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Design of a novel control algorithm for a 6 D.O.F. mobile manipulator based on a robust observer

by

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Abstract: In this paper a control algorithm based on a design technique named "Robust Damping Control" is introduced. A robust observer is further shown to overcome the problem of using velocity sensors that may degrade the system performance. The proposed controller uses only position measurements and is capable of disturbance rejection in the presence of unknown bounded disturbances without requiring the knowledge of its bound. Moreover, we propose an accurate and fast time integration method to solve the dynamic equations of the mobile manipulator system.

The simulation results of a 6 D.O.F. mobile manipulator illustrate the effectiveness of the presented algorithm.

Keywords: robust control, mobile manipulator, generalized-α method, robust observer.

1. Introduction

Mobile manipulators based on their expanded workspace, have found many applications in industry. These intelligent systems have more capability in hazardous situations such as toxic environments and in places that are difficult to handle for humans.

A mobile manipulator is a robotic arm mounted on a moving base. The mobile base (truck) is subject to holonomic or nonholonomic kinematic constraints, which makes control of mobile manipulators very difficult. Moreover, the complex system structure, the highly coupled dynamics between the truck and the mounted manipulator arm, and the mobility of the truck increase the difficulty in designing a controller.

Control of mobile manipulators is a highly challenging research area with great practical significance. In recent years, there has been growing interest in the motion control of mobile manipulators (Kolmanovsky, McClamroch, 1995;

Hootsmanns and Dubowsky, 1991; Tahboub, 1997 a,b; Lin and Goldenberg, 2001, 2002; Minami, Fujiyou and Asakura, 2002).

Tahboub (1997 a,b) introduced a robust controller based on a disturbance observer. Furthermore, he has assumed the mobile manipulator's nonlinear terms as disturbance inputs and designed a linear observer to estimate these terms. But since the complexity of the robotic system increases, separation of nonlinear terms from the robot dynamics becomes a complicated and time consuming task. Hootsmanns and Dubowsky (1991) have developed a control method based on an extended jacobian transpose to compensate for dynamic interactions between the manipulator and the truck. At the same time, some other controllers have also been proposed based on genetic algorithm and neural networks (Lin and Goldenberg, 2001; Minami, Fujiyou and Asakura, 2002). Most of these methods require a precise knowledge of the dynamics of mobile manipulators or they simplify the dynamic model by ignoring complex dynamics, such as vehicle dynamics, payload dynamics, dynamic interactions between the base and the arm, or unknown disturbances.

Recently, Lin and Goldenberg (2002) have presented a robust control method named RDC (Robust Damping Control) to overcome these problems. But in their controller there exist velocity terms and as we know feedback signals from velocity sensors such as a tachometer can have a low SNR (signal-to-noise ratio) and may degrade the performance of the system.

It should be noted that most of these approaches use only simple robotic structures and consequently, the effect of dynamic complexities in the simulations cannot be shown. Most importantly, since the increase of D.O.F. of the mobile manipulator system amplifies the interaction between the truck and its mounted arm, using common algorithms such as "Euler" and "Runge-Kutta" to solve the dynamic equations causes an unwanted decrease in stability and accuracy of the overall system.

In this paper a control algorithm, based on RDC method is introduced using a robust velocity observer. The presented controller uses only position measurements and therefore decreases the required number of sensors and improves the performance of the robotic system. This technique features disturbance rejection in the presence of unknown bounded disturbances without requiring its bound. Moreover, we have also proposed an accurate and fast time integration technique to solve the sophisticated dynamic equations of a 6 D.O.F. mobile manipulator.

The paper is organized as follows. The dynamics of mobile manipulators subject to kinematics constraints and a method to solve it, is developed in Section 2. The robust controller algorithm and its stability characteristics are presented in Sections 3 and 4, respectively. Section 5 presents the simulation results to illustrate the effectiveness of the proposed algorithms.

2. Mobile manipulator dynamic equations

We consider a 6 D.O.F. mobile manipulator as in Fig. 1, in which θ_2, θ_3, θ_4 and θ_5 are joint angles of the robot manipulator, θ_1 is the direction of the truck in the reference coordinate system, and φ_1, φ_2, φ_3 and φ_4 are rotation angles of the mobile manipulator wheels; (x_T, y_T) is the mass center of the truck that is located on the contact point between the truck an its manipulator.

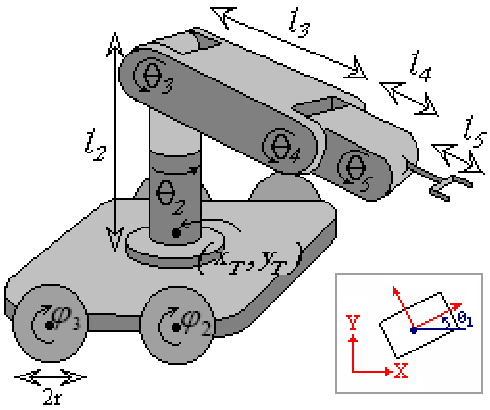


Figure 1. The mobile manipulator considered

In the selected structure, front wheels have differential drive structure. In other words, the difference between the left and right wheel velocities determines the orientation of the truck and each wheel has a separate actuator (τ_L, τ_R).

We have derived dynamic equations of this structure without using any reference, utilizing Lagrange method. However, because of the complexity of these equations and since we want to focus on our control algorithm, we will put aside the details and in the remainder of the paper we use the general form of these equations.

Dynamic equations of a robot without any constraint have the following general structure (Qu and Dawson, 1996):

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + F(q, \dot{q}) = \tau \tag{1}$$

where M is the mass matrix, C is a matrix composed of the Coriolis and centrifugal acceleration terms and F is a vector containing friction and gravitational forces; q and τ are the generalized vector and the input torque vector of the system, respectively, which are defined as follows:

$$\begin{aligned} q &= (x_T, y_T, \theta_1, \varphi_1, \varphi_2, \varphi_3, \varphi_4, \theta_2, \theta_3, \theta_4, \theta_5)^T \\ \tau &= (0, 0, 0, \tau_L, \tau_R, 0, 0, \tau_2, \tau_3, \tau_4, \tau_5)^T \end{aligned} \tag{2}$$

and τ_2, τ_3, τ_4 and τ_5 are torques exerted on the arm joints by the actuators.

If we consider the matrix form of the constraint equations of the system as follows:

$$A(q)\dot{q} = 0, \tag{3}$$

using Lagrange method the general dynamic equations of the mobile manipulator in eq. (1) can be rewritten as follows:

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + F(q, \dot{q}) + A^T(q)\lambda = \tau \tag{4}$$

where λ is the vector of lagrangian factors.

In the subsequent section, we use the generalized-α method to solve equation (4).

2.1. Generalized-α method

In this section, we propose an accurate and fast time integration algorithm to solve complex dynamic equations of a 6 D.O.F. mobile manipulator. This method has better stability characteristics than common numerical techniques such as "Euler" and "Runge-Kutta".

Before using generalized-α numerical algorithm (see Chung and Hulbert, 1993) to solve dynamic equations we should take two stages as follows:

1. Lagrangian factors have to be eliminated from dynamic equations.
2. Some of the system variables must be eliminated via constraint equations.

Therefore, after extraction of the remaining variables we can compute the eliminated variables using the constraint equations.

We have from the constraint equations:

$$\begin{cases} \dot{y}_T = \dot{x}_T \tan\theta_1 \\ \dot{\varphi}_1 = \left[\dfrac{\dot{x}_T}{\cos\theta_1} - \dfrac{\sqrt{2}}{2} l_1 \dot{\theta}_1\right] / r \\ \dot{\varphi}_2 = \left[\dfrac{\dot{x}_T}{\cos\theta_1} + \dfrac{\sqrt{2}}{2} l_1 \dot{\theta}_1\right] / r \\ \dot{\varphi}_3 = \dot{\varphi}_2 \\ \dot{\varphi}_4 = \dot{\varphi}_1, \end{cases} \tag{5}$$

where r is the radius of wheels and l_1 is the distance between truck's center of mass and wheel axis. Therefore, by replacing these equations in the dynamic equations obtained from the elimination of the Lagrangian factors, we can write the following general form:

$$G(q_x, \dot{q}_x, \ddot{q}_x) = M'(q_x)\ddot{q}_x + F'(q_x, \dot{q}_x) = 0\,, \tag{6}$$

in which q_x is defined as:

$$q_x = (x_T, \theta_1, \theta_2, \theta_3, \theta_4, \theta_5)^T. \tag{7}$$

Our final goal in dynamic simulation is to find $q_x = q_x(t)$ such that equation (6) is satisfied and the following initial values are established:

$$\begin{aligned} q_x(0) &= d \\ \dot{q}_x(0) &= v \end{aligned} \tag{8}$$

where d and v are given position and velocity vectors, respectively.

We assume that d_n, v_n and a_n are estimations of $q_x(t_n)$, $\dot{q}_x(t_n)$ and $\ddot{q}_x(t_n)$ in which the index n is the iteration number and we increase it and repeat the algorithm until a predefined tolerance is satisfied. With this assumption we will write d_{n+1} and v_{n+1} as a combination of d_n, v_n, a_n and a_{n+1}. Therefore, we will need an additional equation for computation of a_{n+1} having data from the n^{th} step.

The general form of the Generalized-α algorithm, Chung and Hulbert (1993), is as follows:

$$\begin{aligned} d_{n+1} &= d_n + \Delta t^2 \left((\tfrac{1}{2} - \beta)a_n + \beta a_{n+1}\right) \\ v_{n+1} &= v_n + \Delta t\left((1-\gamma)a_n + \gamma a_{n+1}\right) \\ G(d_{n+1-\alpha_f})a_{n+1-\alpha_m} &+ H(d_{n+1-\alpha_f}, v_{n+1-\alpha_f}) = 0 \\ d_0 &= d \\ v_0 &= v \\ a_0 &= G^{-1}(d)H(d,v), \end{aligned} \tag{9}$$

where:

$$\begin{aligned} d_{n+1-\alpha_f} &= (1-\alpha_f)d_{n+1} + \alpha_f d_n \\ v_{n+1-\alpha_f} &= (1-\alpha_f)v_{n+1} + \alpha_f v_n \\ a_{n+1-\alpha_m} &= (1-\alpha_m)a_{n+1} + \alpha_m a_n. \end{aligned} \tag{10}$$

The second order accuracy of the algorithm is obtained by the condition:

$$\gamma = \frac{1}{2} - \alpha_m + \alpha_f. \tag{11}$$

Moreover, the stability of the system is guaranteed if:

$$\alpha_m \le \alpha_f \le \frac{1}{2} \quad , \quad \beta \ge \frac{1}{4} + \frac{1}{2}(\alpha_m - \alpha_f). \tag{12}$$

In these equations α_m, α_f, γ and β are some constants related to the numerical algorithm and are selected so as to satisfy equation (11) and inequalities (12).

By substituting eq. (9) in eq. (6) we have:

$$G\left(q\left(a_{n+1}\right), \dot{q}\left(a_{n+1}\right), \ddot{q}\left(a_{n+1}\right)\right) = 0. \tag{13}$$

The important point in this algorithm is to find a_{n+1}. For this reason, we first choose an initial assumption such as $a_{n+1} = a_n$ and then modify this assumption using Taylor expansion method to minimize the estimation error of a_{n+1} via a recursive algorithm. Therefore, in each step j we modify $a_{n+1}^{(j)}$ to compute $a_{n+1}^{(j+1)}$ and we repeat this routine until a predefined tolerance is satisfied. In this way, we define:

$$a_{n+1}^{(j+1)} = a_{n+1}^{(j)} + \Delta a_{n+1}^{(j)}. \tag{14}$$

Now we write the Taylor expansion of equation (13) in $a_{n+1}^{(j)}$:

$$G\left(a_{n+1}^{(j)}\right) + \left.\frac{\partial G\left(a_{n+1}\right)}{\partial a_{n+1}}\right|_{a_{n+1}=a_{n+1}^{(j)}} \Delta a_{n+1}^{(j)} \approx 0. \tag{15}$$

By the following definition:

$$J\left(a_{n+1}^{(j)}\right) = \left.\frac{\partial G\left(a_{n+1}\right)}{\partial a_{n+1}}\right|_{a_{n+1}=a_{n+1}^{(j)}} \tag{16}$$

utilizing equations (9) and (10) we can conclude that:

$$J\left(a_{n+1}^{(j)}\right) = \left[\left(1-\alpha_m\right)\frac{\partial G}{\partial \ddot{q}} + \left(1-\alpha_f\right)\gamma\Delta t\frac{\partial G}{\partial \dot{q}} + \left(1-\alpha_f\right)\beta\Delta t^2\frac{\partial G}{\partial q}\right]_{a_{n+1}=a_{n+1}^{(j)}}. \tag{17}$$

Using eqs. (15)-(17) we can compute the modification term of eq. (14) as:

$$\Delta a_{n+1}^{(j)} = -J^{-1}\left(a_{n+1}^{(j)}\right) G\left(a_{n+1}^{(j)}\right). \tag{18}$$

Therefore, equations (14) and (18) bring us enough information to compute $a_{n+1}^{(j+1)}$.

3. A novel robust controller based on robust observer

The robust controller applied to the structure of Fig. 1 has the ability to control the mobile manipulator in the presence of bounded dynamic uncertainties and external disturbances. In addition, rather than determining complex bounding functions as conventional robust control approaches, the RDC compensation terms generate a signal only based on the desired trajectory and sensory data, and thus it does not need the knowledge of the system parameters.

We assume that the dynamic equations of the mobile manipulator in a more general form according to the equation (4) are given as:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + F(q,\dot{q}) + A^T(q)\lambda + \tau_d = E(q)\begin{bmatrix} \tau_v \\ \tau_r \end{bmatrix} \tag{19}$$

where τ_d is due to bounded unknown disturbances including unstructured dynamics, τ_v is the torque vector of mobile platform and τ_r is the torque vector of robot manipulator.

The main idea of this method is to define a Lyapunov function for the mobile manipulator system and to find control laws, which stabilize the system with respect to this Lyapunov function. To define these control laws, we need to prescribe some of the variables:

$$\begin{aligned} q_v &= [x_T, y_T, \theta_1, \varphi_1, \varphi_2, \varphi_3, \varphi_4]^T \\ q_r &= [\theta_2, \theta_3, \theta_4, \theta_5]^T \\ v &= [\dot{\varphi}_1, \dot{\varphi}_2]^T \end{aligned} \tag{20}$$

where q_v is the extended coordinate vector of the base, q_r is the extended coordinate vector of the arm and v is the vector of driving wheel velocities.

Now we can find a full rank matrix $S(q_v)$ such that:

$$\dot{q}_v = S(q_v)v(t). \tag{21}$$

We assume that the reference trajectory from the path planner is available and according to Fig. 2 the desired trajectory for q_r is q_{rd} and the reference trajectory for v is α. Therefore, we define system error variables as follows and our final goal is to design a suitable controller to make these errors vanish:

$$\begin{aligned} z &= v - \alpha \\ e &= q_{rd} - q_r \\ r &= \dot{e} + ke. \end{aligned} \tag{22}$$

Finally, ψ_1 and ψ_2 which are the RDC vectors of the arm and truck, respectively, are defined as (Lin and Goldenberg, 2002):

$$\psi_1 = \begin{bmatrix} \|\dot{\alpha}\| & \|\ddot{q}_{rd} + k(r-ke)\| & \|\alpha\|\|\dot{q}\| & \|\dot{q}_{rd} + ke\|\|\dot{q}\| & \|\dot{q}\| & 1 \end{bmatrix} \tag{23}$$

$$\psi_2 = \begin{bmatrix} \|\ddot{q}_{rd} + k(r-ke)\| & \|S\dot{\alpha} + \dot{S}\alpha\| & \|\dot{q}_{rd} + ke\|\|\dot{q}\| & \|\alpha\|\|\dot{q}\| & \|\dot{q}\| & 1 \end{bmatrix}. \tag{24}$$

From the above definitions, it can be shown that control laws in the following form stabilize the mobile manipulator system (Lin and Goldenberg, 2002):

$$\begin{aligned} \tau_v &= -k_{pv}z - k_1 z\,\|\psi_1\|^2 \\ \tau_r &= k_{pr}r + k_i\int r\,dt + k_2 r\,\|\psi_2\|^2 \end{aligned} \tag{25}$$

Robust observer

As it can be seen, to compute RDC terms we require to measure joint position and velocity values. In most robotic applications, position measurements are available via accurate optical encoders while joint velocity values must be measured by sensors such as tachometers. This may cause noise injection into the system and degrade system performance. So, a robust control algorithm which only needs position measurements, not only decreases the number of sensors to be used, but also improves the dynamic performance of the mobile manipulator.

In the following section we design a robust observer to estimate joint velocities and consider the effect of this observer on system stability.

The block diagram of the proposed novel algorithm containing robust controller and the observer is shown in Fig. 2.

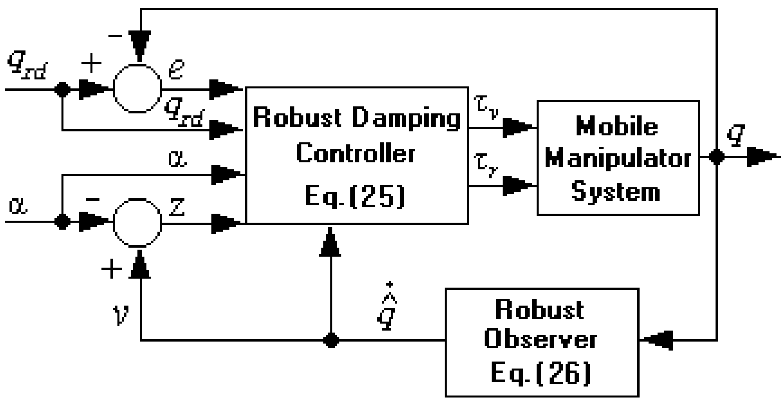


Figure 2. Block diagram of the system

In this regard, we consider the linear and decentralized observer as:

$$\begin{aligned} \dot{\hat{q}} &= y + (K_o + I_n)(q - \hat{q}) \\ \dot{y} &= 2K_o(q - \hat{q}) + 2(q + \hat{q}) \end{aligned} \tag{26}$$

where $\hat{q}$ and $\dot{\hat{q}}$, represent estimates of q and $\dot{q}$ respectively, y is a vector of intermediate variables and K_o is a diagonal gain matrix. By substituting $K_o = k_o I$ into the above equation in the discrete form, we obtain:

$$\begin{aligned} \hat{q}(t + \Delta t) &= \hat{q}(t) + \Delta t \left[y(t) + (k_o + 1) \left(q(t) - \hat{q}(t) \right) \right] \\ y(t + \Delta t) &= y(t) + \Delta t \left[2k_o \left(q(t) - \hat{q}(t) \right) + 2 \left(q(t) + \hat{q}(t) \right) \right]. \end{aligned} \tag{27}$$

Therefore, assuming initial conditions $y(0) = 0$ and $\hat{q}(0) = q(0)$, we can estimate the position vector $\hat{q}\,(t + \Delta t)$. So we can compute joint velocities only by sensing the joint position vector and estimating the position vector in the next time step.

4. Stability analysis of the system with the controller and observer

In this section, we will define some Lyapunov functions for different parts of the system containing the mobile platform, arm and the observer. Then we merge these functions to form a general Lyapunov function.

Let us choose the Lyapunov function of the combined system of base and its mounted arm as in Lin and Goldenberg (2002):

$$V_2 = V_1 + \frac{1}{2}\begin{pmatrix} Sz \\ -r \end{pmatrix}^T M \begin{pmatrix} Sz \\ -r \end{pmatrix}; \quad V_2 \in P.D., \tag{28}$$

where V_1 is the Lyapunov function of the mobile platform and can be designed for a given type of nonholonomic steering system (Kolmanovsky, McClamroch, 1995). Moreover there exists a positive continuous function of time $w_1(t) > 0$ such that $\dot{V}_1 \leq -w_1$.

As we can see in Lin and Goldenberg (2002), by using Lyapunov function V_2 and system's dynamics based on control laws in (25), the following inequality will be fulfilled:

$$\dot{V}_2 < -k_{\min}\left\{\left(\|z\|\,\|\psi_1\| - \frac{\|\Delta_1\|}{2k_1}\right)^2 + \left(\|r\|\,\|\psi_2\| - \frac{\|\Delta_2\|}{2k_2}\right)^2\right\} + \frac{\|\Delta_{\max}\|^2}{2k_{\min}} \tag{29}$$

where:

$$k_{\min} = \min\{k_1, k_2\}$$
$$\|\Delta_{\max}\| = \max\{\|\Delta_1\|, \|\Delta_2\|\} \tag{30}$$

and Δ_1, Δ_2 are vectors containing bounds of system uncertainties (Lin, Goldenberg, 2002).

In (29) $\|\Delta_{\max}\|$ is a bounded quantity, therefore, V_2 decreases monotonically until the solutions reach a compact set determined by the RHS of (29). The size of the residual set can be decreased by increasing $k_{\min}$. According to the standard Lyapunov theory and the extension of the LaSalle theory (Lewis, Abdallah, Dawson, 1993), this demonstrates that without the observer system, the control law (25) may guarantee global uniform ultimate boundedness of all tracking errors.

Observer-based stability analysis

Now by using the same methodology we will show the stability of the proposed algorithm.

Based on eqs. (19) and (26) we have:

$$\begin{cases} \ddot{\hat{q}} = 2k_o(q-\hat{q}) + 2(q+\hat{q}) + (k_o+1)(\dot{q}-\dot{\hat{q}}) \\ \ddot{q} = M^{-1}(q)\tau - M^{-1}(q)N(q,\dot{q}) \\ N(q,\dot{q}) = C(q,\dot{q})\dot{q} + F(q,\dot{q}) + A^T(q)\lambda + \tau_d. \end{cases} \tag{31}$$

If we eliminate lagrangian factors using the constraint equations, since state variables of the system in (31) are decoupled we have the following reduced order system:

$$\begin{cases} \ddot{\hat{q}}_x = 2k_o\left(q_x - \hat{q}_x\right) + 2\left(q_x + \hat{q}_x\right) + \left(k_o + 1\right)\left(\dot{q}_x - \dot{\hat{q}}_x\right) \\ \ddot{q}_x = M_R^{-1}\left(q_x\right)\tau_x - M_R^{-1}\left(q_x\right)N_R\left(q_x, \dot{q}_x\right) \\ N_R\left(q_x, \dot{q}_x\right) = C_R\left(q_x, \dot{q}_x\right)\dot{q}_x + F_R\left(q_x, \dot{q}_x\right) + \tau_d\,. \end{cases} \tag{32}$$

Therefore:

$$\begin{cases} \ddot{q}_x - \ddot{\hat{q}}_x = M_R^{-1}\left(q_x\right)\tau_x + w_2 - 2k_o\left(q_x - \hat{q}_x\right) - \left(k_o + 1\right)\left(\dot{q}_x - \dot{\hat{q}}_x\right) \\ w_2 = -M_R^{-1}\left(q\right)N_R\left(q_x, \dot{q}_x\right) - 2\left(q_x + \hat{q}_x\right) \end{cases} \tag{33}$$

where the vector τ_x is defined as follows:

$$\tau_x = \left(0, 0, \tau_2, \tau_3, \tau_4, \tau_5\right)^T. \tag{34}$$

Now, we define the following error variables:

$$\begin{cases} e = q_{xd} - q_x \\ \hat{e} = q_{xd} - \hat{q}_x \\ \tilde{e} = \hat{e} - e \\ \tilde{r} = \tilde{e} + \dot{\tilde{e}} \end{cases} \tag{35}$$

where q_{xd} is the desired position vector corresponding to the vector q_x.

In this definition, $\hat{e}$ is the estimation of the position error and $\tilde{e}$ is a vector that represents the difference between estimation of the position error and the measured position error.

Since the system without observer is stable, i.e. $\lim_{t\to\infty} e\left(t\right) = 0$, therefore, if the observer error tracks measured error, the overall system will be stable. It means that if the tracking error of the system, $\tilde{e}\left(t\right)$, converges to zero then the overall mobile manipulator system will be guaranteed to be stable.

Using equation (33) we will conclude:

$$\begin{cases} \dot{\tilde{r}} = -k_o\tilde{r} - k_o\tilde{e} + w_3 \\ \dot{\tilde{e}} = \tilde{r} - \tilde{e}\,, \end{cases} \tag{36}$$

where:

$$w_3 = M_R^{-1}\left(q_x\right)\tau_x + w_2. \tag{37}$$

Now to consider the effect of the observer in the system's stability, we define the following Lyapunov function:

$$V_3 = V_2 + k_o\tilde{e}^T\tilde{e} + \tilde{r}^T\tilde{r}; \quad V_3 \in P.D. \tag{38}$$

Differentiating equation (38) yields:

$$\begin{aligned}\dot{V}_3 &= \dot{V}_2 + 2k_o\tilde{e}^T\dot{\tilde{e}} + 2\tilde{r}^T\dot{\tilde{r}} = \dot{V}_2 + 2k_o\tilde{e}^T\left(\tilde{r}-\tilde{e}\right) + 2\tilde{r}^T\left(-k_o\tilde{r} - k\tilde{e} + w_3\right)\\ &= \dot{V}_2 - 2k_o\left\|\tilde{e}\right\|^2 - 2k_o\left\|\tilde{r}\right\|^2 + 2\tilde{r}^T w_3\\ &\le \dot{V}_2 - 2k_o\left\|\tilde{e}\right\|^2 - 2k_o\left\|\tilde{r}\right\|^2 + \frac{2}{\underline{m}}\left\|\tilde{r}\right\|\left\|\tau_x\right\| + 2\tilde{r}^T w_2 \end{aligned} \tag{39}$$

where $\underline{m}$ is the lower bound of uncertainties in the mass matrix.

In order to compute a bound for w_2 we have shown (in the Appendix) that:

$$\left\|N_R\left(q_x,\dot{q}_x\right)\right\| \le \beta_1 + \beta_2\left\|x\right\| + \beta_3\left\|x\right\|^2 \tag{40}$$

$$\left\|\tilde{r}^T\left(q_x+\hat{q}_x\right)\right\| \le \beta_4\left\|z'\right\| + 2\left\|z'\right\|^2 \tag{41}$$

where β_1, β_2, β_3 are positive constant factors and:

$$x = \begin{bmatrix} e\\ \dot{e}\end{bmatrix},\quad z' = \begin{bmatrix}\tilde{e}\\ \tilde{r}\end{bmatrix},\quad \beta_4 = 2\underset{t\in[0,\infty]}{Sup}\left\|q_{xd}\right\|. \tag{42}$$

Using equation (25) and inequalities (39)-(41) we can conclude that:

$$\begin{aligned}\dot{V}_3 \le&\dot{V}_2 - 2k_o\left\|\tilde{e}\right\|^2 - 2k_o\left\|\tilde{r}\right\|^2 + \frac{2}{\underline{m}}\left\|\tilde{r}\right\|(k_{pr}\left\|r\right\| + k_2\left\|r\right\|\left\|\psi_2\right\|^2)+\\ &+2\left\|\tilde{r}\right\|\left[\frac{1}{\underline{m}}\left(\beta_1+\beta_2\left\|x\right\|+\beta_3\left\|x\right\|^2\right)\right] + 4\left(\beta_4\left\|z'\right\| + 2\left\|z'\right\|^2\right).\end{aligned} \tag{43}$$

By substituting inequality (29) in (43) with some simplifications, we can obtain the following result:

$$\begin{aligned}\dot{V}_3 <&- k'_{\min}\left\{\left(\|z\|\|\psi_1\| - \frac{\|\Delta_1\|}{2k_1}\right)^2 + \left(\|r\|\left\|\psi_2\right\| - \frac{\|\Delta_2\|}{2k_2}\right)^2 + 2\left\|\tilde{e}\right\|^2 + 2\left\|\tilde{r}\right\|^2\right\}\\ &+\frac{\|\Delta_{\max}\|^2}{2k_{\min}} + \frac{2}{\underline{m}}\|\tilde{r}\|\left[(k_{pr}\|r\| + k_2\|r\|\|\psi_2\|^2) + (\beta_1+\beta_2\|x\|+\beta_3\|x\|^2)\right]\\ &+4(\beta_4\|z'\| + 2\|z'\|^2)\end{aligned} \tag{44}$$

where:

$$k'_{\min} = \min\left(k_{\min}, k_o\right). \tag{45}$$

This result, according to the standard Lyapunov theory, demonstrates that the mobile manipulator system is globally uniformly stable and all tracking errors will be bounded.

5. Simulation results and conclusion

In the rest of this paper computer simulations are used to demonstrate the performance of the dynamic algorithm, robust controller and observer. Fig. 3 shows a view of the simulator program.

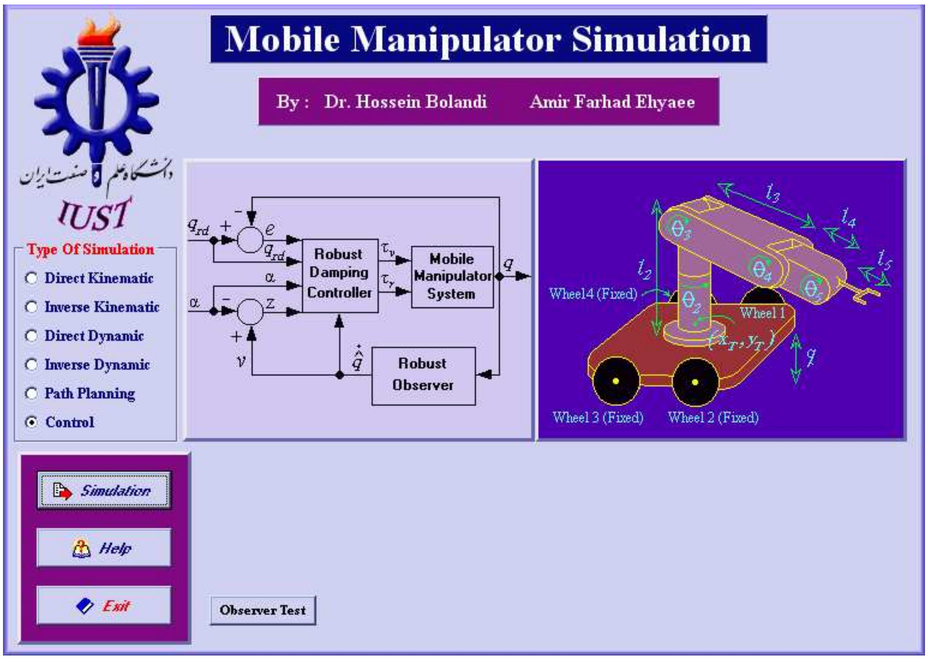
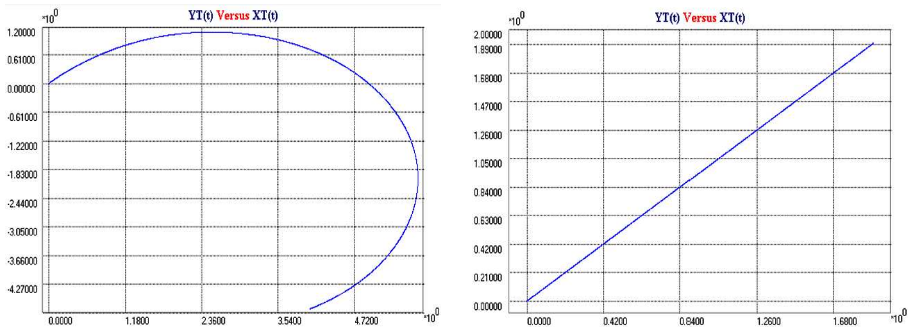


Figure 3. A view of the simulation program

Figs. 4-a and 4-b illustrate the dynamic simulation results for two different input torque vectors. In Fig. 4-a, since a 50 Nm torque is applied to the left wheel, we expect the mobile manipulator to be inflexed from its initial state to the right hand side on an elliptical path. In Fig. 4-b equal torques have been applied to both left and right wheels and therefore the motion trajectory of the robot is a straight line.

The control simulation results have been presented for two different cases: when there is no dynamic parameter uncertainty in the system or in the presence of uncertainty. In the latter case, the uncertainty is considered for the mass of the link, which is connected to the object.

In this simulation, the desired input angles of the arm are considered as sinusoidal signals with frequency of 0.1 KHz, $\pi/8$ phase and amplitude of $\pi/6$. In addition, also desired wheel velocities are assumed to be sinusoidal.



a) τ =(0,0,0,50,0,0,0,0,0,0,0) b) τ =(0,0,0,50,50,0,0,0,0,0,0)

Figure 4. Robot motion in x-y plane

Fig. 5 shows the wheel velocity errors in (rad/sec) and the position error of links in (rad) are shown in Fig. 6 without any plant uncertainty.

At the same time, Fig. 7, for instance, demonstrates the control inputs of left wheel and x-y rotational joint (joint 1) of the mobile manipulator, respectively.

Fig. 8 illustrates the position error of robot joints in (rad) and wheel velocity errors are shown in Fig. 9 in (rad/sec) with plant uncertainty in the mass of the link 5. We suppose that the mass of link 5 suddenly increases to five times of its initial value at time t=7s. As we can see in Figs. 8 and 9 this uncertainty has a little impact on the system accuracy and shows that we have reached the desired goal.

Finally, Fig. 10 shows the desired and actual path of the end effector in three directions (x, y and z). From this figure we can see that the end effector path error is maximum in the "z" direction, specially in the first few seconds of simulation, however, this maximum error is negligible.

In this paper we proposed a robust controller and confirmed its effectiveness by implementing simulations on a 6 D.O.F. mobile manipulator. An important point in this controller is that we must select controller gain factors such that system remains in the stability region. Moreover, in the simulation we cannot unlimitedly increase time intervals, since this will cause instability of the dynamic algorithm and consequently of the control algorithm.

One of the important advantages of the proposed dynamic algorithm is its high accuracy that is necessary for the complex system of mobile manipulator.

In order to acquire a higher reliability of control algorithm in a practical situation, we performed some modifications on the robust damping control algorithm and we illustrated the performance of these modifications in our simulations. Moreover, we designed a robust observer in order to estimate the joint velocity that causes elimination of the velocity sensors, which decrease the signal to noise ratio and degrade the performance of the system.

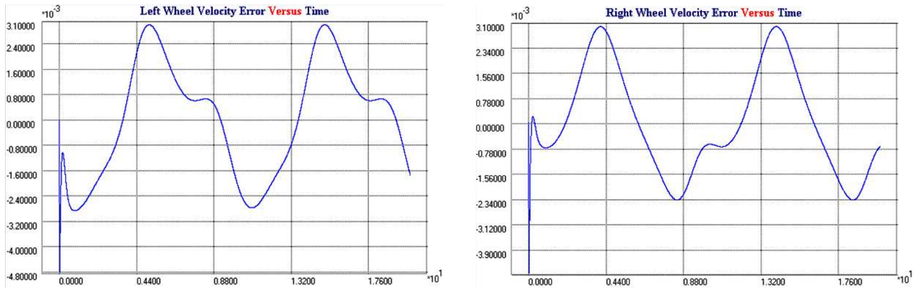


Figure 5. Wheel velocity error without plant uncertainty

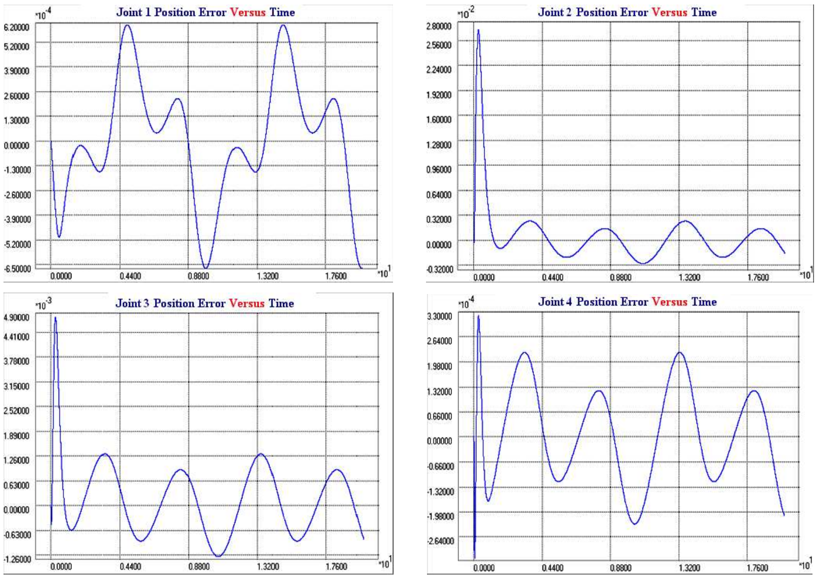


Figure 6. Position error of links without plant uncertainty

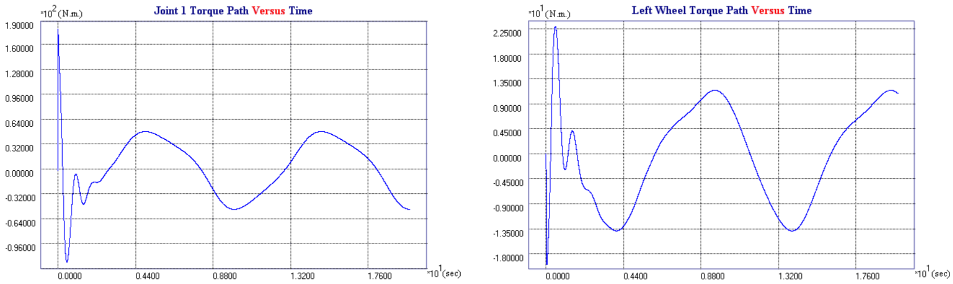


Figure 7. Input control laws

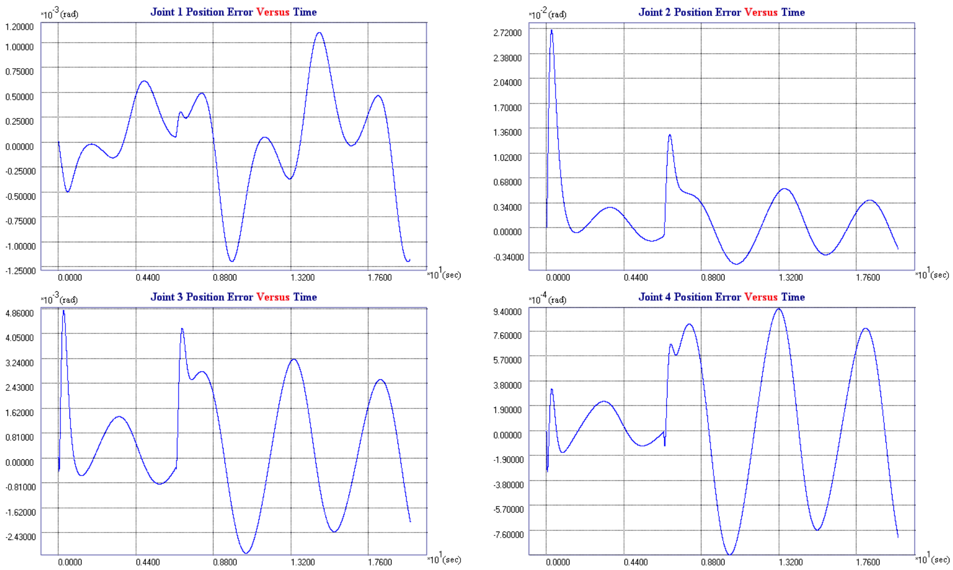


Figure 8. Position error of links with plant uncertainty

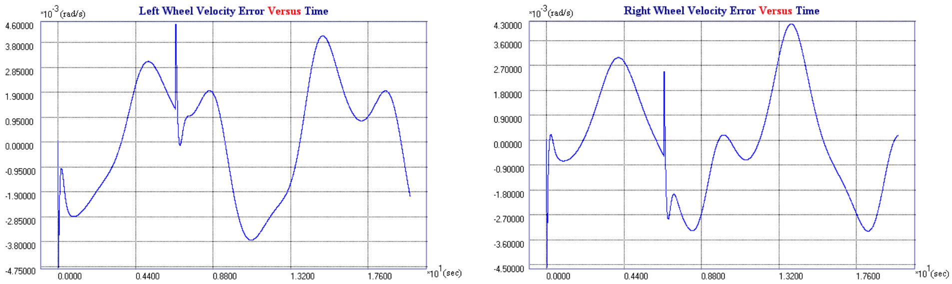


Figure 9. Wheel velocity error with plant uncertainty

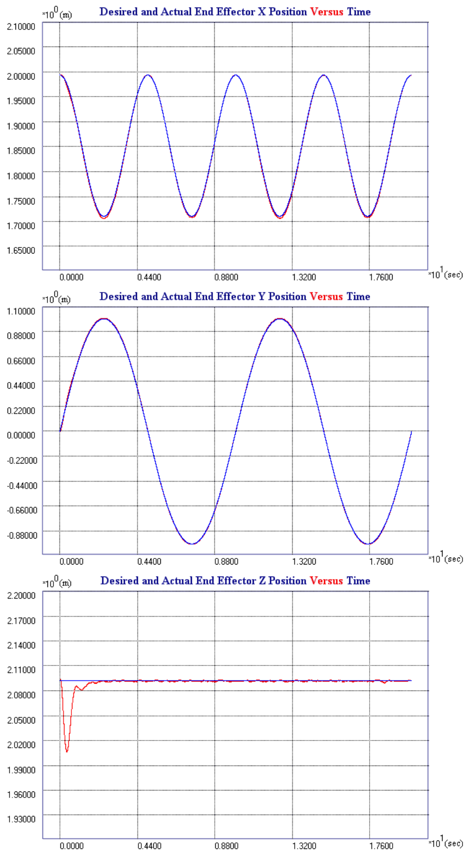


Figure 10. Desired and actual paths of the end effector in the x, y and z directions

Appendix

Proof of Inequality (40)

Assumption 1:

The Coriolis and centripetal term $C_R(q_x, \dot{q}_x)$ is linear in $\dot{q}_x$. Therefore, it follows that:

$$\|C_R(q_x, \dot{q}_x)\| \le \xi_c(q_x)\|\dot{q}_x\| \tag{46}$$

where $\xi_c(q_x)$ is a known, positive definite function of q_x.

Assumption 2:

The friction and gravity term $C_R(q_x, \dot{q}_x)$ is bounded as follows:

$$\|F_R(q_x, \dot{q}_x)\| \le \xi_g(q_x) + \xi_f\|\dot{q}_x\| \tag{47}$$

where $\xi_g(q_x)$ is a known, positive definite function of q_x and ξ_f is a known positive constant.

Assumption 3:

The lumped uncertainty term τ_d is bounded as:

$$\|\tau_d\| \le \xi_t \tag{48}$$

where ξ_t is a known positive constant.

It follows from equation (32) and the above assumptions that:

$$\|N_R(q_x, \dot{q}_x)\| \le (\xi_t + \xi_g(q_x)) + \xi_f\|\dot{q}_x\| + \xi_c(q_x)\|\dot{q}_x\|^2 . \tag{49}$$

From the definitions (35) and (42) we can conclude that:

$$\|\dot{q}_x\|^2 \le \|\dot{q}_x\|^2 + \|q_{xd} - q_x\|^2 = \|\dot{e}\|^2 + \|e\|^2 = \|x\|^2 \tag{50}$$

and then:

$$\|\dot{q}_x\| \le \|x\| . \tag{51}$$

In addition, there exist some positive constants β_1, β_2 and β_3 such that:

$$\begin{aligned} &\|\xi_t + \xi_g(q_x)\| \le \beta_1 \\ &\|\xi_f\| \le \beta_2 \\ &\|\xi_c(q_x)\| \le \beta_3. \end{aligned} \tag{52}$$

Therefore, from (49) we can write:

$$\|N_R(q_x, \dot{q}_x)\| \le \beta_1 + \beta_2\|x\| + \beta_3\|x\|^2 . \tag{53}$$

Proof of Inequality (41)

It follows from the definition (35) that:

$$\left\| \tilde{r}^T \left(q_x + \hat{q}_x \right) \right\| = \left\| \left(\tilde{e} + \dot{\tilde{e}} \right)^T \left(2q_{xd} - \hat{e} - e \right) \right\| \tag{54}$$

and therefore:

$$\left\| \tilde{r}^T \left(q_x + \hat{q}_x \right) \right\| \leq \left\| \left(\tilde{e} + \dot{\tilde{e}} \right)^T 2q_{xd} \right\| + \left\| \left(\tilde{e} + \dot{\tilde{e}} \right)^T \left(\tilde{e} + 2e \right) \right\| . \tag{55}$$

It is obvious that:

$$\left\| q_{xd} \right\| \leq \underset{t \in [0,\infty)}{Sup} \left\| q_{xd} \right\| . \tag{56}$$

Consequently, utilizing the definition (42) since $\left\| \tilde{e} + \dot{\tilde{e}} \right\| \leq \left\| z' \right\|$, we can conclude that:

$$\left\| \tilde{r}^T \left(q_x + \hat{q}_x \right) \right\| \leq \left(2 \underset{t \in [0,\infty)}{Sup} \left\| q_{xd} \right\| \right) \left\| z' \right\| + 2 \left\| z' \right\|^2 . \tag{57}$$

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