_1 \prec t \wedge \forall t^-(t_1 \prec t^- \prec t \rightarrow d(k|t^-, l|t) > d(k|t, l|t)))\wedge$$
$$\exists t_2(t \prec t_2 \wedge \forall t^+(t \prec t^+ \prec t_2 \rightarrow d(k|t, l|t) > d(k|t^+, l|t))) \quad (1)$$

[3]The numbers between brackets refer to the numbers of the formulae presented below.
[4]In the definitions below, we use t^- and t^+ as temporal variables (similarly to t_1 and t_2). The differentiating superscripts simply act as a mnemonic to indicate whether the variable precedes or follows t (as is formally specified by the temporal inequalities).

+: k is moving away from l:

$$\exists t_1(t_1 \prec t \wedge \forall t^-(t_1 \prec t^- \prec t \rightarrow d(k|t^-,l|t) < d(k|t,l|t)))\wedge$$
$$\exists t_2(t \prec t_2 \wedge \forall t^+(t \prec t^+ \prec t_2 \rightarrow d(k|t,l|t) < d(k|t^+,l|t))) \quad (2)$$

0: k is stable with respect to l (all other cases):

$$\exists t_1(t_1 \prec t \wedge \forall t^-(t_1 \prec t^- \prec t \rightarrow d(k|t^-,l|t) < d(k|t,l|t)))\wedge$$
$$\exists t_2(t \prec t_2 \wedge \forall t^+(t \prec t^+ \prec t_2 \rightarrow d(k|t,l|t) = d(k|t^+,l|t))) \quad (3)$$

$$\exists t_1(t_1 \prec t \wedge \forall t^-(t_1 \prec t^- \prec t \rightarrow d(k|t^-,l|t) < d(k|t,l|t)))\wedge$$
$$\exists t_2(t \prec t_2 \wedge \forall t^+(t \prec t^+ \prec t_2 \rightarrow d(k|t,l|t) > d(k|t^+,l|t))) \quad (4)$$

$$\exists t_1(t_1 \prec t \wedge \forall t^-(t_1 \prec t^- \prec t \rightarrow d(k|t^-,l|t) = d(k|t,l|t)))\wedge$$
$$\exists t_2(t \prec t_2 \wedge \forall t^+(t \prec t^+ \prec t_2 \rightarrow d(k|t,l|t) < d(k|t^+,l|t))) \quad (5)$$

$$\exists t_1(t_1 \prec t \wedge \forall t^-(t_1 \prec t^- \prec t \rightarrow d(k|t^-,l|t) = d(k|t,l|t)))\wedge$$
$$\exists t_2(t \prec t_2 \wedge \forall t^+(t \prec t^+ \prec t_2 \rightarrow d(k|t,l|t) = d(k|t^+,l|t))) \quad (6)$$

$$\exists t_1(t_1 \prec t \wedge \forall t^-(t_1 \prec t^- \prec t \rightarrow d(k|t^-,l|t) = d(k|t,l|t)))\wedge$$
$$\exists t_2(t \prec t_2 \wedge \forall t^+(t \prec t^+ \prec t_2 \rightarrow d(k|t,l|t) > d(k|t^+,l|t))) \quad (7)$$

$$\exists t_1(t_1 \prec t \wedge \forall t^-(t_1 \prec t^- \prec t \rightarrow d(k|t^-,l|t) > d(k|t,l|t)))\wedge$$
$$\exists t_2(t \prec t_2 \wedge \forall t^+(t \prec t^+ \prec t_2 \rightarrow d(k|t,l|t) < d(k|t^+,l|t))) \quad (8)$$

$$\exists t_1(t_1 \prec t \wedge \forall t^-(t_1 \prec t^- \prec t \rightarrow d(k|t^-,l|t) > d(k|t,l|t)))\wedge$$
$$\exists t_2(t \prec t_2 \wedge \forall t^+(t \prec t^+ \prec t_2 \rightarrow d(k|t,l|t) = d(k|t^+,l|t))). \quad (9)$$

2. Movement of the second object l, with respect to the position of the first object k at time point t (distance constraint) can be described as in Case (1) with k and l interchanged, and thus will be written concisely:

   −: l is moving towards k (10)

   +: l is moving away from k (11)

   0: l is stable with respect to k (12)-(18)

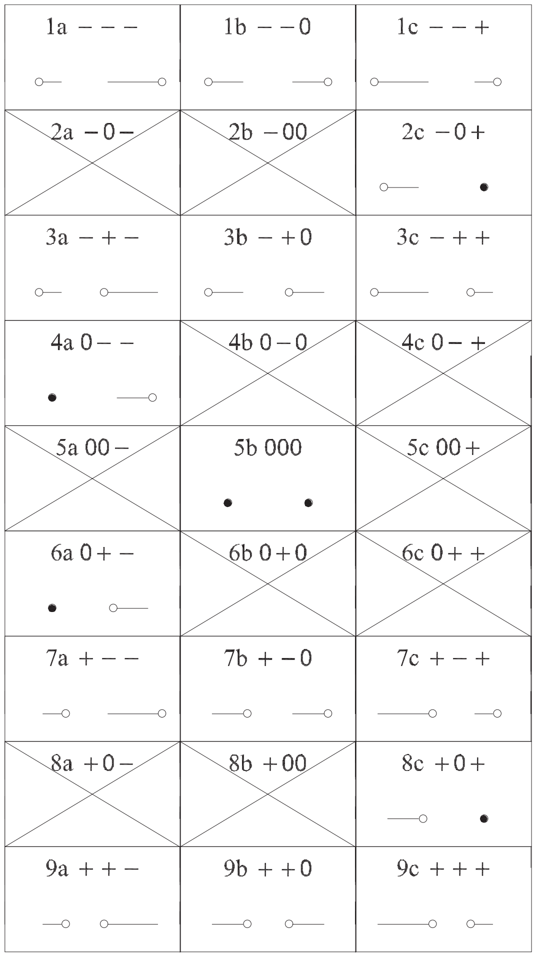

Figure 3. $B1D$-relation icons. The left and the right dot of the $B1D$-relation icon, respectively, represent the positions of k and l. The line segments represent the potential object movements. The line segments show whether an object is moving towards or away from the other. If a dot is filled, then the object is stationary. If a dot is open, then the object may not be stationary.

3. Relative speed of the first object k at time point t with respect to the second object l at time point t (which dually represents the relative speed of the second object with respect to the first object at time point t) (speed constraint):

$$-: \; v_k|t < v_l|t \tag{19}$$

$$+: \; v_k|t > v_l|t \tag{20}$$

$$0:\ v_k|t = v_l|t. \tag{21}$$

We thus can represent a qualitative trajectory pair by a label consisting of three characters, each one standing for the qualitative value of one of the conditions above, ordered as (condition 1, condition 2, condition 3). Theoretically, there are 27 (3^3) so-called $B1D$-relations[5]. However, some of these relations are impossible (crossed-out in Fig. 3) for two objects k and l moving in 1D. It is for example impossible that l is not moving away or toward k, and that k moves slower than l. Relations 2a, 5a and 8a are thus impossible. Therefore, we get only seventeen jointly exhaustive and pairwise disjoint $B1D$ relations. Each icon in Fig. 3 represents one single relation, and therefore is called a relation icon; in this particular case a $B1D$-relation icon. The representations are no more than icons, in which we assume that k is on the left side of l. It is of course possible that k is located on the right side of l. For example, the icon of relation 2c represents not only the situation where k is on the left side of l, with k moving towards l and l being stationary, but also the situation where k is on the right side of l, with k moving towards l and l being stationary. One should be aware that this situation is not the same as relation 4a, because here, l is moving towards k. Note that these differences, which are quite subtle, have no importance to the symmetrical relations 1b, 5b and 9b.

3.2. *QTC* - Basic in Two Dimensions (QTC_{B2D})

The approach for 1D can be successfully used for higher dimensions by denoting the Euclidean distance between a pair of point objects as being the only dimension. This way 2D and even 3D movements can be reduced to 1D movements. Some examples show that this representation can provide a useful abstraction for free trajectory applications:

- Even though a predator and a prey can move in 2D or 3D, the vital question is whether the predator will catch the prey. This will happen as soon as the distance between both has been reduced to zero. Whether this is considered in 1D, 2D or 3D is irrelevant to the qualitative model.
- Even though the path of a boomerang follows a complex 3D trajectory, the main point of interest is its initial flight away from the thrower and its subsequent return, which can be simply represented only using the 1D Euclidean distance between them over time.

To emphasise that we are working on 2D movements, the theory is called the Qualitative Trajectory Calculus - Basic in two dimensions (QTC_{B2D}). In order to clarify what we mean by *is moving towards, is moving away from* and *is stable with respect to*, consider Fig. 4. The definitions for 2D movement of k with respect to l, l with respect to k, and the relative speed of k and l are

[5] In this work, the lexicographical order is maintained in a set of QTC relations.

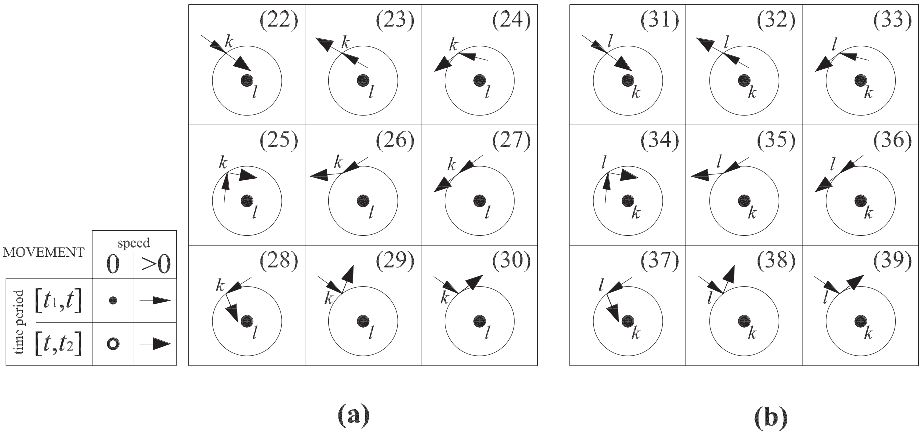

Figure 4. Different cases of motion in QTC_{B2D}.

identical to the definitions for the 1D case[6].

In contrast with QTC_{B1D}, when only 17 of the theoretically possible 27 relations are physically possible, all 27 are possible in QTC_{B2D}; these $B2D$ relations are represented as 27 $B2D$ relation icons in Fig. 5. The reason for all 27 being possible is quite straightforward. In 1D, an object can only move along a straight line, on the other hand in 2D an object can move throughout the complete 2D space, being a higher dimension than the 1D distance, which is the central idea of QTC_{B2D}. Therefore, there is a higher degree of freedom in $B2D$-movements compared to $B1D$ movements, resulting in the increased number of possible relations. In Section 4.3, we give realisations of each of the exclusively 2D relations.

4. Representing specific movements

To clarify the way in which trajectories are represented within QTC_B, it may be helpful to consider some particularly interesting cases of movements.

4.1. Stationary objects

An object k is stationary, if it stands still, at least during an instantaneous temporal interval. An object stands still if it is not moving, thus if its speed is equal to the real number zero. Of course, since we are only measuring distance and speed relative to another object, we cannot infer this directly. In QTC_{B1D}, this will occur if its distance constraint with respect to the other object l has the qualitative value 0 (relations 4a, 5b and 6a in Fig. 3), i.e., if the object is stable

[6]Just as in Fig. 2, the numbers between brackets refer to the numbers of the specific formulae.

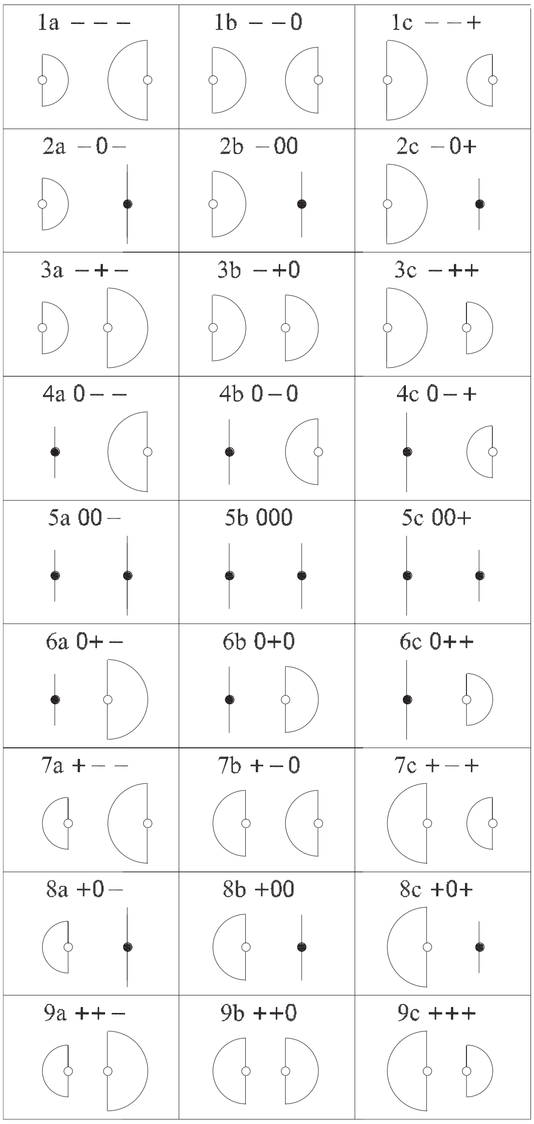

Figure 5. $B2D$-relation icons. The icons contain line segments with the point object in the middle of it. The line segment stands for the opportunity to move to both sides of the point object. If a dot is filled, then the case when the object is stationary is possible. If a dot is open, then the object may not be stationary. The icons also contain crescents with the point object in the middle of its straight border. The crescent stands for an open polygon. If a crescent is used, then the movement starts in the dot and ends somewhere on the curved side of the crescent. It is important that the polygons are not closed. The straight boundary of a crescent is an element of another relation. This is not surprising because QTC_{B2D} is a calculus only containing jointly exhaustive and pairwise disjoint relationships.

with respect to the other object, which occurs if there is an absence of motion to/from the other object as indicated by the first element of the qualitative triple. Thus, for QTC_{B1D}: k is stationary $\leftrightarrow$ k is stable with respect to l. In QTC_{B2D}, we need to make a distinction between an object being stationary (= standing still = speed is zero) and an object being at a stable distance with respect to another object. An object k will be at a stable distance with respect to another object l, if its distance constraint with respect to l has the qualitative value 0 (relations 4a, 4b, 4c, 5a, 5b, 5c, 6a, 6b and 6c in Fig. 5). However, this will not only occur if the object is stationary. It can also occur if k is circling around l, which is presented in Fig. 8, case (i) and will be explained more in detail in section 4.3. Thus for QTC_{B2D}: k is stationary implies k is stable with respect to l, but k is stable with respect to l does not imply k is stationary.

4.2. Circular trajectories

Consider the situations depicted in Fig. 6 where two objects are moving along the same circular path (shown with a thin continuous line). In Fig. 6 case (a), k and l are diametrically opposite at time t. If k moves anywhere below the dashed line (which is perpendicular to the line segment joining k and l), then it will be moving closer to where l is at time t. However, immediately before t, k was moving away from where l is at time t. For this reason, the appropriate qualitative value representing the relationship between k and l is 0 at time t, $-$ immediately before t, and + immediately after t. Dual reasoning applies for the movement of l with respect to the position of k at t. Therefore, the first two characters at t are both 0. It is irrelevant whether the objects are moving clockwise or anticlockwise.

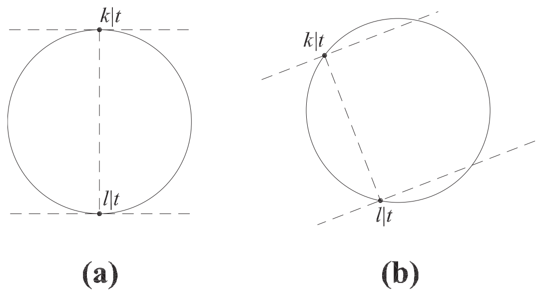

Figure 6. Circular trajectories.

Now assume that k and l are not diametrically opposite at time t (see Fig.6 case (b)). If both objects are moving clockwise, it can be seen that k is moving away from l and l is moving towards k, so the first character is + and the second character is $-$. If the motion were anticlockwise, then the first character would be $-$ and the second character would be +.

4.3. Exclusive $B2D$-relations

In this section, exclusive $B2D$-relations are studied. These are relations that are possible in QTC_{B2D}, but not in QTC_{B1D}: $\{-0-, -00, 0-0, 0-+, 00-, 00+, 0+0, 0++, +0-, +00\}_{B2D}$.

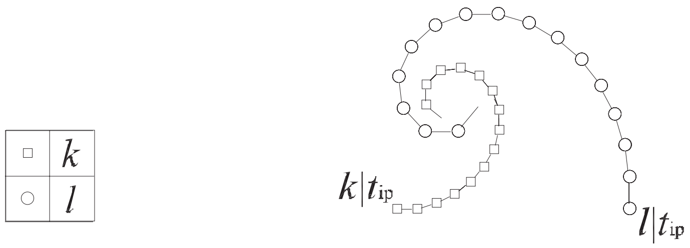

Figure 7. Exclusive $B2D$-relation $(-0-)_{B2D}$.

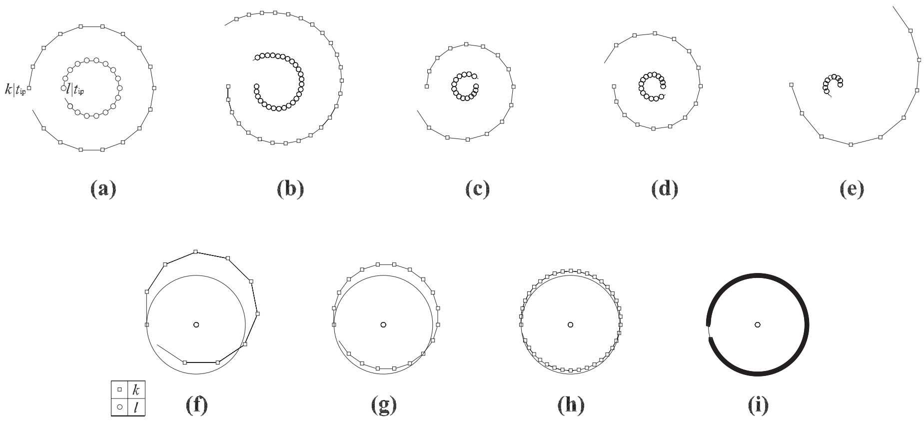

Figure 8. Exclusive $B2D$-relation $(00+)_{B2D}$.

Figs. 7 and 8 represent different trajectory pairs for, respectively, $(-0-)_{B2D}$ and $(00+)_{B2D}$, both being exclusive $B2D$-relations. The purpose of these figures is to show that every exclusive $B2D$-relation has at least one realisation. We do not intend to give an exhaustive overview of all possible realisations. Theoretically, the movement trajectories should be represented via an infinite number of infinitesimal intervals. In order to be able to draw the trajectories, the continuous movement is represented as a discrete movement, with a square/dot for each position at each time point[7] sampled. The squares and dots represent, respectively, the movement of k and l, k being to the left of l at the initial position (t_{ip}). The squares/dots are connected by straight line segments in order to represent the trajectory of each object. One can see that the finer the intervals between time points become, the smoother the trajectory becomes. This would result in a smooth curve when one takes infinitesimal intervals. The end of each path gets a 'finishing line segment', representing the direction of the movement. A non-moving object is represented by means of a single dot.

In Fig. 7, at each point in time, k is moving towards l while l is moving perpendicular to a line drawn between k and l. This is characterised by the relation $(-0-)_{B2D}$. In a similar fashion, (though not shown in a figure) one can construct $(-00)_{B2D}$. In addition, the inverses[8] of these two exclusive $B2D$-relations, namely $(0-+)_{B2D}$ and $(0-0)_{B2D}$ can also be represented like this. Changing $-$ to $+$ in the first two characters, again results in relations that can be constructed in a similar way, namely: $(+0-)_{B2D}$, $(+00)_{B2D}$, $(0++)_{B2D}$, and $(0+0)_{B2D}$.

Now, let us examine the two remaining exclusive relations $(00+)_{B2D}$ and $(00-)_{B2D}$, by working out $(00+)_{B2D}$ in detail. In Fig. 8 cases (a) and (b), k and l start moving in the same direction, respectively both upwards (↑↑) and both downwards (↓↓). In fact, Fig. 8 case (a) represents a special trajectory pair; due to a specific ratio of both object's speed, both objects circle around the same central point, both circles having a different curvature. If both objects start moving in a different direction (Fig. 8 case(c):↑↓, and Fig. 8 case (d):↓↑), a special trajectory pair will occur for both objects having the same speed. Fig. 9 case (a)[9] presents this special trajectory pair, where both objects move in a spiral having the same curvature. A difference in the speed of both objects is reflected in a difference in the curvature of the trajectories of both objects; see difference between Fig. 8 case (d) having a relatively small difference in the speed of both objects, and Fig. 8 case (e) having a relative strong difference. Besides the possibilities that both objects stand still and that both objects are moving, there is a third possibility for $\{00-, 000, 00+\}_{B2D}$, which will occur if only one object is moving. Fig. 8 cases (f), (g), (h), and (i) represent such trajectory pairs. The only difference between these four figures is the interval between two sampled time points. In other words, the only difference is the number of time points necessary to complete a circular movement: 10 equally spaced time points (Fig. 8 case (f)), 20 equally spaced time points (Fig. 8 case (g)), 40 equally spaced time points (Fig. 8 case (h)), and 750 equally spaced time points (Fig. 8 case (i)). In a real continuous movement, the interval between

[7]Note that, notwithstanding the fact that we only need to represent an instantaneous situation, we represent a whole sequence of situations in order to illustrate a possible path described by two objects having the specified relation.

[8]The inverse of a relation $(xyz)_{B2D}$ is the relation $(yxz')_{B2D}$, where z' is 0 if z is 0, + if z is $-$, and $-$ if z is +.

[9]Since relation $(000)_{B2D}$ is not an exclusive $B2D$-relation, it is represented in Fig. 9.

two time points is infinitesimal. Thus, the evolution from Fig. 8 cases (f) to (i) has to be extrapolated, of course resulting in an object circling around the other object. In Fig. 8 case (f), this is not yet the case because the interval is too large. Note here the strong difference between the presented path and the circular path around l represented with a thin line.

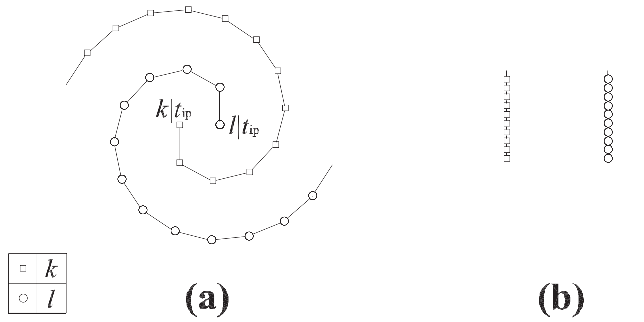

Figure 9. $(000)_{B2D}$ representations being impossible in QTC_{B1D}.

Additionally, Fig. 9 case (b) represents $(000)_{B2D}$ with both objects having the same velocity, resulting in two parallel trajectories. Remember that this relation is also possible in 1D, although only restricted to the trajectory pair where both objects are stationary.

5. Carnivore-prey interaction

In this section, we illustrate the possibilities of how QTC_B might be used for formally representing the qualitative dynamic behaviour of a scenario described in natural language. The example, while not being a complex 'real world' scenario, might be regarded as a 'stepping stone' or an intermediate demonstration of the potential utility of QTC_B. It consists of the evolution of the interaction between a carnivore and its prey. Without going into detail on the possible hunting patterns that might arise between carnivores and prey, clearly the most fundamental question is whether a carnivore catches a prey. In other words, when a carnivore hunts a prey, the topological relationship is typically that of disjointness (DC-relation) until the time that the prey is caught. Therefore, the interaction between one carnivore and a prey can be simplified to the following statement: "a carnivore eats a prey when the distance between both is reduced to zero." Consider the following short story of a lion chasing a zebra: "Once upon a time in Africa… A resting lion sees a resting zebra and starts stalking the zebra. All of a sudden, the zebra gets a glimpse of the lion and tries to escape. The lion reacts and starts pursuing the zebra at a higher speed. After a while, the lion gets tired and is not able to run as fast. The lion realises that he will have to do without food, stops following the zebra and takes a rest. After a while, the zebra is certain that he has evaded the lion, stops running and continues resting."

We now describe the hunt, both informally and with annotations in QTC_{B2D}:

Ex1: a resting lion sees a resting zebra and starts stalking the zebra:

$$\{(000) \rightsquigarrow (-0+)\}_{B2D};$$

Ex2: all of a sudden, the zebra gets a glimpse of the lion and tries to escape:

$$\{(-0+) \rightsquigarrow (-++) \rightsquigarrow (-+0) \rightsquigarrow (-+-)\}_{B2D};$$

Ex3: the lion reacts and starts pursuing the zebra at a higher speed:

$$\{(-+-) \rightsquigarrow (-+0) \rightsquigarrow (-++)\}_{B2D};$$

Ex4: after a while, the lion gets tired and is not able to run as fast:

$$\{(-++) \rightsquigarrow (-+0) \rightsquigarrow (-+-)\}_{B2D};$$

Ex5: the lion realises that he will have to do without food, stops chasing the zebra and takes a rest:

$$\{(-+-) \rightsquigarrow (0+-)\}_{B2D};$$

Ex6: after a while, the zebra is certain that he has evaded the lion, stops running and continues resting:

$$\{(0+-) \rightsquigarrow (000)\}_{B2D}.$$

The story is represented by the following so-called 'conceptual animation', being a sequence of QTC-relations:

$$\{(000) \rightsquigarrow (-0+) \rightsquigarrow (-++) \rightsquigarrow (-+0) \rightsquigarrow (-+-) \rightsquigarrow (-+0) \rightsquigarrow (-++) \rightsquigarrow (-+0) \rightsquigarrow (-+-) \rightsquigarrow (0+-) \rightsquigarrow (000)\}_{B2D}.$$

6. Conclusions and directions for further work

In this work, we addressed the question on how to handle qualitative changes between moving objects, if there is no change in the topological relationship. The Qualitative Trajectory Calculus - Basic (QTC_B) was created in order to be able to handle the relation between pairs of continuously moving point objects, being constantly disjoint. The calculus is based on few primitives, making it elegant and theoretically simple (Cohn, 1995).

A fundamental difference between many traditional approaches in qualitative spatial reasoning and QTC is that the former start from a static relationship (e.g. object k is disconnected from object l) and QTC starts from a dynamic change, i.e., a changing distance over time between a pair of objects (e.g. object k is moving towards object l and object l is moving away from object k).

Without delving into the domain of linguistics, we have shown that QTC_B has potential to present concepts from spatio-temporal events described in natural language, as demonstrated in the 'prey-predator' example. We have shown that particular kinds of movements can be represented by the use of QTC_B, e.g. a circular movement, a parallel movement and a spiral movement. Less attention has been paid to these two central concepts in the study area of moving objects, being a major difference between our research and many other researches conducted in the field of moving objects over the last years.

Since the remoteness from familiar or intuitive processes makes the quantitative approach hard for reasoning (Sharma, 1996), we focused on the qualitative calculus. A major task in qualitative reasoning is qualitative simulation. In contrast to conventional simulation, which makes use of quantitative data to predict precisely what will happen, qualitative simulation does not include precise quantitative information (Weld, de Kleer, 1990, p.84). Therefore, in many situations, qualitative simulation will be ambiguous. Of vital importance is the limitation of the number of possible behaviours produced by qualitative simulation. Further research is needed to find out whether the QTC approach can be used for qualitative simulation, both from a theoretical point of view as well as from the application point of view.

Additionally, following Cohn (1996), we are convinced that qualitative and quantitative reasoning are complementary techniques, and both need to be integrated. An interesting research question could be, "how can qualitative and quantitative information be integrated, for example if some of the information between k and l is known quantitatively and some qualitatively"?

This work focuses on individual movements of objects. Much research has been done concerning aggregated data. We believe that there should also be a meeting point where the aggregated and the individual levels can merge, using advantages of both techniques generating new hybrid methods.

We have built a calculus supporting movements of objects. We have been working at the theoretical level of GIScience and qualitative spatio-temporal reasoning. This study is part of a larger research question that can be formulated as "how to describe motion adequately within a qualitative calculus, both to obtain a tool for data and knowledge representation and for querying spatio temporal data". A full answer to this question needs, besides the fundamental aspects belonging to spatio-temporal reasoning and GIScience, an exhaustive study of technical aspects more related to (spatio-temporal) database research, increasing general performance by the use of efficient algorithms and access methods for computing intensive query operations. Although automation will raise many other questions, we do strongly believe that this work can form the basis for an implementation into an information system, resulting in a GISystem being able to collect, store, analyse, transform and display information about continuously moving objects.

GISystems are now widely deployed in many different applications. Sometimes a purely static view of the world is sufficient, perhaps in urban planning systems. However, GISystems are also used in situations where mobile objects are present and an integral part of the domain, including traffic networks, agricultural vehicle or shipping movements, livestock movements, and logistics amongst others. We envisage that *QTC* calculi may be of use in representing the movements of entities in such domains qualitatively. As has been demonstrated in Fernyhough, Cohn and Hogg (2000), generic behaviours can be learned and recognised using qualitative calculi. By analysing and displaying system behaviours at an abstract qualitative level, a GISystem may be more effective in supporting human decision making. Further experimentation in realistic domains is required however to test this hypothesis.

Acknowledgements

The research work of Nico Van de Weghe and Philippe De Maeyer is funded by the Research Foundation - Flanders, Research Project G.0344.05, and that of Anthony Cohn by the EPSRC under grant EP/C014707/1.

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