Control and Cybernetics

vol. **34** (2005) No. 4

Less conservative results for the exponential stability of uncertain time-delay systems

by

Yeong-Jeu Sun

Department of Electrical Engineering
I-Shou University
Kaohsiung, Taiwan 840, R.O.C.
e-mail: yjsun@isu.edu.tw

Abstract: In this paper, global exponential stability of a class of uncertain systems with multiple time delays is investigated. Simple delay-independent criterion is derived to guarantee the global exponential stability of such systems. The main result is sharper than the recent result reported in the literature. Two numerical examples are also provided to illustrate the main result.

Keywords: delay-independent criterion, exponential stability, time-delay systems.

1. Introduction

In recent years, time delayed systems has been an active area of research; see, for example, Brierley et al. (1982), Chen et al. (2002), Chukwu (1992), Elmali et al. (2000), Hale (1977), Hmamed (1986, 1991), Jalili and Esmailzadeh (2001), Kamen (1980), Kapila et al. (2000), Mori et al. (1981, 1982), Pan et al. (2002), Sun et al. (1996, 1997), Sun (2002), and the references therein. This is due to theoretical interests as well as to the possibility of developing a powerful tool for practical system analysis and control design, since delays are often encountered in various engineering systems, such as the turbojet engine, the microwave oscillator, chemical engineering systems, the rolling mill, the ship stabilization, systems with lossless transmission lines, and manual control. Frequently, it is a source of generation of oscillations and a source of instability in many systems. Besides, stability of the trivial solution of the nonlinear time delayed systems depends upon stability of the trivial solution of its linear approximation.

Depending on whether the stability criterion itself contain the delay argument as a parameter, the stability criteria for time delayed systems can be classified into two categories, namely delay-independent criteria and delay-dependent criteria. Recently, there have been a number of interesting developments in the search for the delay-dependent stability criteria for time delayed systems with or without uncertainties; see, for example, Hale and Huang (1993), Olgac and Sipahi (2002), Sipahi and Olgac (2003, 2004), and Sun et al. (1997). However, in many practical time delayed systems, time delay appearing in the systems is constant but its true value is not exactly known. In this case only delay-independent criteria can be used to check the stability of time delayed systems. In the past, there have been a number of interesting developments in studying the delay-independent stability criteria for time delayed systems, with or without uncertainties; see, for example, Brierley et al. (1982), Hmamed (1986, 1991), Kamen (1980), Mori et al. (1981, 1982), Ping and Cao (2004), Stepan (1989), Sun et al. (1996, 1997), and Wu and Ren (2004). The purpose of this paper is to search for a sharper delay-independent criterion under which the global exponential stability of a class of uncertain systems with multiple time delays can be guaranteed. Moreover, we prove that the criterion we propose is less restrictive than those given in recent work.

This paper is organized as follows. The problem formulation and main result are presented in Section 2. Two examples are provided in Section 3 to illustrate the main result. Finally, conclusion is presented in Section 4.

2. Problem formulation and main result

For convenience, we define notations and abbreviations that will be used throughout this paper:

$N :=$ the set of natural numbers,

$\Re :=$ the set of real numbers,

$\Re^+ :=$ the set of all nonnegative reals,

$C :=$ the set of all complex numbers,

$\Re^{m\times n} :=$ the set of all real m by n matrices,

$C^{m\times n} :=$ the set of all complex m by n matrices,

$j := \sqrt{-1}$,

$I_n :=$ identity matrix of dimension $n \times n$,

$\det(A) :=$ the determinant of the matrix A,

$\mathrm{Re}[\lambda] :=$ the real part of the complex number λ,

$\lambda_i(A) :=$ the i-th eigenvalue of the matrix A,

$\lambda_{\max}(A) :=$ the maximum eigenvalue of the matrix A with real eigenvalues,

$A^* :=$ the conjugate transpose of the matrix A,

$\rho(A) :=$ the spectral radius of $A \in C^{n\times n}$,

$\|A\| :=$ the induced Euclidean norm of $A \in C^{n\times n}$; $\|A\| = \max_i [\lambda_i(A^*A)]^{1/2}$

$\kappa(A) :=$ the condition number of the nonsingular matrix $A \in C^{n\times n}$; $\kappa(A) = \|A\| \cdot \left\|A^{-1}\right\|$,

$\mu(A) :=$ the matrix measure of $A \in C^{n\times n}$; $\mu(A) = \frac{1}{2}\lambda_{\max}[A^* + A]$,
$\delta(A) := \left[\rho^2\left(\frac{A+A^*}{2}\right) + \rho^2\left(\frac{A-A^*}{2}\right)\right]^{0.5}$, with $A \in C^{n\times n}$,
$\theta(A) := \min\{\|A\|, \delta(A)\}$ with $A \in C^{n\times n}$,
$\underline{m} := \{1, 2, \cdots, m\}$,
$\overline{m} := \{0, 1, \cdots, m\}$.

Consider the following uncertain system with multiple time delays:

$$\dot{x}(t) = (A_0 + \Delta A_0)x(t) + \sum_{i=1}^{m}(A_i + \Delta A_i)x(t - h_i), \quad \forall\, t \geq 0; \tag{1a}$$

$$x(t) = \phi(t), \quad t \in [-H, 0], \tag{1b}$$

where $x \in \Re^n$ is the state vector, $A_i \in \Re^{n\times n}$, $\forall i \in \overline{m}$, $h_i > 0$, $\forall\, i \in \underline{m}$, with $H := \max_{1\leq i\leq m} h_i$, represent the constant delay arguments, $A_i \in \Re^{n\times n}$, $\forall\, i \in \overline{m}$ represent uncertain constant matrices, and $\phi(t)$ is a given continuous vector-valued initial function.

Before presenting the main result, we make an assumption as follows.

(A1) There exist nonnegative constants a_i's, $\forall\, i \in \overline{m}$, such that

$$\|\Delta A_i\| \leq a_i, \quad \forall\, i \in \overline{m}.$$

Further, let us also introduce some lemmas, which will be used in the proof of the main result.

LEMMA 2.1 (Desoer and Vidyasagar, 1975) *Let $A \in C^{n\times n}$ and $B \in C^{n\times n}$. Then we have*

(i) $\max_i \mathrm{Re}[\lambda_i(A)] \leq \mu(A) \leq \|A\|$*;*

(ii) $\mu(A + B) \leq \mu(A) + \mu(B)$.

LEMMA 2.2 *Let $A \in C^{n\times n}$, $h > 0$, and $s \in C$. Then we have*

$$\mu(e^{-hs}A) < \theta(A)e^{\alpha h}, \quad \forall\ \mathrm{Re}[s] > -\alpha, \quad \textit{with} \quad \alpha > 0.$$

Proof. By Lemma 2.1, it is easy to see that

$$\mu(e^{-hs}A) \leq \|e^{-hs}A\| \leq \|e^{-hs}\| \cdot \|A\| < e^{\alpha h} \cdot \|A\|, \quad \forall\ \mathrm{Re}[s] > -\alpha. \tag{2}$$

In addition, one has

$$\mu(e^{-hs}A) = \lambda_{\max}\left[\frac{1}{2}(e^{-h(a+bj)}A + e^{-h(a-bj)}A^*)\right]$$
$$= \lambda_{\max}\left[\left(\frac{A+A^*}{2}\right)e^{-ha}\cos(hb) - j\left(\frac{A-A^*}{2}\right)e^{-ha}\sin(hb)\right],$$

$\forall\, s := a + bj$,

with $a,\ b \in \Re$. Applying Weyl inequality (Franklin, 1968) with Lemma 2.1 to the foregoing equation yields

$$\begin{aligned}\mu\left(e^{-hs}A\right) \leq & \lambda_{\max}\left[\left(\frac{A+A^*}{2}\right)e^{-ha}\cos\left(hb\right)\right] \\ & + \lambda_{\max}\left[-j\left(\frac{A-A^*}{2}\right)e^{-ha}\sin\left(hb\right)\right] \\ \leq & \rho\left[\left(\frac{A+A^*}{2}\right)e^{-ha}\cos\left(hb\right)\right]+\rho\left[-j\left(\frac{A-A^*}{2}\right)e^{-ha}\sin\left(hb\right)\right] \\ < & e^{\alpha h}\left|\cos\left(hb\right)\right|\cdot\rho\left(\frac{A+A^*}{2}\right)+e^{\alpha h}\left|\sin\left(hb\right)\right|\cdot\rho\left(\frac{A-A^*}{2}\right) \\ \leq & e^{\alpha h}\cdot\delta\left(A\right), \quad \forall\, s:=a+bj, \quad \text{with} \quad a>-\alpha \quad \text{and} \quad b\in\Re. \end{aligned} \tag{3}$$

From (2) and (3), we conclude that

$$\mu\left(e^{-hs}A\right) < \min\left\{e^{\alpha h}\left\|A\right\| \quad e^{\alpha h}\cdot\alpha\left(A\right)\right\} = e^{\alpha h}\cdot\theta\left(A\right), \quad \forall\ \mathrm{Re}[s] > -\alpha.$$

This completes the proof. ■

A simple delay-independent criterion for the exponential stability of (1) has been provided in the Theorem 1 of Sun et al. (1996), stated as follows:

LEMMA 2.3 (Sun et al., 1996) *The system (1) satisfying (A1) is exponentially stable provided that there exists an invertible matrix $T \in C^{n\times n}$ such that the following inequality is satisfied:*

$$\mu\left(T^{-1}A_0T\right)+\sum_{i=1}^{m}\left\|T^{-1}A_iT\right\|+\kappa\left(T\right)\sum_{i=0}^{m}a_i<0.$$

Based on the Razumikhin-type theorem, a delay-independent criterion for the global asymptotic stability of (1) has been provided in the Corollary 1 of Sun et al. (1997), namely:

LEMMA 2.4 (Sun et al., 1997) *The system (1) satisfying (A1) is globally asymptotically stable provided that there exist a nonsingular matrix T such that*

(i) A_0 is a Hurwitz matrix;

(ii) $\dfrac{1}{\lambda_{\max}(P)} > \kappa(T)\cdot\sum_{i=0}^{m}a_i+\sum_{i=1}^{m}\|T^{-1}A_iT\|$

where $P>0$ is the unique solution to the following Lyapunov equation

$$(T^{1-}A_0T)^TP+P(T^{-1}A_0T)=-2I. \tag{4}$$

Now we present the main result, a delay-independent criterion, for the global exponential stability of the system (1).

Theorem 2.1 *The system (1) satisfying (A1) is globally exponentially stable provided that there exists an invertible matrix $T \in C^{n\times n}$ such that the following inequality is satisfied:*

$$\mu(T^{-1}A_0T) + \sum_{i=1}^{m} \theta(T^{-1}A_iT) + \kappa(T) \sum_{i=0}^{m} a_i < 0. \tag{5}$$

In this case, there is $\mathrm{Re}\,[s] \leqslant -\beta$*, where* s *is any root of the characteristic equation of system (1) and* $\beta > 0$ *is the unique positive root of the following equation*

$$\mu(T^{-1}A_0T)+\beta+\kappa(T)\cdot a_0+\left(\sum_{i=1}^{m} e^{h_i\beta}\theta(T^{-1}A_iT)\right)+\kappa(T)\left(\sum_{i=1}^{m} a_i e^{h_i\beta}\right) = 0. \tag{6}$$

Proof. Let $x(t) = Ty(t)$. Thus, from (1), we have

$$\dot{y}(t) = T^{-1}\left(A_0 + \Delta A_0\right) T\, y(t) + \sum_{i=1}^{m} T^{-1}\left(A_i + \Delta A_i\right) T\, y\left(t - h_i\right), \quad \forall\, t \geqslant 0. \tag{7}$$

Define

$$f(x) := \mu\left(T^{-1}A_0T\right)+x+\kappa\left(T\right)\cdot a_0+\left(\sum_{i=1}^{m} e^{h_i x}\theta\left(T^{-1}A_iT\right)\right)+\kappa\left(T\right)\left(\sum_{i=1}^{m} a_i e^{h_i x}\right). \tag{8}$$

Since $f(x)$ is strictly increasing, $f(0) < 0$, and $f(\infty) = \infty$, there exists a $\beta > 0$ such that (6) is satisfied. It follows that

$$\mu\left(T^{-1}A_0T\right)+x+\kappa\left(T\right)\cdot a_0+\left(\sum_{i=1}^{m} e^{h_i x}\theta\left(T^{-1}A_iT\right)\right)+\kappa\left(T\right)\left(\sum_{i=1}^{m} a_i e^{h_i x}\right) \leqslant 0,$$
$$\forall\, x \leqslant \beta.$$

From Lemma 2.1 and Lemma 2.2, it can be readily obtained that

$$\begin{aligned}
&\beta + \mathrm{Re}\left[\lambda_i\left[T^{-1}(A_0+\Delta A_0)T + \sum e^{-h_i s}\cdot T^{-1}(A_i+\Delta A_i)T\right]\right] \\
&\le \beta + \max_i\, \mathrm{Re}\left[\lambda_i\left[T^{-1}(A_0+\Delta A_0)T + \sum e^{-h_i s}\cdot T^{-1}(A_i+\Delta A_i)T\right]\right] \\
&\le \beta + \mu\left[T^{-1}(A_0+\Delta A_0)T + \sum_{i=1}^{m} e^{-h_i s}\cdot T^{-1}(A_i+\Delta A_i)T\right] \\
&\le \beta + \mu[T^{-1}A_0T] + \mu[T^{-1}\Delta A_0T] + \left(\sum_{i=1}^{m} \mu[e^{-h_i s}\cdot T^{-1}A_iT]\right) \\
&\quad + \left(\sum_{i=1}^{m} \mu[e^{-h_i s}\cdot T^{-1}\Delta A_iT]\right)
\end{aligned}$$

$$
\begin{aligned}
&\leq \beta + \mu[T^{-1}A_0T] + \|T^{-1}\Delta A_0 T\| + \left(\sum_{i=1}^{m} \mu[e^{-h_i s} \cdot T^{-1}A_iT]\right) \\
&\quad + \left(\sum_{i=1}^{m} \|[e^{-h_i s} \cdot T^{-1}\Delta A_i T]\|\right) \\
&\leq \beta + \mu[T^{-1}A_0T] + \kappa(T)a_0 + \left(\sum_{i=1}^{m} e^{h_i\beta}\theta(T^{-1}A_iT)\right) \\
&\quad + \left(\sum_{i=1}^{m} e^{h_i\beta}\kappa(T)a_i\right) \\
&\leq 0, \qquad \text{for every}\, \mathrm{Re}[s] > -\beta. \qquad (9)
\end{aligned}
$$

From (9), it can be deduced that

$$
\mathrm{Re}\left[\lambda_i\left[T^{-1}\left(A_0 + \Delta A_0\right)T + \sum_{i=1}^{m} e^{-h_i s} \cdot T^{-1}\left(A_i + \Delta A_i\right)T\right]\right] \leqslant -\beta,
$$
$$
\forall\ \mathrm{Re}[s] > -\beta,
$$

which implies that

$$
\det\left\{sI_n - \left[T^{-1}\left(A_0 + \Delta A_0\right)T + \sum_{i=1}^{m} e^{-h_i s} \cdot T^{-1}\left(A_i + \Delta A_i\right)T\right]\right\} \neq 0,
$$
$$
\forall\ \mathrm{Re}[s] > -\beta.
$$

Thus, we conclude that the system (7) is globally exponentially stable. By the relation between (1) and (7), we also conclude that the system (1) is globally exponentially stable with $\mathrm{Re}[s] \leq -\beta$, where s is any root of the characteristic equation of system (1). This completes the proof. ■

REMARK 2.1 *Obviously, it can be readily obtained that*

$$
\begin{aligned}
\mu(T^{-1}A_0T) + \sum_{i=1}^{m} \theta(T^{-1}A_iT) + \kappa(T)\sum_{i=0}^{m} a_i &\leq \mu(T^{-1}A_0T) \\
+ \sum_{i=1}^{m} \|T^{-1}A_iT\| + \kappa(T)\sum_{i=0}^{m} a_i\,.
\end{aligned}
$$

Hence we conclude that the result of Theorem 2.1 is less conservative than that of Theorem 1 in Sun et al. (1996).

REMARK 2.2 *From (8), it is easy to see that $f(-\mu(T^1A_0T)) \geq 0$. It follows that the value of β can be directly evaluated by using the Newton's method (Buchanan and Turner, 1992) in $f(x) = 0$ with the starting value $x_1 = -\mu(T^{-1}A_0T)$.*

REMARK 2.3 *Theorem 2.1 provides only a sufficient condition guaranteeing the global exponential stability of system (1). A possible way of choosing matrix T such that the condition of Theorem 2.1 is satisfied is stated in the following. Let us define*

$$E(T) = \mu(T^{-1}A_0T) + \sum_{i=1}^{m} \theta(T^{-1}A_iT) + \kappa(T)\sum_{i=0}^{m} a_i.$$

In order to satisfy the condition of Theorem 2.1, it is necessary to choose the matrix T such that $E(T)$ is small. An adjustment of the matrix T can be performed by applying the particle swarm optimization (Kennedy and Eberhart, 1995). It should be noted, however, that the algorithm of the particle swarm optimization does not guarantee that we can always find a matrix T such that the condition of Theorem 2.1 is satisfied.

REMARK 2.4 *Consider the following system*

$$\dot{x}(t) = A_0x(t) + A_1x\left(t - h_1\right) + A_2x\left(t - h_2\right),$$

where $x \in \Re^3$. This is the special case of system (1) with two time delays, $\Delta A_i = 0$, $\forall i \in \overline{2}$, and $n = 3$. In this case, a delay-dependent stability criterion, which provides also a necessary and sufficient condition, has been proposed in Sipahi and Olgac (2004) to guarantee the asymptotic stability of above system. Obviously, in this case, the result in Sipahi and Olgac (2004) is less conservative than Theorem 2.1 in this paper.

3. Illustrative examples

In this section, we provide two examples to illustrate the main results.

EXAMPLE 3.1 A model of predator-prey interactions (Chukwu, 1992) is given by

$$\dot{x}(t) := (A_0 + \Delta A_0)x(t) + (A_1 + \Delta A_1)x(t - h_1) + (A_2 + \Delta A_2)x(t - h_2), \quad (10)$$

where $x(t) := [x_1(t) \quad x_2(t)]^T$, $A_0 = \begin{bmatrix} -7.5 & -0.5 \\ -0.5 & -7.5 \end{bmatrix}$, $A_1 = \begin{bmatrix} -1.5 & 0.5 \\ -0.5 & -2.5 \end{bmatrix}$, $A_2 = \begin{bmatrix} -1.3 & -0.5 \\ 0.5 & -0.3 \end{bmatrix}$ with $\|\Delta A_0\| = \|\Delta A_1\| = \|\Delta A_2\| = 1$ and $h_i \in \Re^+$, $\forall i \in \overline{2}$.

In addition,
$y_1(t)$:= the population at time t of a species of fish called prey,
$y_2(t)$:= the population at time t of another species of fish called the predator,
$y_{1,eq}$:= the nominal population of the prey,
$y_{2,eq}$:= the nominal population of the predator,

$x_1(t) := y_1(t) - y_{1,eq}$,
$x_2(t) := y_2(t) - y_{2,eq}$.

Comparing (10) with (1), one has $m = 2$. Comparison of (A1) with (10) yields $a_0 = a_1 = a_2 = 1$. In addition, from (4) and (5), by letting $T = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$, it can be obtained that

$$P = \begin{bmatrix} 1/7 & 0 \\ 0 & 1/8 \end{bmatrix}, \qquad \lambda_{\max}(P) = 1/7,$$

$$\mu(T^{-1}A_0T) + \sum_{i=1}^{m} \theta(T^{-1}A_iT) + \kappa(T)\sum_{i=0}^{m} a_i = -0.058 < 0.$$

Consequently, by Theorem 2.1, the system (10) is globally exponentially stable. Finally it can be easily deduced that

$$\lim_{t\to\infty} \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \lim_{t\to\infty} \begin{bmatrix} x_1(t) + y_{1,eq} \\ x_2(t) + y_{2,eq} \end{bmatrix} = \begin{bmatrix} y_{1,eq} \\ y_{2,eq} \end{bmatrix},$$

in view of $\lim_{t\to\infty} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = 0$.

It is noted that the exponential stability of system (10) cannot be guaranteed by Theorem 1 in Sun et al. (1996) in view of $\mu(T^{-1}A_0T) + \sum_{i=1}^{2} \|T^{-1}A_iT\| + \kappa(T)\sum_{i=0}^{2} a_i = 0.005 > 0$. Furthermore, the asymptotic stability of system (10) cannot be guaranteed by the Corollary 1 in Sun et al. (1997) in view of $\frac{1}{\lambda_{\max}(P)} = 7 < 7.005 = \kappa(T)\cdot\sum_{i=0}^{2} a_i + \sum_{i=1}^{2} \|T^{-1}A_iT\|$.

Example 3.2 Consider the following uncertain system with multiple time delays:

$$\dot{x}(t) := (A_0 + \Delta A_0)x(t) + (A_1 + \Delta A_1)x(t-h_1) + (A_2 + \Delta A_2)x(t-h_2), \quad (11)$$

where

$$A_0 = \begin{bmatrix} -7.5 & 0 & 0.5 \\ 0 & -8 & 0 \\ 0.5 & 0 & -7.5 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & \sqrt{2}/2 & 0 \\ 0 & 0 & 0 \\ 0 & -\sqrt{2}/2 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 2 & \sqrt{2}/2 & 1 \\ 3\sqrt{2}/2 & 1 & -\sqrt{2}/2 \\ -1 & 3\sqrt{2}/2 & 0 \end{bmatrix},$$

with $\|\Delta A_0\| = \|\Delta A_1\| = \|\Delta A_2\| = 1$ and $h_i \in \Re^+$, $\forall i \in \overline{2}$. Comparing (11) with (1), one has $m = 2$. Comparison of (A1) with (11) yields $a_0 = a_1 = a_2 = 1$.

In addition, from (4) and (5), by letting $T = \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 1 \\ \sqrt{2}/2 & -\sqrt{2}/2 & 0 \end{bmatrix}$, it can be obtained that

$$P = \begin{bmatrix} 1/7 & 0 & 0 \\ 0 & 1/8 & 0 \\ 0 & 0 & 1/8 \end{bmatrix}, \quad \lambda_{\max}(P) = \frac{1}{7},$$

$$\mu(T^{-1}A_0T) + \sum_{i=1}^{m} \theta(T^{-1}A_iT) + \kappa(T) \sum_{i=0}^{m} a_i = -0.127 < 0.$$

Consequently, by Theorem 2.1, the system (11) is globally exponentially stable.

It is noted that the exponential stability of system (11) cannot be guaranteed by Theorem 1 in Sun et al. (1996) in view of $\mu(T^{-1}A_0T) + \sum_{i=1}^{2} \|T^{-1}A_iT\| + \kappa(T) \sum_{i=0}^{2} a_i = 0.664 > 0$. Furthermore, the asymptotic stability of system (11) cannot be guaranteed by the Corollary 1 in Sun et al. (1997) in view of $\frac{1}{\lambda_{\max}(P)} = 7 < 7.166 = \kappa(T) \cdot \sum_{i=0}^{2} a_i + \sum_{i=0}^{2} \|T^{-1}A_iT\|$.

4. Conclusions

In this paper, the global exponential stability of a class of uncertain systems with multiple time delays has been considered. A delay-independent criterion has been derived to guarantee the global exponential stability of such systems. It has been shown that the main result is sharper than the recent result reported in the literature. Two numerical examples have also been provided to illustrate the main result. It is interesting to search for another sharper criterion for the exponential stability of uncertain systems with multiple time delays. This constitutes an interesting future research problem.

Acknowledgments

The author would like to thank the National Science Council of Republic of China for supporting this work under grant 91-2213-E-214-008.

References

Brierley, S.D., Chiasson, J.N., Lee, E.B. and Zak, S.H. (1982) On stability independent of delay for linear systems. *IEEE Trans. Automat. Contr.* **27**, 252-254.

Buchanan, J.L. and Turner P.R. (1992) *Numerical Methods and Analysis.* McGraw-Hill, New York.

Chen, C.F., Pan, S.T. and Hsieh, J.G. (2002) Stability of nominally stable uncertain singularly perturbed systems with multiple time delays. *Control and Cybernetics* **31**, 5-16.

Chukwu, E.N. (1992) *Stability and Time-Optimal Control of Hereditary Systems.* Academic Press, New York.

Desoer, C.A. and Vidyasagar, M. (1975) *Feedback Systems: Input-Output Properties.* Academic Press, New York.

Elmali, H., Renzulli, M. and Olgac, N. (2000) Experimental comparison of delayed resonator and PD controlled vibration absorbers using electromagnetic actuators. *ASME Journal of Dynamic Systems, Measurement and Control* **122**, 514-520.

Franklin, J.N. (1968) *Matrix Theory.* Prentice-Hall, Englewood Cliffs.

Hale, J.K. (1977) *Theory of Functional Differential Equations.* Academic Press, New York.

Hale, J.K. and Huang, W. (1993) Global geometry of the stable regions for two delay differential equations. *Journal of Mathematical Analysis and Applications* **178**, 344-362.

Hmamed, H. (1986) On the stability of time-delay systems: new results. *International Journal of Control* **43**, 321-324.

Hmamed, A. (1991) Further result on the delay-independent asymptotic stability of linear systems. *International Journal of Systems Science* **22**, 1127-1132.

Jalili, N. and Esmailzadeh, E. (2001) Optimum active vehicle suspensions with actuator time delay. *ASME Journal of Dynamic Systems, Measurement and Control* **123**, 54-61.

Kamen, E.W. (1980) On the relationship between zero criteria for two-variable polynomials and asymptotic stability of delay differential equations. *IEEE Trans. Automat. Contr.* **25**, 983-984.

Kapila, V., Tzes, A. and Yan, Q. (2000) Closed-loop input shaping for flexible structures using time-delay control. *ASME Journal of Dynamic Systems, Measurement and Control* **122**, 454-460.

Kennedy, J. and Eberhart, R. (1995) Particle swarm optimization. *IEEE International Conference on Neural Networks*, Leuven, Belgium, **4** (27), 1942-1948.

Mori, T., Fukuma, N. and Kuwahara, M. (1981) Simple stability criteria for single and composite linear systems with time delays. *International Journal of Control* **34**, 1175-1184.

Mori, T., Fukuma, N. and Kuwahara, M. (1982) On an estimate of the delay rate for stable linear delay systems. *International Journal of Control* **36**, 95-97.

Olgac, N. and Sipahi, R. (2002) An exact method for the stability analysis of time delayed LTI systems. *IEEE Trans. Automat. Contr.* **47**, 793-797.

PAN, S.T., CHEN, C.F. and HSIEH, J.G. (2002) D-stability for a class of discrete descriptor systems with multiple time delays. *Control and Cybernetics* **31**, 79-89.

PING, H. and CAO, D.Q. (2004) Algebraic stability criteria of linear neutral systems with multiple time delays. *Applied Mathematics and Computation* **155** (3), 643-653.

SIPAHI, R. and OLGAC, N. (2003) Active vibration suppression with time delayed feedback. *ASME Journal of Vibration and Acoustics* **125**, 384-388.

SIPAHI, R. and OLGAC, N. (2004) Complete stability map of third order LTI, multiple time-delay systems. *IFAC Time Delay Workshop 2004*, Leuven, Belgium.

STEPAN, G. (1989) *Retarded Dynamical Systems: Stability and Characteristic Function.* John Wiley & Sons Inc., New York.

SUN, Y.J. (2002) Global stabilizability of uncertain systems with time-varying delays via dynamic observer-based output feedback. *Linear Algebra and its Applications* **353**, 91-105.

SUN, Y.J., HSIEH, J.G. and HSIEH, Y.C. (1996) Exponential stability criterion for uncertain retarded systems with multiple time-varying delays. *Journal of Mathematical Analysis and Applications* **201** (2), 430-446.

SUN, Y.J., HSIEH, J.G. and YANG, H.C. (1997) On the stability of uncertain systems with multiple time-varying delays. *IEEE Trans. Automat. Contr.* **42** (1), 101-105.

WU, S.B. and REN, G.X. (2004) A note on delay-independent stability of a predator-prey model. *Journal of Sound and Vibration* **275** (1-2), 17-25.