

Preface to Special Issue on Shape Optimization

In Shape Optimization the following three fundamental theoretical issues are considered:

- existence of solutions in a possibly wide class of feasible domains;
- fixing necessary optimality conditions, allowing for an effective determination of the optimal domains;
- development of numerical methods for approximate determination of optimal shapes by solving the optimality conditions or by optimizing the shape functionals under given constraints, with the use of appropriate computational methods, based, e.g., on the level set concepts.

The articles set forth in this issue concern various topics, being yet representative for the contemporary shape optimization and its applications (in the theory of inverse problems, for instance). In particular, the papers by: G. Allaire, F. de Gournay, F. Jouve and A.-M. Toader; S. Amstutz, I. Horchani and M. Masmoudi; C. Graczykowski and T. Lewiński; X. Guo, K. Zhao, and M. Yu Wang; M. Hintermüller; as well as A.A. Novotny, R.A. Feijóo, C. Padra and E. Taroco; deal with topology optimization in the classical and modern frameworks.

An essential question in shape optimization is to develop effective numerical methods, this question being addressed in the papers by: G. Allaire et al.; S. Amstutz et al.; M. Dambrine and G. Vial; J.-A. Désidéri and J.-P. Zolésio; K. Eppler and H. Harbrecht; X. Guo et al.; J. Haslinger, J. Málek and J. Stebel; M. Hintermüller; A.A. Novotny et al; J.R. Roche.

The papers by D. Bucur; M.C. Delfour and J.-P. Zolésio; W. Horn and J. Sokołowski provide theoretical contribution to modeling and theory of existence of solutions for problems involving a lack of regularity.

Two survey papers are included in the volume: the presentations of the current research in Denmark (M. Bendsøe, E. Lund, N. Olhoff and O. Sigmund) and in France (A. Henrot and J. Sokołowski).

In more detail we can describe the content of the volume, from the point of view of real-life applications, in the following manner: Some papers tackle the problems of optimal shape design of linearly elastic bodies subject to non-classical loadings, like deformation-dependent loading; of Michell structures (here the oldest, original Michell cantilever supported on a boundary of a circle is revisited); thin Kirchhoff plates; optimal design of fluid flow as well as applications in acoustics. Other papers concern inverse problems: detection of

cracks, obstacles and voids from the point of view of possible applications in the electrical impedance tomography. One of the papers (J. Haslinger et al.) solves the problem of optimal control of the velocity profile in the paper manufacturing process; another one (Horn and Sokołowski) solves the problem of optimal control of vibrations of a membrane by a device made of a shape memory alloy.

The next group of papers are devoted to the methods of solving the shape optimization problems. M. Hintermüller puts forward a new algorithm for a selected class of shape optimization problems and shows its convergence. J.R. Roche considers the question of control of convergence of the Newton method. The papers set forth reflect the contemporary trends in the field; the aim is to achieve global solutions, taking into account multi-connected candidates of strongly irregular boundaries. To this end the authors deal with sensitivity of shape functionals with respect to small changes of the boundaries or small changes of the domain, taking into account appearance and vanishing of holes or cavities. This sensitivity is assessed by using several admissible techniques of computing topological derivatives of the shape functionals. The process of achieving the solution can be carried out by using the level set method. A simultaneous usage of both these ideas can be noted in the works by G. Allaire et al. and X. Guo et al.

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